A Multigranular Linguistic Content-Based Recommendation Model

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Recommendation systems are a clear example of an e-service that helps the users to find the most suitable products they are looking for, according to their preferences, among a vast quantity of information. These preferences are usually related to human perceptions because the customers express their needs, taste, and so forth to find a suitable product. The perceptions are better modeled by means of linguistic information due to the uncertainty involved in this type of information. In this article, we propose a *content-based recommendation model* that will offer a more flexible context to improve the final recommendations where the preferences provided by the sources will be modeled by means of linguistic variables assessed in different linguistic term sets. The proposal consists of offering a multigranular linguistic context for expressing the preferences instead of forcing users to use a unique scale. Then the content-based recommendation model will look for the most suitable product(s), comparing them with the customer(s) information according to its resemblance.

1. INTRODUCTION

Almost all areas related to human beings have been impacted by the Internet in past years; this fact has led to the appearance of new markets, services, and much information available for users. This explosive growth means that one of the main problems Internet users face is the vast quantity of information they can find on the network, most of it being useless for their aims. Therefore, different e-services have risen to help these users to easily and quickly reach their desired target.

Recommendation systems are a class of software[1,2] that has emerged recently as an e-service within the domain of e-commerce[3] to help customers to obtain some information or find a product for which they are searching, according to their preferences, needs, or taste, hiding or removing the useless information that exists in the Web sites. Companies such as Amazon or the *Los Angeles Times* use recommendation systems to assist users in their searches.

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INTERNATIONAL JOURNAL OF INTELLIGENT SYSTEMS, VOL. 22, 419–434 (2007)
 Published online in Wiley InterScience (www.interscience.wiley.com). • DOI 10.1002/int.20207

The purpose of these systems is to recommend the most suitable items according to the customer's desires. These systems have been classified into three categories: (i) *collaborative filtering systems*,[4–7] (ii) *content-based filtering systems*,[8–10] and *hybrid content-based and collaborative recommendation systems*.[11–13]

This article focuses on content-based recommendation systems that achieve their recommendations following a three-step process:

1. Gather the preference information from different information sources.[14,15]
2. Filter the items measuring the similarity between customer preferences and the description of the items.
3. Rank and choose which are the most attractive items for the customers.

In the gathering process, each customer provides his preferences to the recommendation system, defining a *source profile* that contains the opinions about his necessities according to his own perceptions regarding the items for which he is searching. This type of information is subjective, because it is related to the customer's own perceptions and usually involves uncertainty. So the information provided by these sources is usually vague, incomplete, and imprecise. Due to these features, the use of the fuzzy linguistic approach[16] to manage this type of information is very suitable.[17] However, most of the current recommendation systems force the sources to express their preferences using just one numerical scale.[18] This fact leads to a lack of expressiveness for the sources and to a lack of precision in the recommendations made by the system.

In this article, we propose a new model to improve the effectiveness of the recommendations given by content-based recommendation systems. It consists of offering the customers a multigranular linguistic context[19] in which the user's preferences and the descriptions of the features of the items will be modeled. Therefore, the sources can express their information using linguistic assessments instead of numerical ones. In addition, each source can choose his or her own linguistic term set to provide his or her preferences according to his or her knowledge about the items. Also, the item features provided by the experts will be assessed by means of linguistic labels that can be conducted in different linguistic term sets. To deal with the multigranular linguistic information in the proposed recommendation model, we shall use the fuzzy linguistic approach[16] to model the input information and fuzzy tools, such as fuzzy measures of comparison[20] to filter the products in order to rank and recommend them.

The proposal for a multigranular linguistic content-based recommendation model is given in the followings steps (Figure 1):

1. *Acquisition of the user profile and the item features*: the source profile is an information structure to gather the information about the customer's needs, tastes, areas of interest, and so forth, and the item features are the characteristics of the items to be recommended that are stored in a database and provided by experts. In this model the customers as the experts will provide their preferences by means of linguistic information that can be conducted in different linguistic term sets.

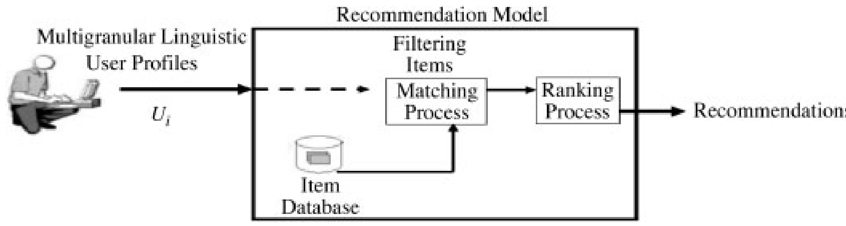

Figure 1. A multigranular linguistic content-based recommendation model.

