A Web-Based Consensus Support System Dealing with Heterogeneous Information

Francisco Mata, Juan Carlos Martínez, and Rosa Rodríguez

Abstract. The study of the consensus is an important research field in Decision Making. Several authors have addressed the analysis of the consensus processes from different points of view (techniques, models, information domains, etc.). In this contribution we show a novel web application about a consensus support system to carry out consensus reaching processes with heterogenous information, i.e., the decision makers may use different information domains (particulary, numerical, interval-valued and linguistic assessments) to express their opinions. The software application has the following main characteristic: i) it automatizes virtual consensus reaching processes in which experts may be situated in different places, ii) experts may use information domains near their work areas to provide their preferences and, iii) it is able to run on any computer and operating system. This application may be seen as a practical development of a theoretical research on the consensus modelling. It could be used by any organization to carry out virtual consensus reaching processes.

1 Introduction

Group decision making (GDM) problems may be defined as decision situations where several people (commonly called decision makers or experts) try to reach

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a common solution to a problem from their opinions or preferences. So, given a set of alternatives, experts try together to find the best alternative to solve the problem.

In the literature we can see several attempts to solve decision problems where experts use the same domain of information to express their preferences [11, 18, 21]. However, in many cases it may be advisable that experts express their points of view through domains more consistent with either the nature of the alternatives or their area of expertise. For example, experts of different departments of a company (marketing, accounting, psychology, ...) may prefer to express their opinions using a domain of information closer to their knowledge fields. Moreover, in decision problems we can deal with alternatives whose nature is quantitative and others whose nature is qualitative. The first ones can be assessed by means of precise values. However, when alternatives are related to qualitative aspects, it may be difficult to qualify them using precise values. In such cases, where the uncertainty is present, other types of assessments as interval-valued [23, 30] or linguistic values [11, 32] could be more suitable. In such circumstances, we can consider the decision problems are defined into a heterogenous context.

Usually GDM problems have been resolved through selection processes where the experts get the best set of alternatives from the preferences expressed by themselves [9, 26]. However it may happen that some experts consider that their preferences were not taken into account to obtain the solution and therefore may disagree with that solution. To avoid this situation, it is advisable to carry out a consensus process (see Fig. 1) where experts discuss and change their preferences to reach enough agreement before making the selection process [7, 12, 15, 19].

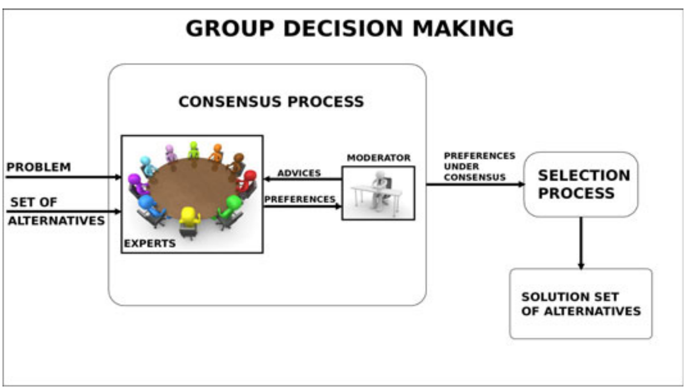


Fig. 1 Group decision making process

The consensus is an interesting research area in decision making that has been approached from different points of view [1, 2, 4, 5, 8, 10, 15, 19, 28, 31, 33]. It can be defined as a mutual-agreement state among the members of a group where all opinions have been expressed and listened to the satisfaction of the group [27]. The consensus-building process is a dynamic and iterative process consisting of several rounds, where experts express and discuss their points of view in order to achieve a common solution. Traditionally this process is coordinated by a human moderator, which calculates the consensus among experts in each round using different consensus measures [17, 22]. If the agreement is not enough, the moderator encourages experts to change their preferences further from the group's opinion in an effort to make them closer in the next consensus round [5, 34].

Roughly speaking, the consensus processes of the real world have two main characteristics:

a) A human moderator, who may become a controversial figure because experts may have complaints about his/her lack of objectivity. Moreover, in heterogeneous contexts, he/she may have problems to understand all the different information domains and scales in a proper way.
b) The physical presence of the experts in the same place. It is true that many consensus sessions are carried out by using the services given by new technologies as for example the videoconference, but the on-line presence of the expert is required. This is a problem since it is very difficult to get together experts or the cost is too high.

Several authors have proposed different models to approach the consensus reaching processes [4, 6, 7, 12, 16, 24, 28] but none of them has finally been implemented as a software solution to carry out virtual consensus processes. Therefore, taking into account the lack of software developments to tackle the consensus problems and the characteristics of the consensus reaching processes introduced previously, the aim of this chapter is to present a novel web application about a web consensus support system (WCSS). This application is supported on the last web technologies to accomplish consensus processes with heterogenous information and without limitations of the human moderator and the physical presence of the experts. This web application allows that experts, who are in different places, can participate in virtual consensus sessions using a simple browser and internet connection.

The chapter is structured as follows. In the Section 2, GDM problems defined in a heterogeneous setting as well as the process to unify heterogeneous information are introduced. In the Section 3, we present the consensus reaching model implemented in the web application. Finally, Section 4 shows the features of the WCSS.

2 Preliminaries

In this section, firstly we shall introduce the GDM problems with heterogeneous information. In the following, we shall briefly review the heterogeneous preferences unification process used by the WCSS to unify the preferences given by the experts.

2.1 Group Decision Making Problems in Heterogenous Settings

GDM problems are decision situations in which several individuals or experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, provide their preferences on a set of alternatives,

$X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, to derive a solution (an alternative or set of alternatives). Depending on different factors as the knowledge degree on the alternatives and/or the features of the problem, experts may use different structures to express their preferences.

In fuzzy contexts, a popular way to provide the experts' preferences are the fuzzy preference relations [18, 29]. A preference relation may be defined as a matrix $\mathbf{P_i} \subset X \times X$,

$$\mathbf{P_i} = \begin{pmatrix} p_i^{11} & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & p_i^{nn} \end{pmatrix}$$

where the value $\mu_{P_{e_i}}(x_l, x_k) = p_i^{lk}$ is meant as the preference degree of the alternative x_l over x_k given by the expert e_i.

