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FLINTSTONES: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions

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ABSTRACT

Uncertainty in real world decision making problems not always has probabilistic nature, in such cases the use of linguistic information to model and manage such an uncertainty has given good results. The adoption of linguistic information implies the accomplishment of processes of computing with words to solve linguistic decision making problems. In the specialized literature, several computational models can be found to carry out such processes. However, there is a shortage of software tools that develop and implement these computational models. The 2-tuple linguistic model has been widely used to operate with linguistic information in decision problems due to the fact that provides linguistic results that are accurate and easy to understand for human beings. Furthermore, another advantage of the 2-tuple linguistic model is the existence of different extensions to accomplish processes of computing with words in complex decision frameworks. Due to these reasons, in this paper a fuzzy linguistic decision tools enhancement suite so-called *Flintstones* is proposed to solve linguistic decision making problems based on the 2-tuple linguistic model and its extensions. Additionally, the *Flintstones* website is also presented, this website has been deployed and includes a repository of case studies and datasets for different linguistic decision making problems. Finally, a case study solved by *Flintstones* is illustrated in order to show its performance, usefulness and effectiveness.

1. Introduction

Decision making processes are one of the most frequent mankind activities in daily life. In order to solve decision making problems, usually, human beings, experts, provide either their knowledge about a set of different alternatives in a given activity to make a decision by means of reasoning processes [2,14,29,34,48,49,58]. Generally, the modeling of such knowledge by linguistic information in decision making is motivated because these situations are defined under uncertainty that has a non-probabilistic nature. In such cases, experts feel more comfortable providing their knowledge by using terms close to human beings cognitive model. Fuzzy logic and fuzzy linguistic approach provide tools to model and manage such a uncertainty by means of linguistic variables [67], improving the flexibility and offering reliability of the decision models in different fields [12,16,33,61].

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Fig. 1. CWW paradigm.

The use of linguistic information involves the need to operate with linguistic variables. Computing with Words (CWW) is a paradigm based on a procedure that emulates human cognitive processes to make reasoning processes and decisions in environments of uncertainty and imprecision [68]. In this paradigm the objects of computation are words or sentences from a natural language and results are also expressed in a linguistic expression domain that, usually, corresponds to the initial linguistic domain. To do so, a computational scheme (see Fig. 1), which includes a translation phase and a retranslation phase, has been defined in such paradigm [18,40,42,64].

The linguistic preference modeling in decision making can be managed by means of CWW processes. However, there are some decision situations that define complex frameworks in which to carry out CWW processes could be not enough. These complex frameworks are briefly detailed below:

- *Heterogeneous frameworks*: Decision problems where each expert may express his/her assessments in different expression domains, depending on the level of knowledge, experience or the nature of criteria that characterized the set of alternatives. Therefore, the assessments are expressed with non-homogeneous information such as, numerical, interval or linguistic [26,32,46,50].
- *Multi-granular linguistic frameworks*: Decision problems with multiple experts or multiple criteria in which appear linguistic information assessed in multiple linguistic term sets with different granularity. Therefore, the assessments of the problem are represented in multiple linguistic scales [5,13,21,25,28].
- *Unbalanced linguistic frameworks*: Decision problems in which it is necessary to assess preferences with a greater granularity on one side of the linguistic scale regarding the another one. Hence, linguistic terms of the scale are neither uniformly nor symmetrically distributed. Therefore, experts express their assessments in an unbalanced linguistic scale [1,4,22,57].

Different linguistic computational models for decision making have been introduced in the literature [8–10,59,63]. However, the 2-tuple linguistic model [23,24] has been compared with them and it has been showed as the most appropriate model in linguistic decision making, considering the computing with words paradigm [24,52]. The main advantage of the 2-tuple linguistic model is its computational model that offers linguistic results in the original linguistic domain in a precise way.

Furthermore, the 2-tuple linguistic model has been extended to perform processes of CWW in complex decision frameworks [13,21,22,25,26,39] and have been successfully applied in different fields such as sustainable energy [15], recommender systems [51], sensory evaluation [16,38], personnel selection [30], quality of service [17], performance appraisal [12], vendor selection problem [3], soft consensus [27,47,69] or software project selection [71]. Given that the 2-tuple linguistic model and its extensions keep the CWW scheme showed in Fig. 1, together with its own features and extensions make of it a flexible and adaptable model to solve decision making problems in all type of decision frameworks.

Notwithstanding there are many linguistic computational models and a lot of applications solved by using them, there is a lack of software tools to solve linguistic decision problems carrying out CWW processes. In [36] was proposed *Decider*, a linguistic decision support system that develops and implements a fuzzy multicriteria group decision making method. *Decider* can deal with complex decision frameworks and has been applied to different evaluation problems [33,35,55,70]. To do so, *Decider* uses a method that unifies the information into triangular fuzzy numbers, which are aggregated to obtain a closeness coefficient for each alternative. This method considers the distance measure between the fuzzy group assessment of each alternative and both a group ideal solution and a group negative ideal solution. The weakness of *Decider* is that the proposed method provides closeness coefficients expressed in the unit interval and, therefore, cannot be considered inside of the CWW paradigm (see Fig. 1). Due to this fact, the proposed method lacks the retranslation phase and computed closeness coefficients cannot be easily interpreted.

In [6] was proposed *jFuzzyLogic*[1] that is an open source Java library which offers a fuzzy inference system. Although the library has been extended to handle decision problems with linguistic information by means the linguistic 2-tuple model and some extensions, this library is far from being a complete tool focused on solving decision problems with linguistic and complex frameworks.

Beyond the scope of linguistic decision making, three interesting software tools can be found. First, *DECERNS* (Decision Evaluation in Complex Risk Network Systems) [66] that is a web-based spatial decision support system for multi-criteria analysis of a wide range of spatially-distributed alternatives. Second, the *decision deck project* [41] that offers open source software tools which develops multicriteria decision aid techniques to support complex decision aid processes.

[1] https://salty.unice.fr/wiki/salty-public/Deliverables.

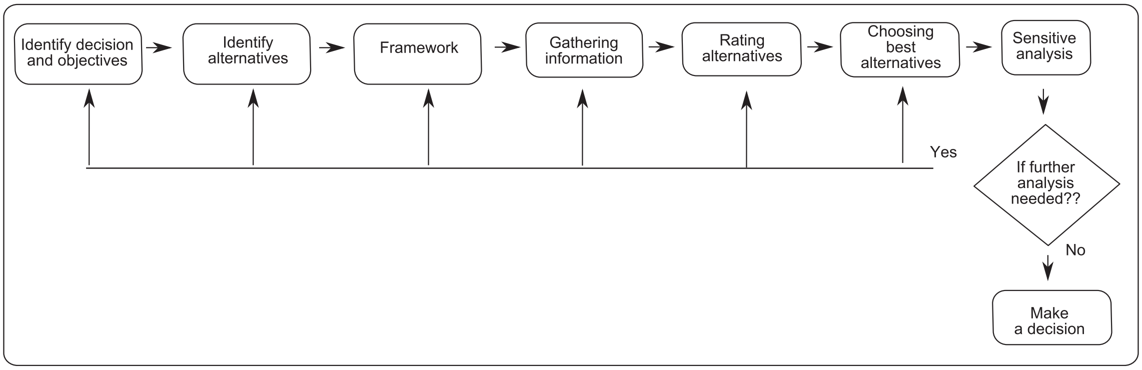


Fig. 2. Decision resolution scheme.