2. *Filtering items*: to find the most interesting items for the customers, the recommendation model will filter the items, comparing each one in its database with the customer needs (profiles) by means of fuzzy measurements of comparison.
3. *Making a recommendation*: the model will rank the item according to its similarity with the customer profile such that the top ranked items will be recommended to the customer.

This article is structured as follows. Section 2 makes a brief review of the different recommendation models that can be found in the specialized literature. Section 3 presents a multigranular content-based recommendation model, and Section 4 shows a simple example. The article concludes in Section 5.

2. RECOMMENDATION SYSTEMS

The current recommendation systems can be classified by attending to the processes and the sources of information that are used to achieve the recommendations. The information used by these systems may be provided from different types of sources, and there exist, at least, the following five types[14]:

1. a person's expressed preferences or choices among alternative products
2. preferences for product attributes
3. other people's preferences or choices
4. expert judgments
5. individual characteristics that may predict preferences

According to which sources are involved, how does the recommendation system deal with the information gathered in order to produce a recommendation? There are three main classes of recommendation systems:

1. ***Collaborative filtering systems***[4–7] *use explicit and implicit preferences from many users to filter and recommend objects to a given user, ignoring the representation of the objects*. In the simplest case, these systems predict a person's preference as a weighted sum of other people's preferences, in which the weights are proportional to correlations over a common set of items evaluated by two people. Collaborative filtering algorithms were first introduced by Golberg et al.[4] This type of system is used by the *Los Angeles Times*, CRAYON, and Tango to customize online newspapers, by Bostondine to recommend restaurants in and around Boston, by Sepia Video Guide to make customized video recommendations, by Movie Critic, Moviefinder, and Morse to recommend movies, and by barnesandnoble.com to recommend books.

2. ***Content-based filtering systems***[8–10] *filter and recommend the items by matching user query terms with the index term used in the representation of the items, ignoring data from other users*. Several commercial systems have been offered by PersonalLogic, Frictionless Commerce, and Active Research that use self-explicated importance ratings and/or attribute trade-offs to make their recommendations.
3. ***Hybrid content-based and collaborative recommendation systems.***[11–13] This new class has emerged between the content-based and collaborative recommendation systems and its aim is to smooth out the disadvantages of each one of them. *The usual way to hybridize both classes is to make a two-level filter algorithm, where, first, an algorithm is used (the content-based filtering algorithm) to obtain an initial set of items and then a second algorithm is used (the collaborative filtering algorithm) to filter and recommend items from the initial set*. Applications of hybrid-based recommendation systems on the Web include search tools such as Google and Inquirus 2 that combine results of both content searches and collaborative recommendations. However, these systems are more complex and have new design problems to resolve to handle efficiently all the information available.

This article is focused on content-based recommendation models.

3. A MULTIGRANULAR LINGUISTIC CONTENT-BASED RECOMMENDATION MODEL

Our proposal for a multigranular linguistic content-based recommendation model is presented here. To make a recommendation for a customer, this type of recommendation model will consider information from the following sources of information:

- A person expresses preferences among alternative products: *customer profile*.
- Preferences are provided for product attributes: *item features.*

We ignore information from the other sources enumerated in Section 2.

The proposal of this article to improve the recommendations of the content-based model is that both the *customer profiles* and the *item features* can be assessed by means of multigranular linguistic information; this means different customers or experts can use different linguistic term sets to provide their assessments. And the recommendation process in this context for a content-based recommendation model will consist of the following phases (Figure 2):

1. *Acquisition of the customer profiles and item features*: in this phase the item features are added to the item database if they have not been already stored and the customer preferences are gathered into a source profile.
2. *Filtering items*: this type of recommendation model filters the possible items to recommend, computing a measure of similarity between the customer profile and each item stored in the database. In this case the recommendation model compares fuzzy numbers, due to the fact that the profiles and the item features are assessed by means of linguistic labels.
3. *Making a recommendation*: the recommendation model chooses the most suitable items for a customer according to their similarity with his or her profile, that is, the top ranked items according to its similarity with the customer profile.