Assuming $p_i^{lj} \in [0,1]$, then:

1. $p_i^{lj} = 1$ indicates the maximum degree of preference of x_l over x_j
2. $0.5 \leq p_i^{lj} \leq 1$ indicates a definitive preference of x_l over x_j
3. $p_i^{lj} = 0.5$ indicates the indifference between x_l and x_j

The fuzzy preference relations may satisfy some of the following properties:

- Reciprocity: $p_i^{lj} + p_i^{jl} = 1, \forall l, j$
- Completeness: $p_i^{lj} + p_i^{jl} \geq 1, \forall l, j$
- Max-Min Transitivity: $p_i^{lk} \geq Min(p_i^{lj}, p_i^{jk}), \forall l, j, k$
- Max-Max Transitivity: $p_i^{lk} \geq Max(p_i^{lj}, p_i^{jk}), \forall l, j, k$
- Restricted Max-Min Transitivity: $p_i^{lj} \geq 0.5, p_i^{lk} \geq 0.5 \Rightarrow p_i^{lk} \geq Min(p_i^{lj}, p_i^{jk})$
- Restricted Max-Max Transitivity: $p_i^{lj} \geq 0.5, p_i^{lk} \geq 0.5 \Rightarrow p_i^{lk} \geq Max(p_i^{lj}, p_i^{jk})$
- Additive Transitivity: $p_i^{lj} + p_i^{jk} - 0.5 = p_i^{lk}, \forall l, j, k$

The ideal situation in a GDM problem is that all experts have a wide knowledge about the alternatives and provide their opinions in a numerical precise scale. However, in many cases, experts belong to distinct research areas and/or may have different levels of knowledge about the alternatives. In these situations, experts may prefer to express their preferences by means of different information domains and we can consider the problem is defined in a heterogeneous context.

In this chapter we deal with heterogenous GDM problems where experts provide their preferences by means of fuzzy preference relations with numerical, interval-valued or linguistic assessments. Under these circumstances, a process to unify these assessments on an unique information domain is necessary.

2.2 Unification of Heterogeneous Information

In GDM problems with heterogeneous information, there are no standard operators to operate directly with the preferences expressed in different information domains.

Herrera et al. proposed in [14] to unify all experts' preferences into a common utility space called basic linguistic term set (BLTS), $S_T = \{s_0, ..., s_g\}$ (see Fig. 2). They defined different transformation functions to transform each numerical, interval-valued and linguistic preference value into a fuzzy set on the BLTS, $F(S_T)$:

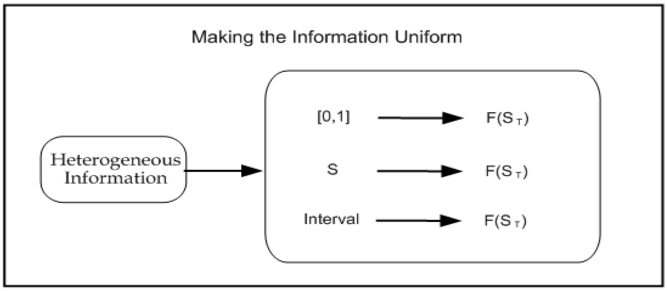


Fig. 2 Unification process of heterogeneous information

Assuming that the WCSS is supported on these transformation functions to unify the experts' preferences, we shall briefly review these functions. More details and examples can be consulted in [14].

2.2.1 Transforming Numerical Values into Fuzzy Sets

To transform a numerical value in $[0,1]$ into a fuzzy set on S_T, we use the following function. Let ϑ be a numerical value, $\vartheta \in [0,1]$, and $S_T = \{s_0, \ldots, s_g\}$ the BLTS. The function τ_{NS_T} that transforms a numerical value ϑ into a fuzzy set on S_T is defined as [13]:

$$\tau_{NS_T} : [0,1] \rightarrow F(S_T)$$
$$\tau_{NS_T}(\vartheta) = \{(s_0, \alpha_0), ..., (s_g, \alpha_g)\}, s_i \in S_T \, and \, \alpha_i \in [0,1]$$
$$\alpha_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & if \, \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & if \, a_i \leq \vartheta \leq b_i \\ 1, & if \, b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & if \, d_i \leq \vartheta \leq c_i \end{cases}$$

Remark 1: We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, are represented by a parametric function (a_i, b_i, d_i, c_i). As a particular case, triangular membership functions with $b_i = d_i$.

2.2.2 Transforming Interval-Valued into Fuzzy Sets

To transform an interval-valued into a fuzzy set on S_T, we use the following function. Let $I = [\underline{i}, \overline{i}]$ be an interval valued in $[0,1]$ and $S_T = \{s_0, \ldots, s_g\}$ the BLTS. Then, the function τ_{IS_T} that transforms the interval-valued I into a fuzzy set on S_T is defined as:

$$\tau_{IS_T} : I \rightarrow F(S_T)$$
$$\tau_{IS_T}(I) = \{(s_k, \alpha_k^i) / k \in \{0,...,g\}\},$$
$$\alpha_k^i = \max_y \min\{\mu_I(y), \mu_{s_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions associated with the interval-valued I and terms s_k, respectively.

2.2.3 Transforming Linguistic Terms into Fuzzy Sets

To transform a linguistic value in S into a fuzzy set on S_T, we use the following function. Let $S = \{l_0, \ldots, l_p\}$ be a linguistic term set and $S_T = \{s_0, \ldots, s_g\}$ the BLTS, such that, $g \geq p$. Then, the function τ_{SS_T} that transforms $l_i \in S$ into a fuzzy set on S_T is defined as:

$$\tau_{SS_T} : S \rightarrow F(S_T)$$
$$\tau_{SS_T}(l_i) = \{(s_k, \alpha_k^i) / k \in \{0,...,g\}\}, \forall l_i \in S$$
$$\alpha_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and s_k, respectively.

Remark 2: If S is chosen as S_T, then the fuzzy set that represents a linguistic term will be **0** except the value of the ordinal of the linguistic label that will be **1**.

3 A Consensus Support System Model for GDM Problems with Heterogeneous Information

Several theoretical models have been proposed in the literature to approach consensus processes [6, 7, 16, 20, 24, 28]. In this section we present the comprehensive consensus support system model implemented in the web application. The aim of this model is to automate consensus processes without the direct intervention of the human moderator. The model has the following main features:

1. It is able to carry out consensus processes in heterogeneous GDM problems with numerical, interval-valued and linguistic assessments. Experts provide their opinions by means of preference relations.
2. It uses two types of measurements,*consensus degrees* to check the level of agreement reached among the experts in each round and *proximity measures* to measure the distance among the experts' individual preferences and the collective preference.
3. It has a guided advice generator to suggest the changes of preferences to experts in order to increase the level of agreement in each new consensus round. A set of advice rules based on both measurements is used to guide the direction of the changes of the experts' preferences.

The CSS model consists of the following phases depicted in Fig. 3:

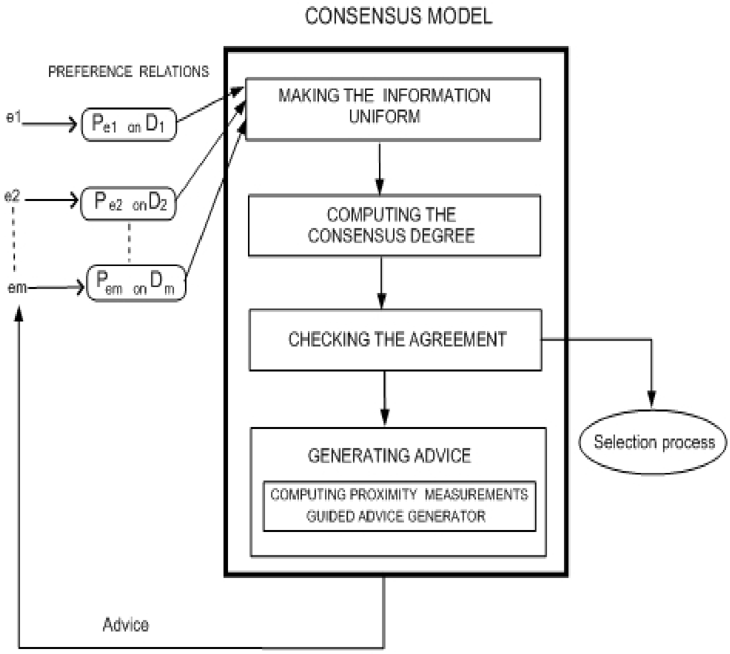


Fig. 3 A consensus support system model with heterogeneous information

A brief explanation about every phase is presented in the following subsections.