Finally, *LINGO*[2] is a comprehensive tool designed to help build and solving linear, nonlinear, and integer optimization models,

In spite of different software tools to deal with decision and linguistic decision problems, it is clear that there is a lack of software tools within the CWW paradigm to solve these decision problems. The aim of this paper is to present a decision software suite called *Fuzzy LINguisTic deciSion TOols eNhancEment Suite* (*Flintstones*),[3] based on the 2-tuple linguistic model and its extensions in order to solve decision problems defined in linguistic and complex frameworks, offering linguistic results that facilitate their understandability. Furthermore, not only this paper introduces a linguistic decision software but also presents the *Flintstones* website in which different releases can be download together with a repository of case studies and datasets for different decision making problems with linguistic and complex frameworks that can be solved by using *Flintstones*. In order to show its performance, usefulness and effectiveness, a case study defined in a multi-granular linguistic framework is solved step by step by *Flintstones*.

The paper is structured as: Section 2 provides a revision about the decision scheme as well as the foundations of the 2-tuple linguistic model and the different extensions based on this model for CWW in complex frameworks. Section 3 presents the linguistic decision method implemented and developed by *Flintstones* as well as its architecture and the technologies used. In Section 4, the website of the proposed software suite that includes a repository of case studies and datasets is presented as well as the resolution of a case study with *Flintstones*. Finally, in Section 5, conclusions and future works are pointed out.

2. Preliminaries

In this section, we first provide a brief revision of a general decision scheme and the use in it of linguistic information that will be adapted by *Flintstones*. We then provide a review of the foundations of the 2-tuple linguistic model and its extensions to solve linguistic decision making problems with linguistic and complex frameworks.

2.1. Decision scheme

In [7] was proposed a common decision resolution scheme that has been adapted or extended according to the needs of the decision situations [20,49,54]. The common decision resolution scheme consists of following eight phases [7] (see Fig. 2):

- *Identify decision and objectives*.
- *Identify alternatives*.
- *Framework:* The structure and elements of the decision problem are defined: experts, criteria, etc.
- *Gathering information:* The information provided by experts is collected, according to the defined framework.
- *Rating alternatives:* The gathered information provided by experts is aggregated to obtain a collective value for each alternative. Therefore, in this phase, it is necessary to carry out a solving process in order to compute the collective assessments for the set of alternatives, using appropriate aggregation operators. Other authors call this phase *aggregation phase* [54].
- *Choosing the alternative/s:* Normally, the highest collective assessment corresponds to the best alternative [31] that is selected to solve the decision making. To do so, it may use a choice function that assigns a choice degree for each alternative [19]. This phase is also called *exploitation phase* [54].

[2] http://www.lindo.com/.
[3] http://serezade.ujaen.es/flintstones/.

- *Sensitive analysis*: The information computed is analyzed. If the information is not good enough to make a decision, it is necessary to return to the previous phases in order to make a depth analysis.
- *Make a decision*: The information obtained from the previous decision analysis can be used to make a decision.

The common decision resolution scheme may include linguistic information to model and manage decision situations under non-probabilistic uncertainty. This fact implies the need to operate with linguistic terms in order to compute linguistic assessments for alternatives, according to the CWW scheme (Fig. 1).

In line with our aims in this paper, the 2-tuple linguistic model and its extensions will be implemented by *Flintstones* due to the fact that provide adequate computational models to deal with linguistic information in decision problems defined in linguistic and complex frameworks.

2.2. 2-Tuple linguistic representation model and its extensions

In this section, the representation and computational models for 2-tuple values are reviewed. A brief revision of extensions of the 2-tuple linguistic model to carry out CWW processes in complex frameworks is then provided.

2.2.1. 2-Tuple linguistic representation model

This model was presented in [23] to avoid the loss of information and improve the precision in processes of CWW when the linguistic term set has an odd value of granularity, being triangular-shaped, symmetrical and uniformly distributed its membership functions.

The 2-tuple linguistic model represents the information by means of a pair of values (s, α), where s is a linguistic term with syntax and semantics, and α is a numerical value that represents the *symbolic translation*. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a numerical value in its interval of granularity.

Definition 1 [23]. The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the difference of information between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $S = \{s_0, \ldots, s_g\}$ which indicates the index of the closest linguistic term in S.

This model defines a set of functions to facilitate the computational processes with 2-tuple linguistic values [23].

Definition 2 [23]. Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. The function $\Delta_S : [0, g] \longrightarrow \langle S \rangle$, is defined by:

$$\Delta_S(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = round(\beta), \\ \alpha = \beta - i, \end{cases} \tag{1}$$

where $round(\cdot)$ assigns to β the integer number $i \in \{0, 1, \ldots, g\}$, closest to β.

Proposition 1. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple linguistic value. There is always a function Δ_S^{-1} such that from a 2-tuple linguistic value, it returns its equivalent numerical value $\beta \in [0, g]$ as $\Delta_S^{-1}(s_i, \alpha) = i + \alpha$.*

Remark 1. It is obvious that the conversion of a linguistic term into 2-tuple linguistic value consists of adding a value 0 as symbolic translation.

2.2.2. 2-Tuple linguistic computational model

The 2-tuple linguistic representation model has a linguistic computational model associated based on Δ_S^{-1} and Δ_S in order to accomplish CWW processes in a precise way:

- *Comparison of 2-tuple linguistic values*. The comparison of linguistic information represented by 2-tuple linguistic values is carried out according to an ordinary lexicographic order.
  Let (s_k, α_1) and (s_l, α_2) be two 2-tuple linguistic values, each one representing a counting of information.
  - If $k < l$, then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
  - If $k = l$, then
    1. If $\alpha_1 = \alpha_2$, then (s_k, α_1) and (s_l, α_2) represent the same information.
    2. If $\alpha_1 < \alpha_2$, then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
    3. If $\alpha_1 > \alpha_2$, then $(s_k, \alpha_1) > (s_l, \alpha_2)$.

- *Negation operator of a 2-tuple linguistic value*. The negation operator over a 2-tuple linguistic value is defined as: $Neg(s_i, \alpha) = \Delta_S(g - (\Delta_S^{-1}(s_i, \alpha)))$, being $g + 1$ the cardinality of S.
- *2-Tuple linguistic aggregation operators*. The 2-tuple linguistic aggregation operator consists of obtaining a value that summarizes a set of 2-tuple linguistic values. Therefore, the result of an aggregation process of a set of 2-tuple linguistic values must be a 2-tuple linguistic value. Several 2-tuple linguistic aggregation operators have been proposed in the literature [23,44,45,60,62,65].

2.3. 2-Tuple linguistic model extensions for linguistic complex frameworks

Decision making situations under uncertainty can define linguistic complex frameworks (multi-granular linguistic, heterogeneous, unbalanced linguistic) that need more than just a linguistic domain to model all information involved in the decision problem. In such contexts, the extensions of the 2-tuple linguistic model can perform processes of CWW in these complex frameworks, obtaining satisfactory results in linguistic decision problems.

These extensions follow the CWW paradigm and share a common process for rating alternatives in the decision scheme in a proper way (see Fig. 3). A further detailed overview of these extensions can be found in [39]. To facilitate the understanding of the decision solving methods in *Flintstones*, in the following subsections are briefly reviewed the different 2-tuple linguistic extensions designed to deal with complex decision frameworks.