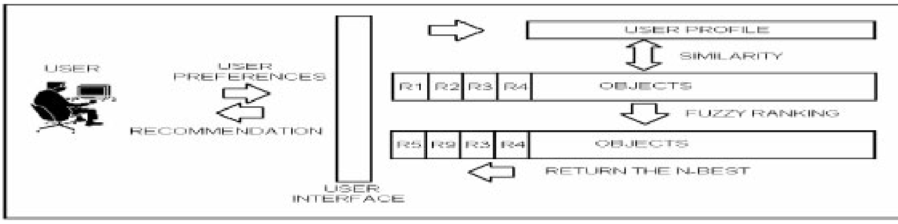

Figure 2. Content-based recommendation model.

In the following subsections we shall present these stages in detail and how this model works in order to recommend the most suitable products to the customers.

3.1. Acquisition of the User Profiles and the Item Features

The aim of this phase is to gather the information with regard to products in which the customer u is interested. To do so, let $C = \{c_1, \ldots, c_k, \ldots, c_l\}$ be a set of criteria or attributes that the customer uses to describe his needs, preferences, and tastes about the items in which he is interested.

The recommendation model will have a set of items or products (item database) $A = \{a_1, \ldots, a_j, \ldots, a_n\}$ that can be recommended, where each *item* is described by a set of values, *item features*, for each criterion, c_k, provided by some experts, where these values will be linguistic labels that could be assessed in different linguistic term sets.

So the recommendation system describes each item, a_j, by means of a vector of item features, $F_{a_j} = \{v_1^j, \ldots, v_k^j, \ldots, v_l^j\}$, $j = 1, \ldots, n$, where each criterion, c_k, is assessed (see Table I), v_k^j being the linguistic value that describes the criterion, c_k of the object, a_j. The item features are usually provided by experts in the type of items contained in A (films, books, toys, etc.).

The customer profiles are provided by each customer, u, who wants to obtain a recommendation about which are the most suitable items in the item database, according to his preferences.

Therefore the customer, u, in order to obtain a recommendation from the system will provide his profile, P_u, by means of a utility vector that expresses his preferences, needs, and tastes with regards to the items he is looking for:

$$P_u = \{p_1^u, \ldots, p_k^u, \ldots, p_l^u\}$$

Table I. Item features in the database of items.

	c_1	...	c_l
a_1	v_1^1	...	v_l^1
...	...	...	...
a_n	v_1^1	...	v_l^n

where $p_k^u \in S_{uk}$ is the linguistic value that the customer, u, wishes for the criterion, c_k, of the item he is looking for according to his taste, preference, or need and S_{uk} is the linguistic term set chosen by the customer, u, to assess his preference about the criterion, c_k.

Different customers can have different perceptions about their own preferences or tastes, or the same customer can even have different knowledge about his preferences in different criteria. Hence, the proposed recommendation model offers the possibility that customers conduct their preferences in different linguistic term sets according to their knowledge. So, this proposal offers to the customers and the experts a flexible multigranular linguistic context instead of forcing all of them to provide their preferences or knowledge in the same scale. Therefore, each source of information can choose its own linguistic term set to provide its knowledge.

3.2. Filtering Items

Once the system has the user profile, the recommendation model will have the following:

1. A user profile $P_u = \{p_1^u, \ldots, p_l^u\}$ with the user preferences provided by the user u, which are described by means of linguistic labels assessed in S_{uk}.
2. A set of products $A = \{a_1, \ldots, a_n\}$ described linguistically by means of their item features $F_{a_j} = \{v_1^j, \ldots, v_l^j\}$ and stored in the item database.

To find the most suitable items for a customer u, the recommendation model filters the items by choosing the most similar ones. To do so, the recommendation model will compare the user profile, P_u, with the item features of all the items in the database by means of a matching process to obtain a similarity measurement that indicates the items in the database closest to the customer profile.

In our case, the information that represents the user profiles as the item features are linguistic labels whose semantics are given by means of fuzzy numbers, so to carry out this matching process measures are needed of comparison between fuzzy numbers. In the following subsections we briefly review this type of measure and present the matching process used by our recommendation model to obtain the similarity between the items and the customer profile.

3.2.1. Measures of Comparison

The comparison of objects is a common task in many fields such as psychology, physical sciences, image processing, clustering, and deductive reasoning. Generally, these comparisons are based on measures of the difference and similitude between two objects. In the literature we can find different types of comparison measures[20,21]:

1. *Measures of satisfiability*: these measures correspond to a situation in which a reference object or a class is considered and it is necessary to decide if a new object is compatible with it or satisfies it.
2. *Measures of resemblance*: a measure of resemblance is used for a comparison between the descriptions of two objects, at the same level of generality, to decide if they have *enough* common characteristics.
3. *Measures of inclusion*: considering a reference object like in the measures of satisfiability. It is determined how important the common characteristics of A and B are, with regard to A.
4. *Measures of dissimilarity*: this is another kind of measure that does not assess the similitude but the difference. This measure is based on the concept of distance between two fuzzy sets.