3.1 Making the Information Uniform

As we have said before, there are no standard operators to manage heterogeneous information directly, so we have to transform all preferences into an unique domain in order to operate with this information. By using the transformation functions introduced in Sect. 2.2, the model transforms all experts' preferences into fuzzy sets. So, assuming each fuzzy set, $\tilde{p}_i^{lk} = (s_0, \alpha_{i0}^{lk}), \ldots, (s_g, \alpha_{ig}^{lk})$, is represented by means of its respective membership degrees, $(\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, the result of this phase is a matrix of fuzzy sets, $\tilde{\mathbf{P}}_\mathbf{i}$:

$$\tilde{\mathbf{P}}_\mathbf{i} = \begin{pmatrix} \tilde{p}_i^{11} = (\alpha_{i0}^{11}, \ldots, \alpha_{ig}^{11}) & \cdots & \tilde{p}_i^{1n} = (\alpha_{i0}^{1n}, \ldots, \alpha_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ \tilde{p}_i^{n1} = (\alpha_{i0}^{n1}, \ldots, \alpha_{ig}^{n1}) & \cdots & \tilde{p}_i^{nn} = (\alpha_{i0}^{nn}, \ldots, \alpha_{ig}^{nn}) \end{pmatrix}$$

3.2 Computing the Consensus Degree

The consensus degree evaluates the level of agreement achieved among the experts in each consensus round. So, if experts' preferences are similar, the consensus degree will be high, otherwise, if preferences are very different, the consensus degree will be low. To compute the level of agreement, a consensus matrix is obtained by aggregating the values that represent the similarity or distance among the experts' preferences, comparing each other.

The distance between two preferences $\tilde{p}_i^{lk}$ and $\tilde{p}_j^{lk}$ is computed by means of the similarity function $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ measured in the unit interval $[0,1]$ [16]:

$$s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) = 1 - \left| \frac{cv_i^{lk} - cv_j^{lk}}{g} \right| \tag{1}$$

The cv_i^{lk} is the central value of the fuzzy set:

$$cv_i^{lk} = \frac{\sum_{h=0}^{g} index(s_h^i) \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}}, \tag{2}$$

and represents the centre of gravity of the information contained in the fuzzy set $\tilde{p}_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, where $index(s_h^i) = h$ and α_{ih}^{lk} are the membership degrees. The range of this central value is the closed interval $[0,g]$.

The closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ to 1 the more similar preferences p_i^{lk} and p_j^{lk} are, while the closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ to 0 the more distant p_i^{lk} and p_j^{lk} are.

Once the similarity function has been introduced, the consensus degree is computed according to the following steps:

1. Central values of all fuzzy sets are calculated:

$$cv_i^{lk};\ \forall\, i = 1,\ldots,m;\ \ l,k = 1,\ldots,n\ \wedge\ l \neq k. \tag{3}$$

2. For each pair of experts e_i and e_j $(i < j)$, a *similarity matrix* $SM_{ij} = \left(sm_{ij}^{lk}\right)$ is calculated, where,

$$sm_{ij}^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}). \tag{4}$$

3. A *consensus matrix* CM is obtained by aggregating all the similarity matrices at the level of pairs of alternatives,

$$CM = \begin{pmatrix} cm^{11} & \cdots & cm^{1n} \\ \vdots & \ddots & \vdots \\ cm^{n1} & \cdots & cm^{nn} \end{pmatrix}$$

where,

$$cm^{lk} = \phi(sm_{12}^{lk}, sm_{13}^{lk}, \ldots, sm_{1m}^{lk}, sm_{23}^{lk}, \ldots, sm_{2m}^{lk},\ \ldots, sm_{(m-1)m}^{lk})$$

for l,k $\in \{1,\ldots,n\}$.

As we can deduce from (1), if two preferences $\tilde{p}_i^{lk}$ and $\tilde{p}_j^{lk}$ are very similar, the similarity value between them, $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$, is high and therefore we can consider the agreement is high too. Taking into account this assumption, in order to evaluate the level of agreement among experts, the model computes the similarity values between each expert's preference with respect to the rest of experts' preferences and it aggregates them. So, the more similar the preferences are the more high aggregated values are and therefore the agreement will be higher. In this model, we use the arithmetic mean as the aggregation function ϕ, although, other aggregation operators could be used according to other points of view (majority concept, penalization techniques, etc.) [25].

4. The consensus degree is computed in three different levels: pairs of alternatives, alternatives and relations. In this way, we can know in a precise way the level of agreement in each pair of alternatives and so to identify the pairs as well as the alternatives in which there exists greater disagreement.

Level 1. Consensus on pairs of alternatives, cp^{lk}, represents the agreement on the pair of alternatives (x_l, x_k). They are obtained from the consensus matrix CM,

$$cp^{lk} = cm^{lk}, \ \forall l,k = 1,\ldots,n \ \wedge \ l \neq k.$$

Values of cp^{lk} close to 1 mean a greater agreement. This measurement allows to identify those pairs with a poor level of agreement.

Level 2. Consensus on alternatives, ca^l, represents the agreement on the alternative x_l,

$$ca^l = \frac{\sum_{k=1,\, l\neq k}^{n} (cp^{lk} + cp^{kl})}{2(n-1)}. \tag{5}$$

Level 3. Consensus on relations or global consensus, cr, represents the global agreement among all experts' preferences,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}. \tag{6}$$

The model uses this value to check the level of agreement achieved in each round. If cr is high enough then the system stops the consensus process.

Assuming the computing of the consensus degree is a key phase of the model, an example is presented to facilitate the understanding.