2.3.1. Multi-granular linguistic frameworks

Usually, in decision situations with multiple criteria or several experts, the preferences are expressed in multiple linguistic term sets with different granularity, considering the imprecision and uncertainty of the related information. These decision situations define a multi-granular linguistic framework [5,28] and require an adequate solving process to manage such frameworks. Three extensions based on the 2-tuple linguistic model have been proposed to deal with multiple linguistic scales [13,21,25] that are reviewed below.

2.3.1.1. Fusion approach for managing multi-granular linguistic information. This extension was presented in [21] and provides a total flexible linguistic framework because it does not impose any limitation related with the granularity of each linguistic term set as well as the shape of the fuzzy membership functions of each linguistic term.

The description of this extension in the rating process is the following one (see Fig. 4):

1. *Unification process*. The multigranular information is unified into a specific linguistic domain called *Basic Linguistic Term Set* (BLTS) and noted as $S_{BLTS} = \{s_i, i = 0, \ldots, g\}$, which is selected with the aim of keeping as much knowledge as possible (see [21]). This BLTS might be a linguistic term set fixed in the framework with the condition that this set can be represented by a 2-tuple linguistic value. A linguistic term $s_l^k \in S^k$, such that $S^k = \{s_l, l = 0, \ldots, g_k\}$ and $g_k \leqslant g$, is unified into fuzzy sets in the BLTS by using the transformation function $\varphi_{S^k S_{BLTS}} : S^k \to F(S_{BLTS})$ defined as:

$$\varphi_{S^k S_{BLTS}}(s_l^k) = \sum_{i=0}^{g}(s_i/\gamma_i), \tag{2}$$

where $\gamma_i = \max_y \min\{\mu_{s_l}(y), \mu_{s_i}(y), i = 0, \ldots, g\}$.

2. *Aggregation process*. The information expressed in multiple linguistic scales has been unified into fuzzy sets in the BLTS. Therefore, in this process, the computations are directly carried out on fuzzy sets by using the fuzzy arithmetic [11].
3. *Retranslation process*. In this process, the results expressed into fuzzy sets, $F(S_{BLTS})$, are transformed into 2-tuple linguistic values in the BLTS by the function $\chi : F(S_{BLTS}) \to \langle S_{BLTS} \rangle$ that is defined as:

$$\chi\Big(\{(s_0, \gamma_0), (s_1, \gamma_1), \ldots, (s_g, \gamma_g)\}\Big) = \Delta_S\left(\frac{\sum_{i=0}^{g} i\gamma_i}{\sum_{i=0}^{g} \gamma_i}\right) = (s, \alpha) \in \langle S_{BLTS} \rangle. \tag{3}$$

It is noteworthy that the transformations in which fuzzy sets are involved can imply a lack of information. Therefore, the 2-tuple linguistic results obtained in the rating process can be inaccurate.

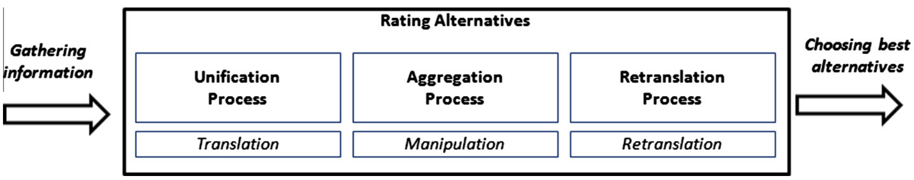


Fig. 3. Common schema in rating process of extensions based on the 2-tuple linguistic model.

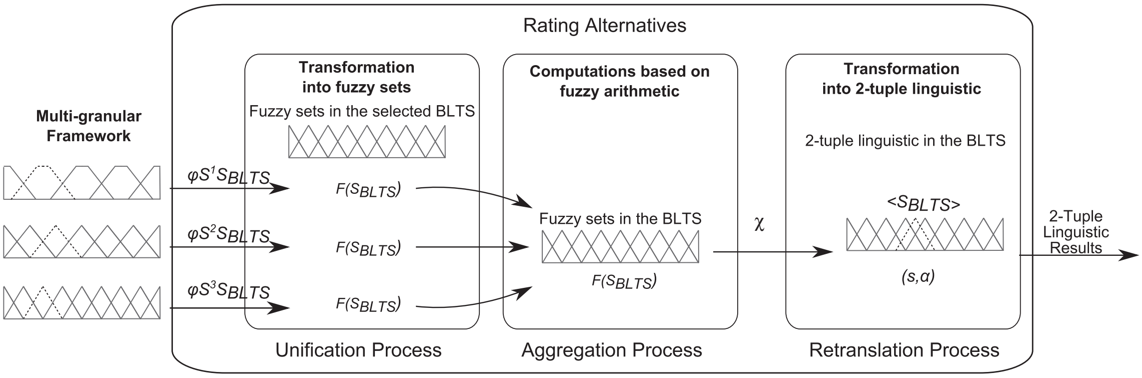


Fig. 4. Rating process in fusion approach for managing multi-granular linguistic information.

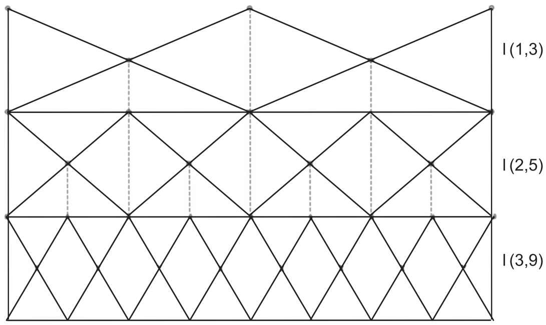


Fig. 5. Linguistic hierarchy with 3 levels.

2.3.1.2. Linguistic hierarchies. Linguistic hierarchies was proposed in [25] to overcome drawbacks related to the accuracy and the expression domain of the linguistic results of the previous extension [21].

A linguistic hierarchy is defined by the union of its levels $l(t, n(t))$. Each level t corresponds to a linguistic term set denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$ with a granularity of uncertainty of $n(t)$.

$$LH = \bigcup_t l(t, n(t)). \tag{4}$$

In order to ensure the accomplishment of the CWW processes without loss of information, each level must be a linguistic term set symmetrically and uniformly distributed and the level $t+1$ must keep the former modal points of the level t (see Fig. 5). To do so, the granularity of the level $t+1$ is obtained from the granularity of its predecessor as:

$$n(t+1) = (2 \cdot n(t)) - 1. \tag{5}$$

The rating process of this extension to operate with information expressed in multiple linguistic scales is described as follows (see Fig. 6):

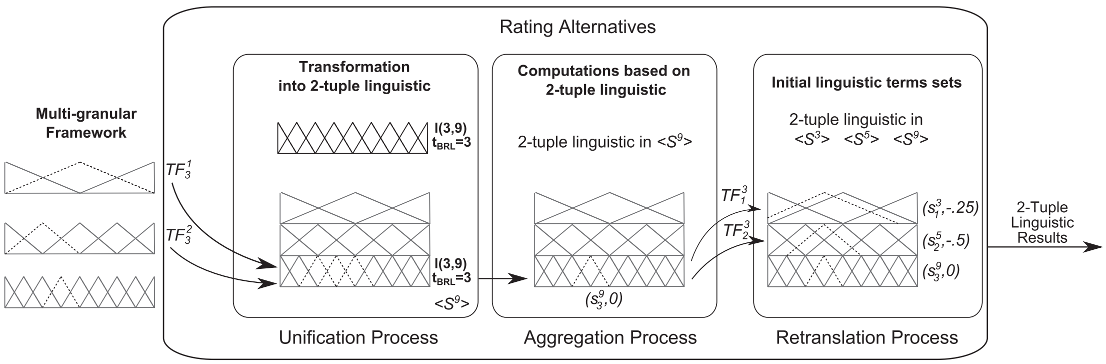


Fig. 6. Rating process in linguistic hierarchies.