The proposal to compare the customer profile and the item features consists of using measures of resemblance. In Ref. 22 a measure of resemblance was proposed that is easy to manage and compute. This one has been widely used in the literature to carry out this kind of processes:

$$\textbf{M1:}\ D(A,B) = \sup_x \min(f_A(x), f_B(x)) \tag{1}$$

When this measure is applied to two fuzzy sets, A and B, some knowledge is obtained about their similarity, where the greater the value is, the more similarity there is.

3.2.2. Matching Process

The recommendation model will compare the customer profile $P_u = \{p_1^u, \ldots, p_l^u\}$ and the item features $F_{a_j} = \{v_1^j, \ldots, v_l^j\}$ of each item, $\{a_j, j = 1, \ldots, n\}$ in the database. This comparison will compute the similarity, R_j^u, between the customer profile P_u and item features $F_{a_j} = \{v_1^j, \ldots, v_l^j\}$ of all the items in the item database:

$$R_j^u = Similarity(P_u, F_{a_j}), \quad j = 1, \ldots, n \tag{2}$$

Due to the fact that the assessments belong to the customer profile and the item features are linguistic values whose semantics are fuzzy numbers, the similarity will be computed by means of the resemblance measure presented in Equation (1). With r_k^j being the resemblance between the linguistic value, p_k^u, assigned to the criterion, c_k, of the customer profile, P_u, and its correspondent linguistic value, v_k^j, assigned to the criterion, c_k, in the a_j item features, F_{a_j},

$$r_k^j = D(p_k^u, v_k^j) = \sup_x \min(f_{p_k^u}(x), f_{v_k^j}(x)) \tag{3}$$

where $f_{p_k^u}$ is the membership function of the linguistic value assigned to the criterion, c_k, in P_u, and $f_{v_k^j}$ is the membership function of the linguistic value assigned to the criterion, c_k, in F_{a_j}.

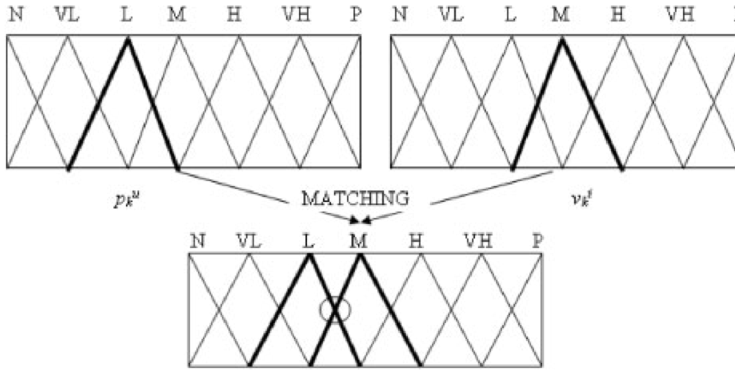

Figure 3. Matching process: computing the resemblance.

The similarity measure between a user profile and each item is a fuzzy set, $R_j^u = (r_1^j, \ldots, r_l^j)$, where each component, r_k^j, is computed by Equation (3):

$$R_j^u = Similarity(P_u, F_j) = (r_1^j, \ldots, r_l^j)$$

For example, let us consider the item feature, $\boldsymbol{v_k^j} = \boldsymbol{M}$, of the item, a_j, and criterion, c_k, and $\boldsymbol{p_k^u} = \boldsymbol{L}$, the customer's preference to the same criterion, in its profile, P_u. Its resemblance, r_k^j, will be (see Figure 3)

$$r_k^j = \sup_x \min(L, M) = 0.5$$

3.3. Making a Recommendation

The objective of a recommendation model is to determine which are the most suitable products for a customer. So far, the model has computed the similarity, R_j^u, between all the items, a_j, and the customer profile, P_u. In this phase of the recommendation process, the similarity can be interpreted as a preference, due to the fact that the greater the value is, the more suitable the item is.

Therefore, to achieve the objective, it is necessary to rank the items according to their similarity with the user profile, but the similarity values computed are expressed by means of fuzzy sets. Therefore, to rank them and recommend the most suitable items, this recommendation model will use the three-step ranking process presented in Ref. 19:

1. to build a preference relation from the measures of similitude
2. to compute a nondominance degree (NDD) for each item
3. to rank the items according to the NDD and recommend the top n ranked

In the following, we present in further detail each step of the recommendation phase.