Example 1. . *Let e_1, e_2 and e_3 be three experts who use numerical, linguistic and interval-valued assessments respectively. Their preferences are the following* [1]*:*

[1] Notice that the properties of the preference relations defined in Sect. 2.1 have been smoothed in order to make more flexible the entrance of experts' preferences.

$$\mathbf{P_1} = \begin{pmatrix} - & .5 & .8 & .4 \\ .3 & - & .9 & .3 \\ .3 & .2 & - & .4 \\ .9 & .8 & .5 & - \end{pmatrix}; \; \mathbf{P_2} = \begin{pmatrix} - & H & VH & M \\ L & - & H & VH \\ VL & N & - & VH \\ L & VL & N & - \end{pmatrix}$$

$$\mathbf{P_3} = \begin{pmatrix} - & [.7,.8] & [.65,.7] & [.8,.9] \\ [.3,.35] & - & [.6,.7] & [.8,.85] \\ [.3,.35] & [.3,.4] & - & [.7,.9] \\ [.1,.2] & [.2,.4] & [.1,.3] & - \end{pmatrix}$$

After carrying out the unification process, experts' preferences have been transformed into matrices of fuzzy sets:

$$\tilde{\mathbf{P}}_\mathbf{1} = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,.19,.81,0,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,.19,.81,0,0,0,0) \\ (0,.19,.81,0,0,0,0) & (0,.81,.19,0,0,0,0) & - & (0,0,.59,.41,0,0,0) \\ (0,0,0,0,0,.59,.41) & (0,0,0,0,.19,.81,0) & (0,0,0,1,0,0,0) & - \end{pmatrix}$$

$$\tilde{\mathbf{P}}_\mathbf{2} = \begin{pmatrix} - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) & (0,0,0,1,0,0,0) \\ (0,0,1,0,0,0,0) & - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) \\ (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - & (0,0,0,0,0,1,0) \\ (0,0,1,0,0,0,0) & (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - \end{pmatrix};$$

$$\tilde{\mathbf{P}}_\mathbf{3} = \begin{pmatrix} - & (0,0,0,0,.81,.81,0) & (0,0,0,.12,1,.19,0) & (0,0,0,0,.19,1,.41) \\ (0,.19,1,.12,0,0,0) & - & (0,0,0,.41,1,.19,0) & (0,0,0,0,.19,1,.12) \\ (0,.19,1,.12,0,0,0) & (0,.19,1,.41,0,0,0) & - & (0,0,0,0,.81,1,.41) \\ (.41,1,.19,0,0,0,0) & (0,.81,1,.41,0,0,0) & (.41,1,.81,0,0,0,0) & - \end{pmatrix}$$

From these matrices, the model computes the global consensus degree cr applying the following steps:

1. *Central values. Applying (2), the model computes the central values of the fuzzy sets:*

$$cv_1 = \begin{pmatrix} - & 3 & 4.81 & 2.41 \\ 1.81 & - & 5.41 & 1.81 \\ 1.81 & 1.19 & - & 2.41 \\ 5.41 & 4.81 & 3 & - \end{pmatrix}; \; cv_2 = \begin{pmatrix} - & 4 & 5 & 3 \\ 2 & - & 4 & 5 \\ 1 & 0 & - & 5 \\ 2 & 1 & 0 & - \end{pmatrix}$$

$$cv_3 = \begin{pmatrix} - & 4.5 & 4 & 5.13 \\ 1.94 & - & 3.86 & 4.94 \\ 1.94 & 2.13 & - & 4.81 \\ 0.86 & 1.81 & 1.18 & - \end{pmatrix}$$

2. *Similarity matrices. The model computes a similarity matrix between each pair of experts by using the distance function (1):*

$$SM_{12} = \begin{pmatrix} - & 0.83 & 0.96 & 0.9 \\ 0.96 & - & 0.76 & 0.46 \\ 0.86 & 0.8 & - & 0.56 \\ 0.43 & 0.36 & 0.5 & - \end{pmatrix}; \; SM_{13} = \begin{pmatrix} - & 0.75 & 0.87 & 0.54 \\ 0.97 & - & 0.74 & 0.47 \\ 0.97 & 0.84 & - & 0.59 \\ 0.24 & 0.5 & 0.69 & - \end{pmatrix}$$

$$SM_{23} = \begin{pmatrix} - & 0.91 & 0.84 & 0.64 \\ 0.99 & - & 0.97 & 0.99 \\ 0.84 & 0.64 & - & 0.97 \\ 0.81 & 0.86 & 0.8 & - \end{pmatrix}$$

3. *Consensus matrix. The model calculates the consensus matrix by aggregating the similarity matrices:*

$$CM = \begin{pmatrix} - & 0.83 & 0.89 & 0.69 \\ 0.97 & - & 0.82 & 0.64 \\ 0.89 & 0.76 & - & 0.71 \\ 0.49 & 0.57 & 0.66 & - \end{pmatrix}$$

4. *Consensus degrees. The model computes the consensus degree at different levels:*

Level 1. Consensus on pairs of alternatives. The element (l,k) of CM represents the consensus degree on the pair of alternatives (x_l, x_k).

Level 2. Consensus on alternatives.

$$ca^1 = 0.79,\; ca^2 = 0.80,\; ca^3 = 0.78,\; ca^4 = 0.63$$

Level 3. Consensus on the relations or global consensus. Finally the consensus degree among experts is,

$$cr = 0.73$$

3.3 Checking the Agreement

In this phase the model controls the level of agreement achieved in the current consensus round. Before starting the process, a minimum consensus threshold, $\beta \in [0,1]$, is fixed, which will depends on the particular problem we are dealing with. When the consequences of the decision are of a transcendent importance, the minimum level of consensus required to make that decision should be logically high, for example $\beta = 0.8$ or higher. At the other extreme, when the consequences are not so transcendental (but are still important) and it is urgent to obtain a solution, a fewer consensus threshold near to 0.5 could be required.

In any case, independently of the value β, when the global consensus cr reaches β, the consensus process stops, and the selection process is applied to obtain the solution. However, there is the possibility that the global consensus will not converge to consensus threshold and the process will get block. In order to avoid this circumstance, the model incorporates a parameter, *Maxcycles*, to limit the number of consensus rounds to carry out. The performance of this phase is shown in Fig. 4.

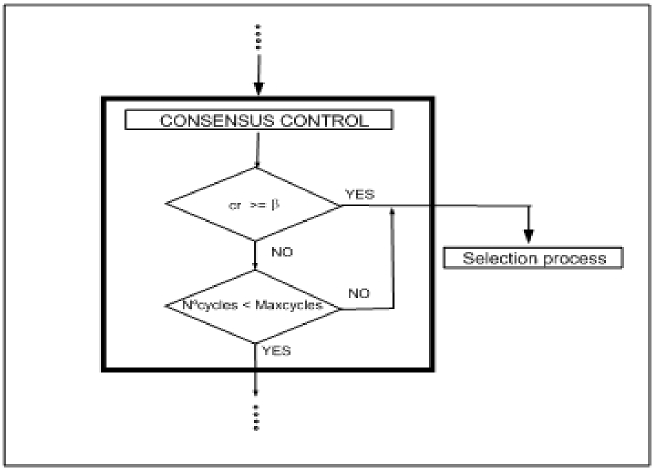


Fig. 4 Consensus control

3.4 Generating Advice

When the agreement is not big enough, i.e. $cr < \beta$, experts should modify their preferences to make them closer and to increase the level of agreement. In order to identify the furthest experts' preferences from the collective opinion, the model computes proximity measurements among experts'preferences and the collective opinion. Once the furthest preferences have been identified, a guided advance generator suggests the changes on these preferences with the objective of increasing the agreement in the next consensus round. Both processes are presented in detail below.