1. *Unification process*. This extension defines the transformation function $TF^t_{t'} : \langle S^{n(t)} \rangle \rightarrow \langle S^{n(t')} \rangle$, designed for transforming linguistic terms from different levels in the *LH* in an accurate way as follows:

$$TF^t_{t'}(s_i^{n(t)}, \alpha) = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha) \cdot (n(t') - 1)}{n(t) - 1}\right). \quad (6)$$

   The linguistic information is unified into a single level of the *LH*, called *Basic Representation Level* and noted as t_{BRL}. This unification is carried out by means of Eq. (6) with $TF^t_{t_{BRL}}$.
2. *Aggregation process*. The information is expressed in 2-tuple linguistic values in the level t_{BRL}. Therefore, the 2-tuple linguistic computation model is carried out to performance CWW processes and to obtain linguistic results in $S^{(n(t_{BRL}))}$ [23].
3. *Retranslation process*. The 2-tuple linguistic results have been expressed in the unified level. These results can be expressed in each initial linguistic term set defined in the framework in a precise way by means of the transformation function defined by Eq. (6) with $TF^{t_{BRL}}_t$.

2.3.1.3. Extended linguistic hierarchies. This extension was proposed in [13] to offer more flexibility in the multi-granular linguistic framework than the extension based on LH [25], keeping accuracy in the CWW processes.

An extended linguistic hierarchy (ELH) does not impose any rule about the granularity of each linguistic term set that is included in the hierarchy and adds a last level that is noted as t^* to keep all the former modal points of the original linguistic scales (see Fig. 7).

The new level t^*, keeps all the information in CWW processes and its granularity is computing using the granularities of the initial linguistic term sets as follows:

$$n(t^*) = lcm(n(1) - 1, n(2) - 1, \ldots, n(m) - 1) + 1, \quad (7)$$

being *lcm* the *least common multiple* and *m* the number of initial linguistic scales. Therefore, an extended linguistic hierarchy is the union of original levels and the new generated level (see Fig. 7).

$$ELH = \left(\bigcup_{t=m}^{t=1} l(t, n(t))\right) \bigcup l(t^*, n(t^*)). \quad (8)$$

The rating process in this extension is similar to the extension dealing with LH, taking into account that the unified level in this case must be t^* (see Fig. 8) in order to ensure the CWW processes without loss of information.

1. *Unification process*. The information expressed in multiple linguistic scales is unified in the new level t^*, using the transformation function $ETF^t_{t^*} : \langle S^{n(t)} \rangle \rightarrow \langle S^{n(t^*)} \rangle$ without loss of information and defined by:

$$ETF^t_{t*} = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t)}, \alpha) \cdot (n(t^*) - 1)}{n(t) - 1}\right). \quad (9)$$

2. *Aggregation process*. The unified information has been represented in 2-tuple linguistic values in the new level t^*. So, the processes of CWW are carried out by using the 2-tuple linguistic computational model [23], obtaining 2-tuple linguistic results in $\langle S^{n(t^*)} \rangle$.
3. *Retranslation process*. In this process, the 2-tuple linguistic results are expressed in each original linguistic term set in a precise way by means of the transformation function $ETF^{t^*}_t$ given by:

$$ETF^{t*}_t = \Delta_S\left(\frac{\Delta_S^{-1}(s_i^{n(t^*)}, \alpha) \cdot (n(t) - 1)}{n(t^*) - 1}\right). \quad (10)$$

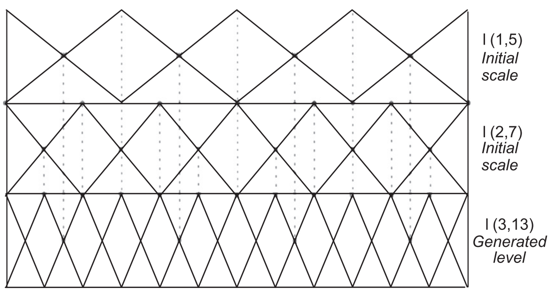


Fig. 7. Extended linguistic hierarchies with two initial levels.

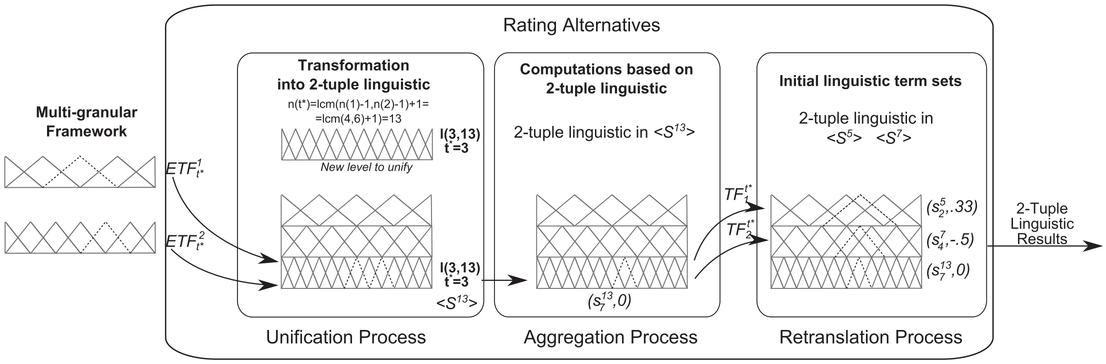


Fig. 8. Rating process in extended linguistic hierarchies.

2.3.2. Heterogeneous frameworks

Similarly to the previous framework, sometimes in decision making situations, experts with diverse background usually may express their preferences in different expression domains such as: numerical domain, interval-valued domain and linguistic domains, considering the imprecision and uncertainty of the related information as well as the nature of assessed criteria. Therefore, these situations define a heterogeneous framework that requires an adequate process to solve the decision problem.