3.3.1. Building a Preference Relation

At this moment the model builds a preference relation, $Q_u = [q_{ij}]$, from the similarity values, R_j^u, of the customer u with the item database in order to rank the items more suitable for the customer according to their similarity with his preferences. To do so, this recommendation model uses an inclusion measure. Let A and B be two fuzzy sets; an inclusion measure, $S(A, B)$, computes how much A is included in B. In the literature different types of inclusion measures can be found[23,24]; but after a study of several of them, we have decided, due to its easy computation and good performance in our model, to use the following one:

$$\textbf{M2: } S(A,B) = \inf_x \min(1 - f_A(x) + f_B(x), 1) \tag{4}$$

where f_A and f_B are the membership functions of A and B, respectively.

The above function computes how much A is included in B, but the recommendation model needs to know how much A covers B to interpret this value as a preference. So, to obtain the preference degree of A over B, q_{AB}, the inclusion of B in A is computed:

$$q_{AB} = S(B, A)$$

Consequently, to build the preference relation, Q_u, we used the inclusion measure (4) to measure how much covers R_i^u to R_j^u, $\forall i, j \in \{1, \ldots, n\}$. The value, q_{ij}, will express the preference degree of R_i^u over R_j^u, and it is computed as

$$q_{ij} = S(R_j^u, R_i^u) = \inf_x \min(1 - f_{R_j^u}(x) + f_{R_i^u}(x), 1) \tag{5}$$

Applying this computation to all the possible pairs, we built the fuzzy preference relation Q^u:

$$Q_u = \begin{pmatrix} q_{11} & \cdots & q_{1j} & \cdots & q_{1n} \\ \cdots & & \cdots & & \cdots \\ q_{i1} & \cdots & q_{ij} & \cdots & q_{in} \\ \cdots & & \cdots & & \cdots \\ q_{n1} & \cdots & q_{nj} & \cdots & q_{nn} \end{pmatrix}$$

An example of computing preference values for Q_u could be the following: let $R_i^u = \{0, 0.5, 1, 0.5, 0\}$ and $R_j^u = \{0, 0.5, 0.5, 0.5, 0\}$ be two fuzzy sets corresponding to the resemblance measures between the user profile P_u and the products features F_{a_i} and F_{a_j}, respectively; the preference degrees obtained are the following (the symbol ^ stands for "min"):

$$q_{ij} = S(R_j^u, R_i^u) = ((1-0+0) \wedge 1) \wedge ((1-0.5+0.5) \wedge 1) \wedge ((1-0.5+1) \wedge 1) \wedge ((1.5+0.5) \wedge 1) \wedge ((1-0+0) \wedge 1) = 1$$

$$q_{ji} = S(R_i^u, R_j^u) = ((1-0+0) \wedge 1) \wedge ((1-0.5+0.5) \wedge 1) \wedge ((1-1+0.5) \wedge 1) \wedge ((1-0.5+0.5) \wedge 1) \wedge ((1-0+0) \wedge 1) = 0.5$$

So, the degree of R_i^u over R_j^u is $q_{ij} = 1$, whereas the degree of R_j^u over R_i^u is $q_{ji} = 0.5$.

3.3.2 Computing the Nondominance Degree

Different choice degrees can be used to rank the items.[25] This recommendation model will use the nondominance degree (NDD) that indicates which item is nondominated by the others.

DEFINITION 1.[25] *Let $Q = [q_{ij}]$ be a fuzzy preference relation defined over a set of alternatives X. For the alternative x_i its nondominance degree, NDD_i, is obtained as*

$$NDD_i = \min_{X_j}\{1 - q_{jt}^s, j \neq i\} \tag{6}$$

where $q_{ji}^s = \max(q_{ji} - q_{ij}, 0)$ represents the degree to which x_i is strictly dominated by x_j. The nondominance degree (NDD) of all products is obtained according to Equation (6). First of all, the recommendation model must compute the strict preference relation, *Q_u^s from Q_u:*

$$Q_u^s = [q_{ij}^s], \text{ where } q_s^{ij} = \max(q_{ij} - q_{ji}, 0)$$

From the previous example, the strict preference values between $q_{ij} = 1$ and $q_{ji} = 0.5$ is

$$q_{ij}^s = \max(q_{ij} - q_{ji}, 0) = \max(1 - 0.5, 0) = 0.5$$

$$q_{ji}^s = \max(q_{ji} - q_{ij}, 0) = \max(0.5 - 1, 0) = 0$$

Finally, the model will use Q_u^s to compute the *NDD* for each product a_j as

$$NDD_j = \min_{j \neq i}\{1 - q_{ij}^s\}$$

3.3.3. *Ranking the Items in Order of Their Recommendation*

Eventually the best products or items are those with a greater *NDD*, that is, the alternatives less dominated by the others, because it means that their similarity to the customer profile is greater than the others.