3.4.1 Computing Collective Preference and Proximity Measurements

The proximity measurements evaluate the distance between the individual experts' preferences and the collective preference. To calculate them, firstly the model obtains a collective preference $\tilde{\mathbf{P}}_c$,

$$\tilde{\mathbf{P}}_\mathbf{c} = \begin{pmatrix} \tilde{p}_c^{11} & \cdots & \tilde{p}_c^{1n} \\ \vdots & \ddots & \vdots \\ \tilde{p}_c^{n1} & \cdots & \tilde{p}_c^{nn} \end{pmatrix}$$

which represents the group's opinion. $\tilde{\mathbf{P}}_{\mathbf{c}}$ is calculated by aggregating the set of individual preference relations $\{\tilde{\mathbf{P}}_{\mathbf{e_1}},\ldots,\tilde{\mathbf{P}}_{\mathbf{e_m}}\}$,

$$\tilde{p}_c^{lk} = \psi(\tilde{p}_1^{lk},\ldots,\tilde{p}_m^{lk}) = (\alpha_{c0}^{lk},\ldots,\alpha_{cg}^{lk})$$

where

$$\alpha_{cj}^{lk} = \psi(\alpha_{1j}^{lk},\ldots,\alpha_{mj}^{lk})$$

and ψ is the average of the membership degrees.

Once the model has obtained the collective preference relation, it computes a proximity matrix, PM_i, for each expert e_i,

$$PM_i = \begin{pmatrix} pm_i^{11} & \cdots & pm_i^{1n} \\ \vdots & \ddots & \vdots \\ pm_i^{n1} & \cdots & pm_i^{nn} \end{pmatrix}$$

by using the similarity function defined in expression (1),

$$pm_i^{lk} = s(\tilde{p}_i^{lk},\tilde{p}_c^{lk}).$$

These matrices contain the necessary information to know the distance of the preferences of each expert with respect to the collective opinion.

The experts' proximities are computed at level of pairs of alternatives, alternatives and relations.

Level 1. Proximity on pairs of alternatives, pp_i^{lk}, represents the proximity between the expert's preference and the collective one on the pair of alternatives (x_l,x_k),

$$pp_i^{lk} = pm_i^{lk}, \ \ \forall l,k = 1,\ldots,n \ \wedge \ l \neq k.$$

Level 2. Proximity on alternatives, called pa_i^l, represents the proximity between the expert's preferences and the collective one on the alternative x_l,

$$pa_i^l = \frac{\sum_{k=1,k\neq l}^{n}(pp_i^{lk}+pp_i^{kl})}{2(n-1)}. \tag{7}$$

Level 3. Proximity on the relation, pr_i, represents the global proximity between the expert's preferences and the collective one,

$$pr_i = \frac{\sum_{l=1}^{n} pa_i^l}{n}. \tag{8}$$

3.4.2 Guided Advice Generator

The purpose of the guided advice generator is to identify the furthest experts' preferences and to suggest how to change them in order to increase the consensus. The model will only suggest increasing or decreasing the current assessments and each expert will decide the ratio of the changes according his/her both experience and the preferences expression domain.

To attain this objective, the guided advice generator uses two types of advice rules: *identification rules* and *direction rules*.

a) *Identification rules (IR)*. These rules identify what experts, alternatives and pairs of alternatives should be changed. In this way, the model only focuses on the preferences in disagreement and will not recommend to change those which the agreement is enough. The model uses three types of rules:

a.1) An identification rule of experts. Experts whose proximity, pr_i, is smaller than the average of proximities have to change some of their preferences.

IR.1. $\forall e_i \in E \cap EXPCH$, then e_i must change his/her preferences, where

$$EXPCH = \{e_i |\ pr_i < \overline{pr}\}$$

and

$$\overline{pr} = \frac{\sum_{i=1}^{m} pr_i}{m}.$$

a.2) An identification rule of alternatives. Alternatives whose consensus degree, ca^l, is smaller than the global consensus, cr, should change some of its pairs.

IR.2. $\forall e_i \in EXPCH$, e_i should change some assessments associated to the pairs that belong to the alternative x_l, such that, $x_l \in ALT$ and $ALT = \{x_l \in X |\ ca^l < cr\}$.

a.3) An identification rule of pairs of alternatives. Pairs of alternatives whose proximity value at level of pairs, pp_i^{lk}, is smaller that the average.

IR.3. $\forall (x_l \in ALT \ \wedge \ e_i \in EXPCH)$, if $(x_l, x_k) \in PALT_i$, then e_i should change the pair p_i^{lk}, where

$$PALT_i = \{(x_l, x_k) \mid x_l \in ALT \ \wedge \ e_i \in EXPCH \ \wedge \ pp_i^{lk} < \overline{pp}^{lk}\}.$$

and

$$\overline{pp}^{lk} = \frac{\sum_{i=1}^{m} pp_i^{lk}}{m}.$$

b) *Direction rules (DR)*. Once the model has identified the pairs of alternatives to be changed, it uses a set of direction rules to suggest the directions of the changes. For each preference value to be changed, the model will suggest increasing or decreasing the current assessment. Now each expert increases or decreases the his/her currents assessments by using his/her respective information domain. The following directions rules are used taking into account the central values of the fuzzy sets of both experts' preferences and collective ones.

DR.1. If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) < 0$ then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k).

DR.2. If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) > 0$ then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k).

DR.3. If $(cv(\tilde{p}_i^{lk}) - cv(\tilde{p}_c^{lk})) = 0$ then the expert e_i should not modify the assessment associated to the pair of alternatives (x_l, x_k).

4 WCSS Features

In this section we present the features of the web consensus support system to carry out virtual consensus processes with heterogeneous information. This web application has been developed by the research group Intelligent Systems Based on Fuzzy Decision Analysis (*Sinbad*[2]) of the University of Jaén. As we said in the Sect. 1, the consensus may be seen as a technique to achieve good results in any GDM problem. If we take as example consensus processes of the real world, the physic presence of the experts (in the same place or by videoconference) is required. In many cases this is a problem since it is difficult to get together experts (different cities or ever countries, engagement books, ...) or the cost is too high for the organization. In these cases, carrying out virtual consensus sessions supported on web technologies with experts participating from different places might be very interesting. Different theoretic consensus models have been proposed in the literature, but very few of them have been implemented and none of them considers the possibility for accomplishing virtual consensus processes.

The characteristics of the web application are the following:

a) It automatizes virtual consensus reaching processes where the moderator's tasks are assumed by the system and the experts may be situated in different places.
b) Ability to manage different information domains (numerical, linguistic and interval-valued assessments).
c) It has been implemented using the last technologies related to the developments of web applications.
d) Multi-user and multi-platform system.
e) Storage of information in databases.
f) Smooth handling and ease of use.