2.3.2.1. Fusion approach for managing heterogeneous information. In [26] was presented an extension to deal with information expressed in different expression domains: numerical domain, interval-valued domain and any linguistic term set. This extension shares the operation of the fusion approach for managing multi-granular linguistic information (reviewed in Section 2.3.1.1), defining two new transformation functions to transform numerical values and interval values into fuzzy sets. Therefore, this extension provides a total flexible framework in which experts can express their preferences by means of different expression domains. The rating process with this extension to deal with heterogeneous frameworks is described as follows (see Fig. 9):

1. *Unification process*. The heterogeneous information is unified into a selected linguistic domain called *Basic Linguistic Term Set* (BLTS) and noted as $S_{BLTS} = \{s_i, i = 0, \ldots, g\}$ that is chosen with the aim of keeping as much knowledge as possible (see [26]). Hence, each assessment is transformed by using an adequate transformation function, according to its expression domain:
   (a) *Numerical domain*. When $v \in [0,1]$, a numerical transformation function $\varphi_{NS_{BLTS}} : ([0,1]) \rightarrow F(S_{BLTS})$ is applied by:

$$\varphi_{NS_{BLTS}}(v) = \sum_{i=0}^{g} (s_i/\gamma_i), \tag{11}$$

   where $\gamma_i = \mu_{s_i}(v) \in [0,1]$ is the membership degree of v to $s_i \in S_{BLTS}$.
   (b) *Interval domain*. When $v \in P([0,1])$, an interval transformation function $\varphi_{IS_{BLTS}} : P([0,1]) \rightarrow F(S_{BLTS})$, is applied by:

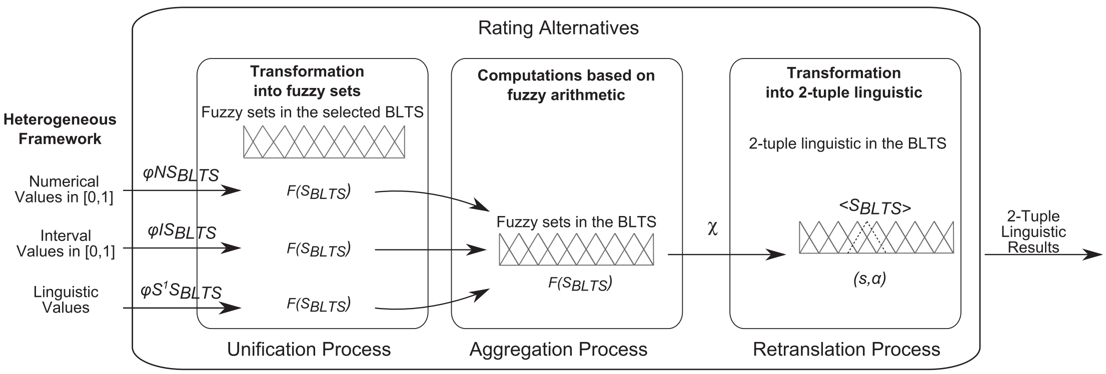


Fig. 9. Rating process in fusion approach for managing heterogeneous information.

$$\varphi_{IS_{BLTS}}(v) = \sum_{i=0}^{g}(s_i/\gamma_i), \tag{12}$$

where $\gamma_l^i = \max_y \min\{\mu_I(y), \mu_{s_l}(y)\}$, with $l = \{0, \ldots, g\}$, being $\mu_I(\cdot)$ and $\mu_{s_i}(\cdot)$ membership functions associated with the interval $I \in P([0,1])$ and the term $s_l \in S_{BLTS}$, respectively.

(c) *Linguistic domain*. When $v \in S^k$, such that $S^k = \{s_0^k, \ldots, s_{g_k}^k\}$ and $g_k < g$, the linguistic transformation function $\varphi_{S^k S_{BLTS}}$ is applied. This function was defined in Eq. (2).

2. *Aggregation process*. In the same way as the fusion approach for managing multi-granular linguistic information, the computations are directly carried out in fuzzy sets by using the fuzzy arithmetic [11].
3. *Retranslation process*. The aggregated results have been expressed in fuzzy sets in the BLTS, $F(S_{BLTS})$. In this process, these results are transformed into 2-tuple linguistic values by the function χ that was defined in Eq. (3). Therefore, the aggregated fuzzy sets are transformed into 2-tuple linguistic values in the BLTS.

As fusion approach for managing multi-granular linguistic information, the fusion approach for managing heterogeneous information can imply a lack of information in the transformations in which fuzzy set are involved.

2.3.3. Unbalanced linguistic frameworks

The unbalanced linguistic frameworks appear in decision situations under uncertainty when is necessary to assess preferences with a greater granularity on one side of the linguistic scale regarding the another one.

2.3.3.1. Fuzzy linguistic methodology to deal with unbalanced linguistic term sets. In [22] was presented a methodology based on LH [25] that provides an algorithm to represent the semantics of an unbalanced linguistic scale as well as a boolean function $Brid(S)$, which is used in CWW processes (see Fig. 10).

This algorithm builds the semantics for an unbalanced linguistic term set $\mathcal{S}$ with a $LH, LH(\langle\mathcal{S}\rangle) = \{(s_{I(i)}^{G(i)}, \alpha), i = \{0, \ldots, g\}, s_{I(i)}^{G(i)} \in \mathcal{S}\}$, being $I(i)$ the function that assigns the index of the label that represents its semantics in the LH and $G(i)$ the function that assigns to each label the granularity of the level in which it is represented.

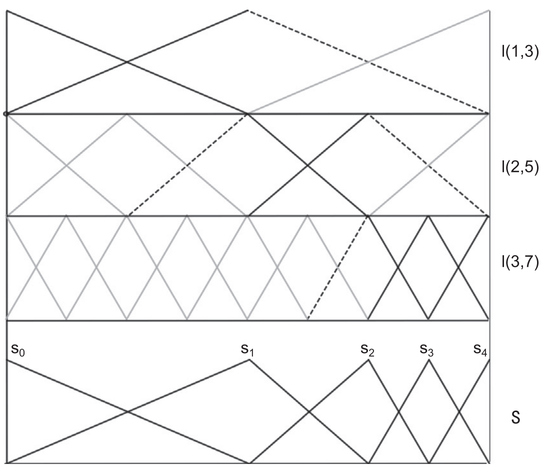
The description of this methodology in the rating process is the following (see Fig. 11):

1. *Unification process*. The semantics of the unbalanced linguistic term set $LH(\langle\mathcal{S}\rangle)$ belongs to different levels of the LH. In this process, the information expressed in the unbalanced linguistic scale is unified at a unified level t_{BRL}. To do so, first, each unbalanced term is transformed into 2-tuple linguistic value in its respective level of the LH by means the transformation function $\mathcal{LH} : \langle\mathcal{S}\rangle \rightarrow LH(\langle\mathcal{S}\rangle)$ defined as:

$$\forall (s_i, \alpha) \in (\mathcal{S} \times [-0.5, 0.5)) \Rightarrow \mathcal{LH} : (s_i, \alpha) = (s_{I(i)}^{G(i)}, \alpha). \tag{13}$$

Second, linguistic terms expressed in different levels of the LH are unified into the unified level t_{BRL}, by using the transformation function $TF_{t_{BRL}}^{t'}$ that was defined in Eq. (6) with $l(t', G(i))$.

2. *Aggregation process*. In this step, the 2-tuple linguistic computation model is performed [23], obtaining 2-tuple linguistic results expressed in $\langle S^{(n(t_{BRL}))}\rangle$.



S	$LH(S)$	$Brid(S)$
$s_0 = Poor$	$s_{I(0)}^{G(0)} = s_0^3$	False
$s_1 = Average$	$s_{I(1)}^{G(1)} = s_1^3$ or $s_{I(1)}^{G(1)} = s_2^5$	True
$s_2 = Good$	$s_{I(2)}^{G(2)} = s_3^5$ or $s_{I(2)}^{G(2)} = s_6^9$	True
$s_3 = Very\ Good$	$s_{I(3)}^{G(3)} = s_7^9$	False
$s_4 = Excellent$	$s_{I(4)}^{G(4)} = s_8^9$	False

Fig. 10. Unbalanced linguistic scale and the table with the related information.