It must be taken into account that there can be several alternatives with the same NDD. These will occupy the same order in the ranking.

Example. Let us suppose we have computed the nondominance choice degree of each alternative:

$$\{NDD_1 = 0.49, NDD_2 = 1, NDD_3 = 0.48, NDD_4 = 1\}$$

So, the solution is a ranking where NDD_2 and NDD_4 are the best alternatives followed by NDD_1 and finally, the worst one is NDD_3.

A general scheme of the recommendation process carried out by this recommendation model can be seen in the Figure 4.

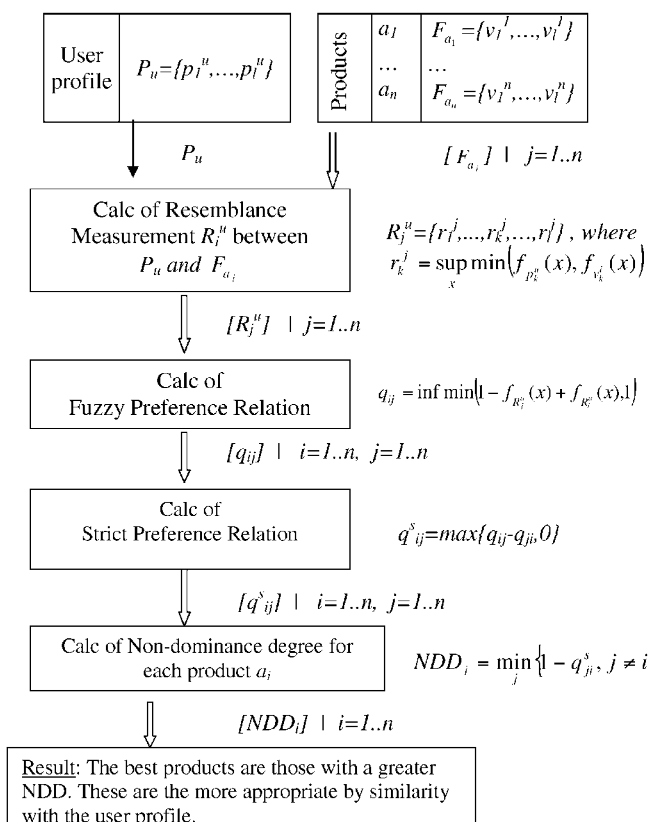

Figure 4. Content-based recommendation model in great detail.

4. EXAMPLE OF A RECOMMENDATION PROCESS

To choose the finest toys for a child is a really hard task, because each child is different and needs different aspects to improve his verbal skills, reasoning, athletic ability, and so forth. Here we apply the multigranular linguistic content-based recommendation model to recommend suitable toys for a child.

The model is guided by several criteria that describe the features of the toys; each criterion will be assessed in a linguistic term set (see Table II) according to the knowledge that the experts have about them:

- *Independent play*: this learning parameter promotes self-esteem and confidence in children. It will be assessed in the linguistic term set C.
- *Mathematical play*: it measures if the children are involved in problem-solving activities, reasoning, and so forth. It will be assessed in the linguistic term set B.
- *Musical play*: the toy engages children in rhythmic musical activity. Music enhances creative skills. It will be assessed in the linguistic term set B.
- *Linguistic play*: it encourages a child's verbal skill. It will be assessed in the linguistic term set B.
- *Motor skill*: it promotes and develops children's physical athletic ability and/or eye–hand coordination. It will be assessed in the linguistic term set C.
- *Cooperative play*: it improves cooperation and interaction with the objective of achieving common goals. It will be assessed in the linguistic term set B.
- *Visual play*: it stimulates the child in visual evaluation and activities that enhance creativity. It will be assessed in the linguistic term set C.
- *Easy to learn how to play*: Some toys need more time than others to learn how to play it. It will be assessed in the linguistic term set A.

These criteria are also used to describe the user profile. To simplify the example we use the same linguistic term sets for each one, but there can be other ones. The semantics of the linguistic term sets are shown in Table II and in Figure 5.

In Table III, we can see the item database that we use in this example.