This system may be considered as an example of research transfer from the theoretical model proposed in Sect. 3 to a real system to solve consensus reaching problems.

This section has been structured in two main parts: system architecture and system functionality. Details of the implementation code have been obviated because we have considered these are out of the scope of this book.

4.1 System Architecture

In this section we briefly describe the main elements of the model along with technologies used to design and to implement the system. As it is depicted in the Fig. 5, the system has been developed following the typical client-server architecture and the object-oriented paradigm. So, the system is hosted in a computer which makes the role of internet server [2]. Remote clients can connect with this server through a web browser. Users send requests that are parsed on the server and consistent answers are sent back to client. This architecture frees the end user of the task of

[2] In this version of the application, Apache Tomcat 6.0 web server has been used since it is open source software, very popular among programmers and it is used as web server in many organizations.

installing the application on his computer. Moreover, it is highly scalable and extensible to add new clients and servers. Note that a database system is also used to store all the information related with the process, that is, information about problems, experts, preferences given in every round, etc.

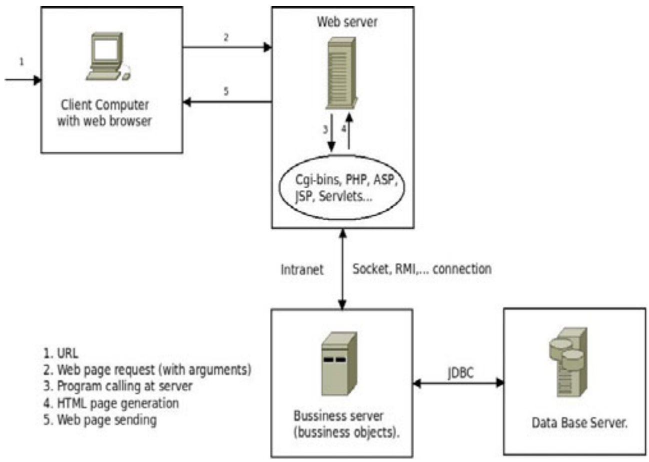


Fig. 5 System architecture

Regarding web technologies and programming languages, the application has been implemented with the Java language, particulary JavaServer Pages (JSP) because allow to generate dynamic web pages by using HTML and XML documents. JSP generate pages that are compiled and executed on the server to deliver an HTML or XML document. The compiled pages and dependent Java libraries use Java bytecode rather than a native software format. In this way the application can be executed within the Java Virtual Machine on any computer and operating system.

Although it is not our intention to explain the implementation details, we think it is interesting to describe the package used to manage the heterogenous information. The design of this package may guide other researchers to define other similar packets in other contexts.

4.1.1 Package for Managing Heterogeneous Information

The Fig. 6 depicts the class diagram to deal with heterogeneous information used by our system. This diagram has been designed taking into account the nomenclature proposed by Unified Modeling Language (UML) [3].

The class diagram is clair, we only focus on the class *Valuation* because it is interesting from of point of view of the heterogeneous information. This class is an abstract class of “Numeric”, “Interval” and “Linguistic” classes, which inherit the unification() operation. This method transforms all experts’ preferences into fuzzy sets according to the expression domain (see Sect. 2.2). As we can see all classes

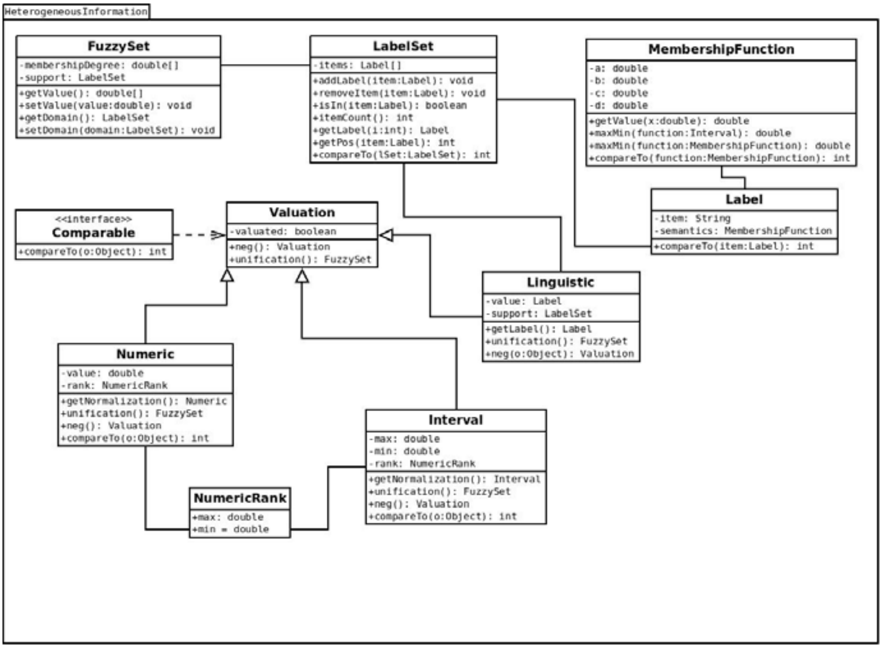


Fig. 6 Class diagram of the heterogeneous information package

have their own attributes (e.g. the attributes of the class Interval are the extremes values of the interval and the range of the possible values) and their methods.

4.1.2 Database

As it seems logic, any consensus process generates a lot information that has to be stored. For example, about the experts we manage personal details, about the problems, we have parameters (consensus threshold, number of rounds), and about the consensus rounds we store the consensus degrees achieved, number of changes, etc. All this information have to be organized properly and should be accessible by the system, administrator and experts, therefore a database [3] is needed. In the Fig. 7 we show the Entity- Relationship diagram used by the system.

It is clearly viewed that admin, expert, alternative, problem, round, etc., are entities in our conceptual schema. Regarding relationships, we describe some examples: in a problem several experts take part and an expert takes part in one or several problems (R1); an expert uses an information type and an information type can be used by several experts (R2); a problem is set on several alternatives and an alternative is included in only one problem (R3); etc.

[3] MySQL 5.0 database system has been used due to it is easy to use, reliable and quite fast.