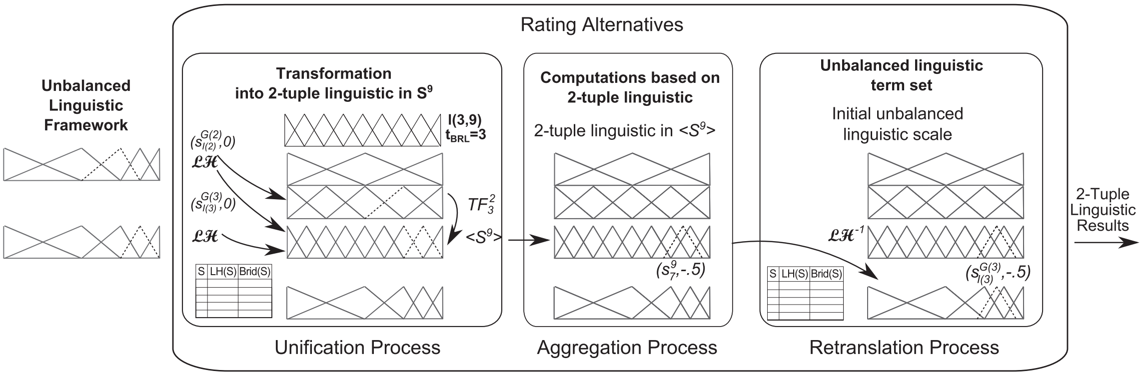


Fig. 11. Rating process with the fuzzy linguistic methodology to deal with unbalanced linguistic term sets.

3. *Retranslation process*. In this process, 2-tuple linguistic results expressed in the unified level are transformed in the unbalanced linguistic scale by means of the transformation function showed in Eq. (14). This transformation function was defined by cases in [22] and is based on the satisfaction of conditions imposed on $LH(\langle\mathcal{S}\rangle)$ and the boolean function $Brid(S)$.

$$\mathcal{LH}^{-1} : \langle S^{t_{BRL}}\rangle \rightarrow \langle\mathcal{S}\rangle. \tag{14}$$

Therefore, 2-tuple linguistic results are expressed in the initial unbalanced linguistic scale.

3. FLINTSTONES: a fuzzy linguistic decision tools enhancement suite

Once the basics have been reviewed, this section introduces a novel decision tools suite called *Flintstones* to solve decision making problems under uncertainty by using the 2-tuple linguistic model and its extensions. To do so, we first present the linguistic decision method that is implemented and developed by *Flintstones*. We then present the architecture and technologies of the software suite.

3.1. Linguistic decision method for Flintstones based on the 2-tuple linguistic model and its extensions

Our aim is to develop a software suite of tools to solve linguistic decision making problems based on the 2-tuple linguistic model and its extensions, dealing with linguistic and complex frameworks. Hence, it is necessary to propose a specific linguistic decision solving method that includes the selection of a solving process based on the 2-tuple linguistic model, according to the framework in which the decision problem is defined.

Such a general linguistic decision scheme is presented in Fig. 12 and it has been adapted from the common decision scheme that was showed in Fig. 2. Following, the proposed decision method is described in further detail, showing the operation of each phase in *Flintstones*:

3.1.1. Framework

This phase defines the framework that includes the elements involved in the decision making problem:

- A finite set of alternatives $X = \{x_1, \ldots, x_h\}$.
- A set of criteria $C = \{c_1, \ldots, c_n\}$ that might be grouped.
- A set of experts $E = \{e_1, \ldots, e_m\}$ that could also be grouped. The set of experts will provide the assessments of the decision problem.

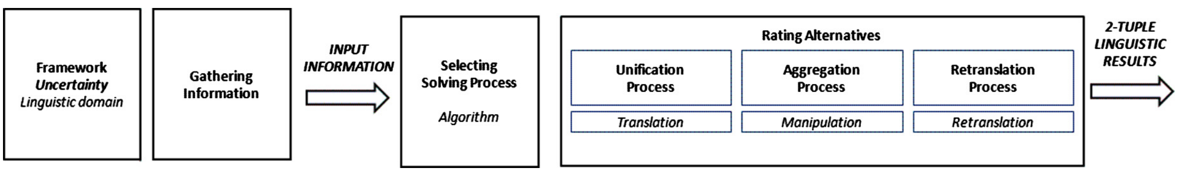


Fig. 12. Linguistic decision method based on the 2-tuple linguistic model and its extensions.

- The set of expression domains $\mathcal{F}$ in which the assessments provided by experts will be expressed, allowing the definition of the following domains of expression: numerical domain (*num*), interval domain (*int*), linguistic domain (*lin*), and an unbalanced linguistic domain defined in a linguistic hierarchy (*linUnb*).
- Fix expression domains that will be used by experts, according to the uncertainty and the nature of criteria as well as the background of each expert.

3.1.2. Gathering information

In this phase, each expert e_k, provides the assessments by means of assessment vectors

$$U^k = (v_{ij}^k : i = 1 \dots n,\ j = 1, \dots, m : v_{ij}^k \in \mathcal{F}).$$

The assessment v_{ij}^k, provided by each expert e_k, for each criterion c_i of each alternative x_j, is expressed in the allocated expression domain in the framework $\mathcal{F}$.

3.1.3. Selecting a solving process

A decision solving process based on the 2-tuple linguistic representation model should be selected to carry out the rating process in the next phase. The selection of the suitable solving process is based on expression domains defined in the framework and the way to obtain the linguistic results. In a decision making problem with a single scale in which its linguistic terms are uniform and symmetrically distributed, the *2-tuple linguistic computational model* will be carried out in the rating process. However, if the decision problem facing a linguistic complex context, the algorithm selects *the fusion approach for managing heterogeneous information* when the information involved in the problem is expressed with non-homogeneous information. In the decision situations in which the information is expressed in an unbalanced linguistic scale built from a linguistic hierarchy, the *fuzzy linguistic methodology to deal with unbalanced linguistic term sets* will be chosen.

There are three extensions to deal with multi-granular linguistic contexts. On the one hand, *the fusion approach* always can be applied due to the fact that it offers a total flexible framework. However, this extension can provide linguistic results with a lack of accuracy. For this reason, when another extension to deal with multi-granular linguistic context can be applied, this will be selected in order to ensure accurate linguistic results. On the other hand, the extension based on *extended linguistic hierarchies* is a generalization of the extension based on *linguistic hierarchies* that includes a new level in the hierarchy to unify the multi-granular information and accomplish the CWW processes. In order to reduce the computations, the linguistic hierarchies will be selected when the multi-granular linguistic context allows its application.