The recommendation process follows the steps presented in Section 3:

1. *Acquisition of the user profiles*. A user provides his profile to obtain a recommendation according to his needs (Table IV). With this information our recommendation model will find those toys that are closer to the user's needs.
2. *Filtering items*. The first step is to compute the similarity between the user profile and every toy, by means of matching process presented in Section 3.2.2 (see Table V). $R^u_{T_1}$ is calculated comparing T_1 and the user profile U:

$$Similarity(T_1, U) = \{D_{Independent\ Play}(N, AV), \ldots, D_{learning}(D, S)\}$$

$$= S_{T_1} = (0, 0.5, 0.5, 0.5, 1.0, 0, 0.5)$$

3. *Making a recommendation*. The last stage is to make a recommendation. To do so, first the model computes a fuzzy preference relation, Q_u, such as was shown in Section 3.3.1 (see Figure 6), where q_{12} represents the preference degree of the toy T_1 over T_2 (or how much similarity degree of T_1 covers T_2). For example, q_{12} is computed as

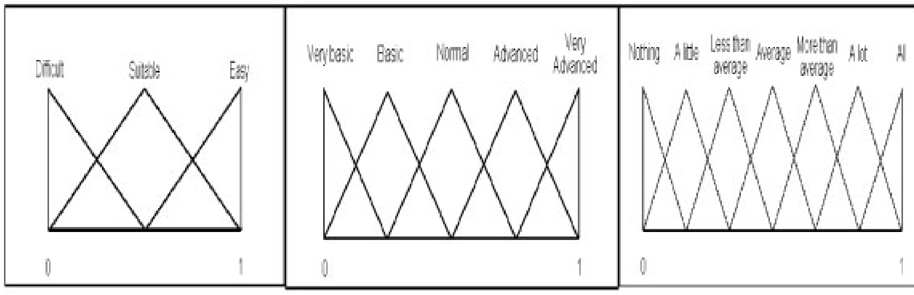

Figure 5. The linguistic term sets A, B, and C.

Table II. Semantics of the linguistic term sets A, B, and C.

Linguistic term set A		Linguistic term set B		Linguistic term set C	
Difficult (D)	(0, 0, 0.5)	Very basic (VB)	(0, 0, 0.25)	Nothing (N)	(0, 0, 0.16)
Suitable (S)	(0, 0.5, 1)	Basic (B)	(0, 0.25, 0.5)	A little (LT)	(0, 0.16, 0.33)
Easy (E)	(0.5, 1, 1)	Normal (N)	(0.25, 0.5, 0.75)	Less than average (LA)	(0.16, 0.33, 0.5)
		Advanced (AD)	(0.5, 0.75, 1)	Average (AV)	(0.33, 0.5, 0.66)
		Very advanced (VA)	(0.75, 1, 1)	More than average (MA)	(0.5, 0.66, 0.83)
				A lot (AL)	(0.66, 0.83, 1)
				All (A)	(0.83, 1, 1)

Table III. Descriptions of toys of our recommendation system in the item database.

Toy	Independent play	Mathematical play	Musical play	Linguistic play	Motor skill	Cooperative play	Visual play	Learning
T_1	N	VB	VB	N	AV	VB	MA	D
T_2	LT	B	VA	VB	AV	B	AL	D
T_3	AV	B	B	AD	LA	N	A	S
T_4	LA	N	N	N	A	AD	AV	S
T_5	AV	AD	VA	AD	AL	VA	LA	D
T_6	AV	VB	N	N	MA	N	N	S
T_7	MA	N	N	VA	AV	AD	AV	S
T_8	AL	VA	N	N	N	N	AV	S
T_9	N	N	B	VA	N	VA	N	D
T_{10}	LT	AD	N	N	MA	N	AL	D

Table IV. User profile.

Independent play	Mathematical play	Musical play	Linguistic play	Motor skill	Cooperative play	Visual play	Learning
AV	B	B	AD	AV	N	A	S

Table V. Similarity degree between user profile and every toy.