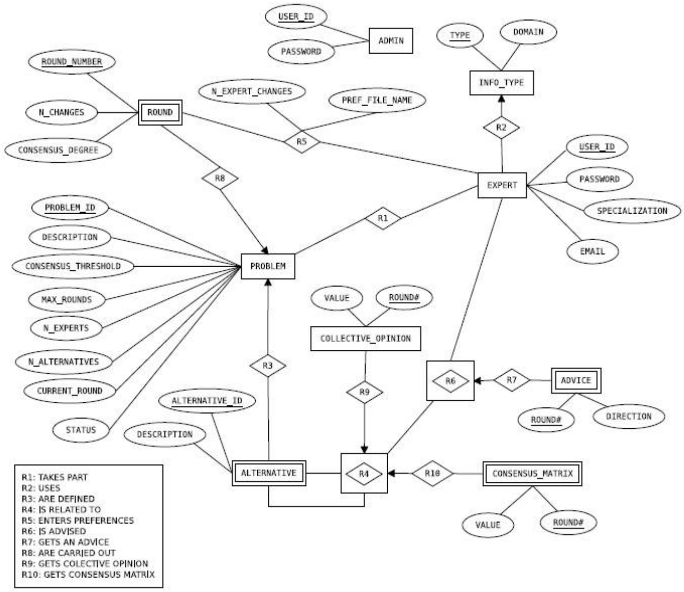


Fig. 7 Entity-Relationship diagram for system database

4.2 System Functionality

In this subsection the system functionality is described by following a structure similar to the consensus model depicted in the Fig. 3. To improve the understanding of the system, we use an example which allows us to see the system performance during a consensus reaching process. So, a food company wants to launch a new type of juice but it has doubts about the best juice taste (lemon, apple, orange and peach taste). To solve this decision, the company decides to consult several experts from different areas (sports, medicine, marketing, ...) who live in different cities and countries. The company management wants to carry out a consensus process among the experts in order to attain a high level of agreement about the preferred taste before deciding the juice to be produced. Due to experts live in different cities, the company decides to use our web application to reproduce a virtual consensus reaching process.

4.2.1 Users Identification

Two different user profiles have been considered in this application from the point of view of the functionality:

- Experts, persons who participate in the consensus process by providing their opinions about a problem.
- Administrators, persons who are in charge of the system performance. They manage all the information concerning the problems, experts, and they check that the consensus process is developed properly.

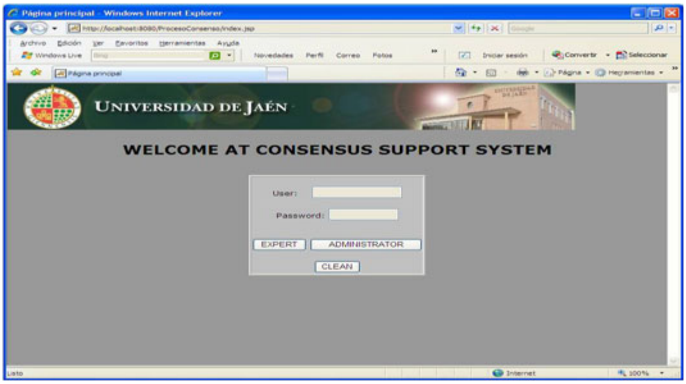


Fig. 8 Access to the system

The system identifies the user profile by means of the typical identification mechanism based on an user's name and password (see Fig. 8.) In the case of experts, the access codes are assigned by the administrator when experts' accounts are created.

4.2.2 Gathering Information

The information about the problems (description, set of alternatives, consensus threshold, ...) is exclusively inserted and/or updated by the administrator such as it is depicted in the Fig. 9. This information is requested by the system when the administrator creates the problem. Note that the information may be changed easily

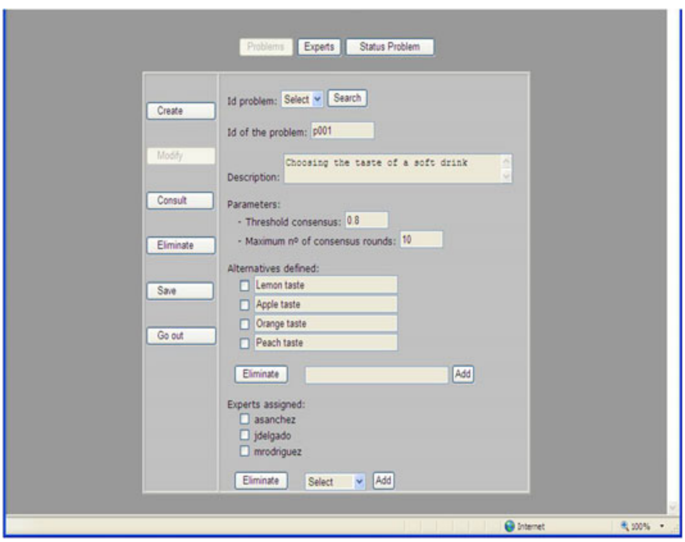


Fig. 9 Information and characteristic about a problem

because all parameters are edited. For example, the administrator can add new alternatives (button Add below list of alternatives) or change the consensus threshold. In this way, the system is more flexible and able to adapt to new situations without having to introduce all the information again.

From the point of view of the experts' opinions, once the expert has been identified by the system, this shows a empty preference relation to be filled out with the expert's preferences.

4.2.3 Computing the Consensus Degree

Once the preferences have been inserted by the experts, the administrator can compute the consensus degree. The result of this action is shown in the Fig. 10 and Fig. 11:

As we can see in the Fig. 10, the system shows the following information:

- General information on the problem.
- Status of the problem. Two status are possible: *progress*, when the consensus process is open or *finished* when the process is over.
- Consensus Matrix. We can see the consensus values on each pair of preferences.
- Consensus degree achieved in each consensus round.
- List of experts and number of changes of preferences (see Fig. 11).
- Preferences given by each expert and round.

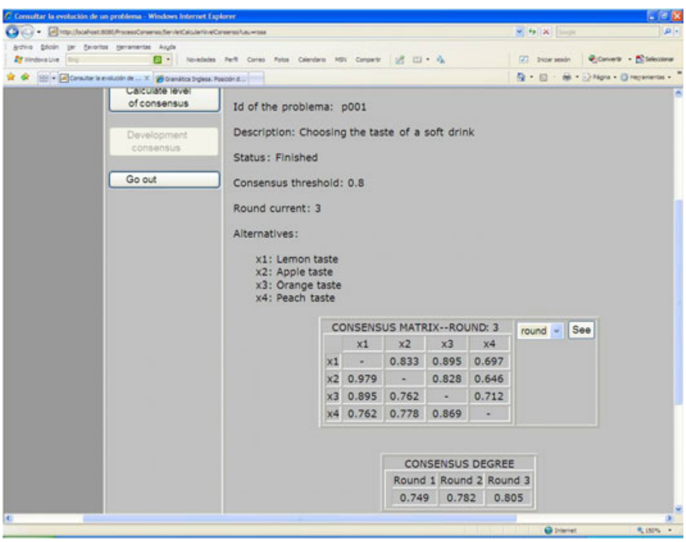


Fig. 10 Status of the consensus process

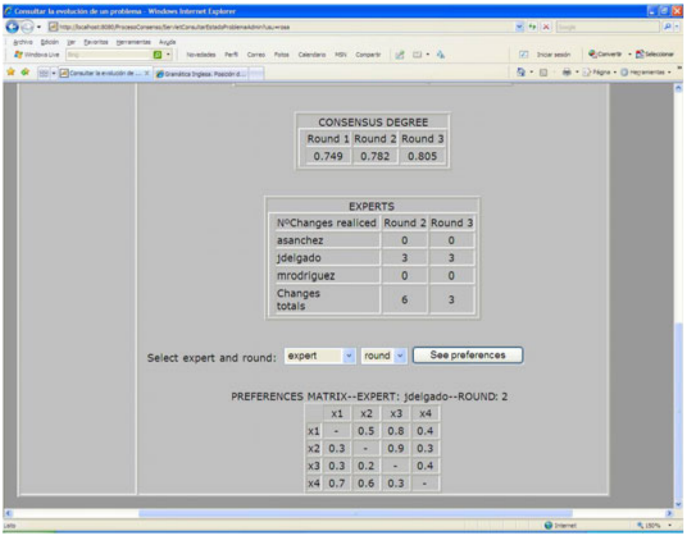


Fig. 11 Status of the consensus process: Expert's preferences

This information allows administrator to know with accuracy the evolution of the consensus reaching process and to accomplish a sensibility analysis in order to extract conclusions or relationship among the system parameters, preferences and changes. The information is stored in the database and can be consulted in the future to study the details of each problem. Note that all mathematic operations to unify the heterogenous information and to compute the consensus degree are transparent for experts and administrator.