Therefore to select the right solving process it has been developed an algorithm that requires the following input information about the framework:

- $edNum \in \{True, False\}$ indicates if a numerical expression domain *num* was defined in $\mathcal{F}$.
- $edInt \in \{True, False\}$ establishes if an interval expression domain *int* was defined in $\mathcal{F}$.
- $edLinUnb \in \{True, False\}$ determines if an unbalanced linguistic domain defined with a *linguistic hierarchy linUnb* was fixed in $\mathcal{F}$.
- $tamEdLinLis \in \mathbb{N}^+$ defines the number of linguistic scales established in the set of expression domains $\mathcal{F}$.
- $edLin = \{card, 2T\}$ describes a linguistic domain *lin* fixed in the framework that is characterized by two values. First, $card \in \mathbb{N}^+$ that indicates the cardinality of the linguistic domain and, the second value, $2T \in \{True, False\}$ that establishes if the linguistic domain can be represented by 2-tuple linguistic values, i.e., the linguistic term set has an odd value of granularity and whose membership functions are triangular-shaped, symmetrically and uniformly distributed in the unit interval.
- $edLinList = \{edLin_i; i = 1, \dots, tamEdLinLis\}$ is a vector of *edLin* that provides information about the linguistic scales established in $\mathcal{F}$.

Algorithm 1 illustrates the proposed procedure to select the suitable solving process for the decision making problem. The algorithm selects the $id \in \{1, 2, 3, 4, 5, 6\}$ that identifies the solving process according to the following references:

1. *2-Tuple linguistic computational model.*
2. *Fusion approach for managing multi-granular linguistic information.*
3. *Linguistic hierarchies.*
4. *Extended linguistic hierarchies.*
5. *Fusion approach for managing heterogeneous information.*
6. *Fuzzy linguistic methodology to deal with unbalanced linguistic term sets.*

Algorithm 1. Algorithm to select the suitable solving process

```
Require: edNum, edInt, edLinUnb, tamEdLinLis, edLinList
Ensure: id
if (edLin[1].2T = true) and (tamEdLinLis = 1) then
   return 1
else if (edNum = true) or (edInt = true) then
   return 5
else if (edLinUnb = true)
   return 6
else
   edLinListShortCard ⇐ short (edLinList,edLinList.card)
   i ⇐ 1
   while i < tamEdLinLis do
      if (edLinListShortCard.edLin[i].2T = false) then
         return 2
      else if (edLinListShortCard[i+1].card ≠ ((edLinListShortCard[i].card)-1)·2+1) then
         return 4
      else
         i ⇐ i + 1
      end if
   end while
   return 3
end if
```

3.1.4. Rating alternatives

The aim of this phase is to obtain a 2-tuple linguistic global assessment for each alternative that is easily interpreted. Taking into account the previous phase, a linguistic assessment for each alternative is computed, using the selected solving process that allows to manage the information expressed in the decision framework. Each solving process follows the common scheme with the following three processes (see Fig. 12):

1. *Unification process*. Decision making problems under uncertainty can be defined in linguistic complex frameworks (multi-granular linguistic, heterogeneous or unbalanced linguistic). As was reviewed in Section 2.3, to manage these frameworks, the extensions based on the 2-tuple linguistic model represent the gathered information using different expression domains. Therefore, the first process is to represent the gathered information into a unified domain.
2. *Aggregation process*. In this second process, the unified information is aggregated selecting aggregation operators for the unified domain in order to obtain a global assessment for each alternative that summarizes its gathered information.
3. *Retranslation process*. A retranslation process is needed to express the global assessment for each alternative in a linguistic expression domain that can be easily interpreted by experts, keeping the CWW scheme (see Fig. 1).

3.2. Architecture and technologies

It is important to show the architecture of *Flintstones* and the technologies used in the software suite of tools. *Flintstones* has been developed using the Rich Client Platform (RCP)[4] that provides a framework to build desktop applications, client applications, with rich functionality.

The main value of RCP is that allows to quickly develop professional applications with native look-and-feel on multiple platforms. Another advantage of RCP is that its components have a high quality and are actively maintained.

Four modules have been built in *Flintstones* (see Fig. 13) in order to separate the decision scheme, the solving processes, the set of aggregation operators and a series of interface representation elements. The details of each type of basic module are described below.

- *Libraries* provide structures and procedures with the aim of supporting the resolution of the decision problem. These libraries include elements such as: experts, criteria, alternatives, assessments and expression domains.
- *Graphical User Interface* (GUI) that allows users to interact with the software suite of tools.
- *Methods* that develop the 2-tuple linguistic computational model and its extensions in order to solve decision making problems with linguistic and complex contexts.

[4] http://www.eclipse.org/home/categories/rcp.php.

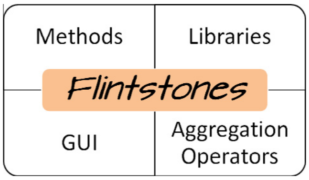


Fig. 13. Modules in *Flintstones*.

- *Operators* implement the set of aggregation operators that can be used to aggregate the information involved in the decision problem. This module includes the most popular aggregation operators: *maximum, minimum, median, arithmetic mean, weighted average and ordered weighted average.*

Flintstones offers several advantages based on its architecture and technologies. The strongest points of the software suite are:

- *Flintstones* has been developed with an RCP based on Java. Therefore, this suite of tools can be used on any machine with Java Virtual Machine (JVM), independently of its operating system.
- The software suite is divided in four separated modules. Due to this fact, it is possible to upgrade the software suite just by making changes in a particular module.
- The structure of *Flintstones* is ready to include new aggregation operators as well as new solving processes in a fast and simple way. As a result, it reduces programming task because it offers the *Libraries module* and *Graphical User Interface module* that includes a full structure, simplifying the integration of new aggregation operators and new extensions based on the 2-tuple linguistic model.

4. *Flintstones* website: case studies and datasets

The development of a software suite of tools is very important but it is not enough if users cannot use it to verify its performance with real datasets in order to make comparisons with either their own proposals or problems. Therefore, we have