T_1	T_2	T_3
$R^u_{T_1} = (0, 0.5, 0.5, 0.5, 1, 0, 0, 0.5)$	$R^u_{T_2} = (0, 1, 0, 0, 1, 0.5, 0.5, 0.5)$	$R^u_{T_3} = (1, 1, 1, 1, 0.5, 1, 1, 1)$
T_4	T_5	T_6
$R^u_{T_4} = (0.5, 0.5, 0.5, 0.5, 0, 0.5, 0, 1)$	$R^u_{T_5} = (1, 0, 0, 1, 0, 0, 0, 0.5)$	$R^u_{T_6} = (1, 0.5, 0.5, 0.5, 0.5, 1, 0, 1)$
T_7	T_8	T_9
$R^u_{T_7} = (0.5, 0.5, 0.5, 0.5, 1, 0.5, 0, 1)$	$R^u_{T_8} = (0, 0, 0.5, 0.5, 0, 1, 0, 1)$	$R^u_{T_9} = (0, 0.5, 1, 0.5, 0, 0, 0, 0.5)$
	T_{10}	
	$R^u_{T_{10}} = (0, 0, 0.5, 0.5, 0.5, 1, 0.5, 0.5)$	

$$\begin{aligned} q_{12} = \inf_x \min(1 - f_{T_2}(x) + f_{T_1}(x), 1) &= ((1 - 0 + 0) \wedge 1) \wedge ((1 - 1 + 0.5) \wedge 1) \\ &\wedge ((1 - 0 + 0.5) \wedge 1) \wedge ((1 - 0 + 0.5) \wedge 1) \wedge ((1 - 1 + 1) \wedge 1) \\ &\wedge ((1 - 0.5 + 0) \wedge 1) \wedge ((1 - 0.5 + 0) \wedge 1) \wedge ((1 - 0.5 + 0.5) \wedge 1) = 0.5 \end{aligned}$$

where $\wedge$ stands for "min".

Now, for each toy T_i, the model calculates its nondominance degree NDD_i. First, the strict preference relation Q_u^s is computed (see Figure 6). Therefore, we compute the nondominance choice degree for each toy:

$$\{NDD_1 = 0.5, NDD_2 = 0.5, NDD_3 = 1, NDD_4 = 0, NDD_5 = 0, NDD_6 = 0,$$
$$NDD_7 = 0.5, NDD_8 = 0, NDD_9 = 0, NDD_{10} = 0\}$$

with NDD_1 being

$$NDD_1 = \min\{(1-0), (1-0.5), (1-0), (1-0), (1-0.5), (1-0), (1-0),$$
$$(1-0), (1-0.5)\} = 0.5$$

$$Q_u = \begin{pmatrix} - & 0.5 & 0 & 0.5 & 0 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0.5 & - & 0 & 0.5 & 0 & 0 & 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & - & 1 & 1 & 1 & 0.5 & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0.5 & 0.5 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & - & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 1 & 0.5 & - & 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 1 & 0.5 & 0.5 & - & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & - & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & - & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 & 0 & 0.5 & 0.5 & 0.5 & - \end{pmatrix} \quad Q_u^s = \begin{pmatrix} - & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & - & 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & - & 1 & 1 & 1 & 0.5 & 1 & 1 & 1 \\ 0 & 0 & 0 & - & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0.5 & 0.5 & - & 0 & 1 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 & 0.5 & 0 & - & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & - \end{pmatrix}$$

Figure 6. Fuzzy preference relation Q_u and the strict preference relation Q_u^s.

Table VI. Ranking of the toys.

First Level	Second Level	Third Level
T_3	T_1, T_2, T_7	$T_4, T_5, T_6, T_8, T_9, T_{10}$

Finally, the model ranks the toys using the nondominance choice degree (Table VI).

The most suitable toy to recommend according to the user profile is T_3; following T_3 the model recommends T_1, T_2, and T_7, and the least suitable toys to recommend are T_4, T_5, T_6, T_8, T_9, and T_{10}. Therefore the recommendation system will recommend to the customer the items in the first level and several of those in the second level, indicating that the first ones are more suitable for the child.

5. CONCLUDING REMARKS

There is more and more need for e-services in different areas nowadays, due to the complexity of several digital tasks and also because sometimes the users need some support to optimize the processes they want to do in information systems.

In this article we have focused our interest on the recommendation systems that help users to find the most suitable products for them according to their preferences, needs, or tastes. Classical recommendation systems force the users to express their information using just one scale of numerical values. However, the customers express information related to their perceptions (qualitative in nature), so it is not very suitable to assess this type of perceptions by means of precise information. We have presented a new recommendation model for content-based systems to offer a greater flexibility to the customers when they provide the information about their needs. This model offers to the users a linguistic context to provide their information because this preference modeling is more suitable for this type of information and, in addition, the model presented offers a multigranular linguistic environment, which means that the users can use different linguistic scales to provide their knowledge. This new recommendation framework will improve the final recommendations due to the fact that the customers have a greater freedom to express their preferences and therefore the recommendations will be more accurate with regard to their initial customer requirements.

Acknowledgment

This work has been partially supported by the Research Project TIC 2002-03348.

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