4.2.4 Checking the Agreement

If the consensus process progresses in the right way, the level of agreement is achieved and the system shows a message reporting about the success of the process (see Fig. 12). Otherwise, the system shows other message reporting about the failure of the consensus process because the maximum number of consensus rounds has been exceeded.

4.2.5 Generating Advice

If the level of agreement is not high enough, each expert may know the changes suggested on his/her preferences in order to increase the agreement in the next consensus round. Before applying the changes, the expert may consult the current state of the problem (see Fig. 13). This window shows information about the number of rounds, current consensus degree, preferences given in each round, etc.

To help expert about the right direction of the changes, the system shows the window of the Fig. 14. As we can see, on the one hand we have the expert's preferences given in the last consensus round and by means of a color code the system

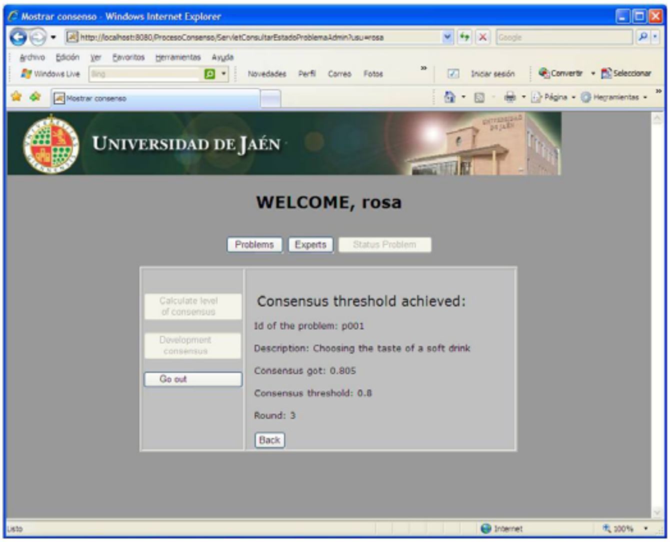
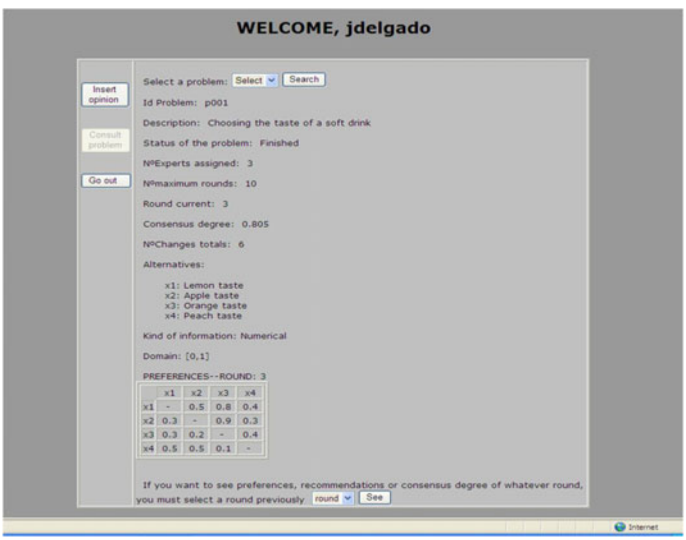


Fig. 12 Consensus has been achieved



	x1	x2	x3	x4
x1	-	0.5	0.8	0.4
x2	0.3	-	0.9	0.3
x3	0.3	0.2	-	0.4
x4	0.5	0.5	0.1	-

Fig. 13 Information about the problem

indicates the direction of the changes. So, values in red color have to be decreased while values in green color have to be increased. The rest of preferences have not to be modified. On the other hand, at the bottom of the window, the system shows a preference relation where the preferences to be changed are empty. In this way the system allows expert to insert new assessments which will be taken into account for the next consensus round.

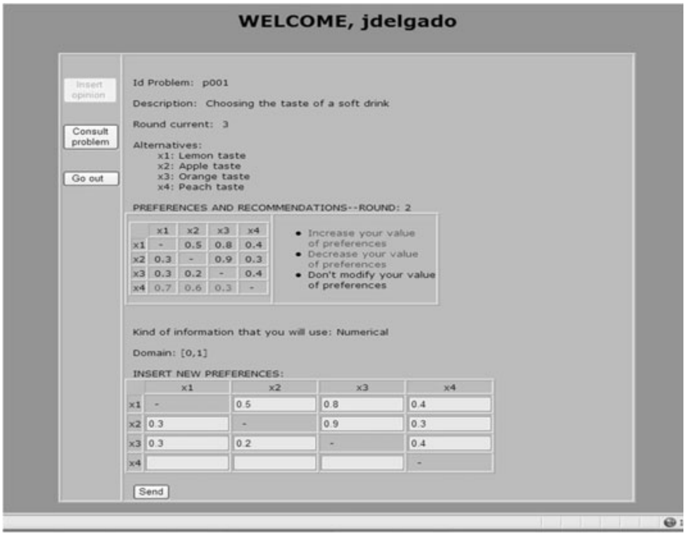


Fig. 14 Changes suggested by the system

Finally, note that the system has a wide repository of warning messages about possible errors (identifiers already used, empty fields of the forms) which makes easy the use of the application.

5 Conclusions

In this chapter we have shown a web consensus support system to automate the consensus processes in GDM problems with heterogeneous information. This WCSS uses a methodology based on transformation functions proposed by Herrera et al. [14] to unify the heterogeneous information. Different approaches to consensus process have been proposed in the literature but very few of them have been implemented as real software applications. Here we present a novel software to carry out virtual consensus reaching processes in which the experts may be situated in different places. Three main features may be highlighted of the system: i) it automatizes virtual consensus reaching process without the direct intervention of human moderator, ii) experts may use numerical, interval-valued and linguistic assessments and iii) it is a multi-platform system able to run on any computer and operating system. The system has been developed following the object-oriented paradigm and uses open source software tools. This system may be seen as a transfer of knowledge from the theoretical research about the consensus process in GDM to a practical development.

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