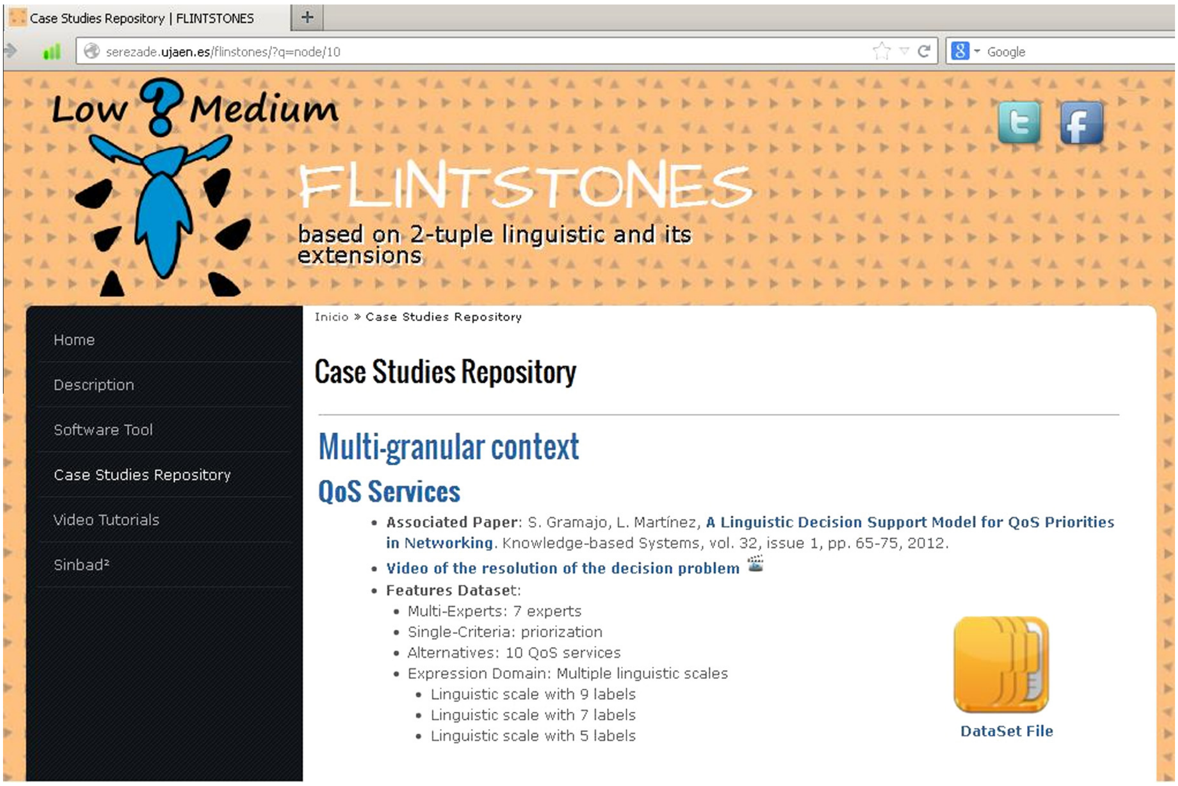


Fig. 14. *Flintstones* website.

Table 1
Repository of case studies and datasets.

Framework	Extension	Application	Year
Multi-granular linguistic	Fusion for multi-granular information	Decision making [13]	2011
	Linguistic hierarchies	Decision making [25]	2001
		Sensory evaluation [38]	2008
	Extended linguistic hierarchies	Decision making [13]	2011
		Quality of services in networking [17]	2012
Heterogeneous	Fusion for heterogeneous information	360-Degree performance appraisal [12]	2013
		Sustainable energy evaluation [15]	2012
		ERP evaluation processes [56]	2009
Unbalanced linguistic	Methodology to deal with unbalanced scales	Sensory evaluation [37]	2009
		Decision making [22]	2008

not only developed *Flintstones* as a decision software suite but also we have deployed the *Flintstones* website[5] that includes a repository of case studies and their corresponding datasets as well as different interesting sections. In this section, we present the *Flintstones* website and a case study is solved in detail by using *Flintstones*.

4.1. Flintstones website

The website has been designed with the aim to publish all *Flintstones* released versions and a repository of case studies with real datasets that can be solved with the software suite of tools. Furthermore, different interesting sections with theoretical foundations as well as video tutorials about the software suite of tools can be found in the website. Each section of the website is described briefly below:

- *Description*. In order to provide the theoretical foundations of *Flintstones*, the main theoretical concepts are briefly introduced in this section of the website. Examples of these concepts are *Computing with words paradigm* or the *2-tuple linguistic representation model*.
- *Software tool*. All *Flintstones* released versions will be available in the website. The current version, v1.0, has all the functionality to create, manage and solve decision making problems with linguistic and complex frameworks. In order to run *Flintstones*, it is only necessary to download and unpack the *zip* file and execute the *flintstones.jar* file. This file can be used on any machine with the JVM, independently of the operating system. The software tool is licensed under the terms of the GNU General Public License.[6]
- *Analysis and design*. Technical aspects related to the analysis and design of the suite are provided in order to offer a complete view of the internal structure of the suite. So, this website section shows the architecture of the suite as well as the package and class diagrams.
- *Case studies repository*. Case studies of decision making problems are available in the website, which are categorized by the type of framework (multi-granular linguistic framework, heterogeneous framework and unbalanced linguistic framework). Each case study is associated with its datasets for *Flintstones* that includes the definition of the framework and the set of assessments provided by experts. Furthermore, each case study is associated with the research paper in which the use of the 2-tuple linguistic representation model or any of its extensions has been successfully applied to it. The repository is alive, new datasets will be included, Table 1 shows a summary of the case studies which are currently incorporated in the repository.
- *Video tutorials*. A set of video tutorials that illustrate the functionality of *Flintstones* are showed in this section. Each basic functionality has been briefly described and has been illustrated in a video tutorial, which can be directly reproduced from the website.

4.2. On the use of Flintstones for a case study

To facilitate the understanding of *Flintstones*, we describe a decision making problem for installing an Enterprise Resource Planning (ERP) that is defined in a multi-granular linguistic framework. Therefore, *Flintstones* will select and run the linguistic decision method proposed in Section 3.1.

4.2.1. Framework

Let us suppose a company which plans to invest a sum of money in the best ERP among four possible alternatives $X = \{x_1, \ldots, x_4\}$ that were selected by departments involved in the process. The final decision lies on a group of four experts $E = \{e_1, \ldots, e_4\}$ that must evaluate the alternatives according to four benefit criteria $C = \{c_1, \ldots, c_4\}$, which are respectively:

[5] http://serezade.ujaen.es/flintstones/.
[6] http://www.gnu.org.

standard degree, *interrelation with other subsystems*, *degree of modularity* and *learning curve for users*. Due to the fact that the decision problem implies imprecision and uncertainty that has not probabilistic nature, the set of criteria will be evaluated in a linguistic domain.

In this case study, the group of experts have different knowledge degree about the set of criteria. Therefore, it seems suitable that experts can express their preferences in two different linguistic term sets based on their own knowledge. A linguistic term set with 5 labels for experts with a less knowledge degree is defined as well as a linguistic term set with 7 labels for experts with a bit more knowledge degree. Both linguistic term sets are distributed symmetrically and uniformly around the central label. The elements involved in framework are included in the datasets of this case study for *Flintstones*, i.e., the group of four experts, the set of four criteria and the set of alternatives as well as the linguistic expression domains and their allocation (see Fig. 15).

4.2.2. Gathering information

The case studies of the repository are associated to datasets which include all assessments (see Fig. 14). In this phase, the assessments provided by experts for each criterion for each alternative are provided, according to expression domains fixed in the framework. The assessment provided by the expert e_1, for the alternative x_2 about the criterion c_2 that are expressed in the linguistic term set with 7 labels is shadowed in Fig. 16.

4.2.3. Selecting a solving process

Here, *Flintstones* carries out Algorithm 1 to select the suitable solving process for the decision making problem, taking into account the expression domains established in the framework. The input information and the selection algorithm is as follows: $edNum = False, edInt = False, edLinUnb = False, tamEdLinLis = 2$ and $edLinList = ((5, True), (7, True))$.

Our case study is defined in a linguistic complex context, particularly in a multi-granular linguistic framework. According to the proposed algorithm, the value *3* is returned that corresponds to the suitable solving process based on *extended linguistic hierarchies*. It is noteworthy that the extension based on *fusion approach for managing multi-granular linguistic information* could also be applied to solve our case study (see Fig. 17).

However, as it was reviewed in Section 2.3, the methodologies based on *fusion of information* can provide linguistic results with loss of information. Therefore, the selected suitable solving process is based on *extended linguistic hierarchies* due to the fact that this extension provides linguistic accuracy results. It is remarkable that the algorithm selects the suitable solving process and allows also use other solving process available to compare results.

Finally, the extension to deal with multi-granular linguistic frameworks called *linguistic hierarchies* cannot be applied because this extension requires a condition of granularity (see Eq. (5)) that the framework does not fulfill.

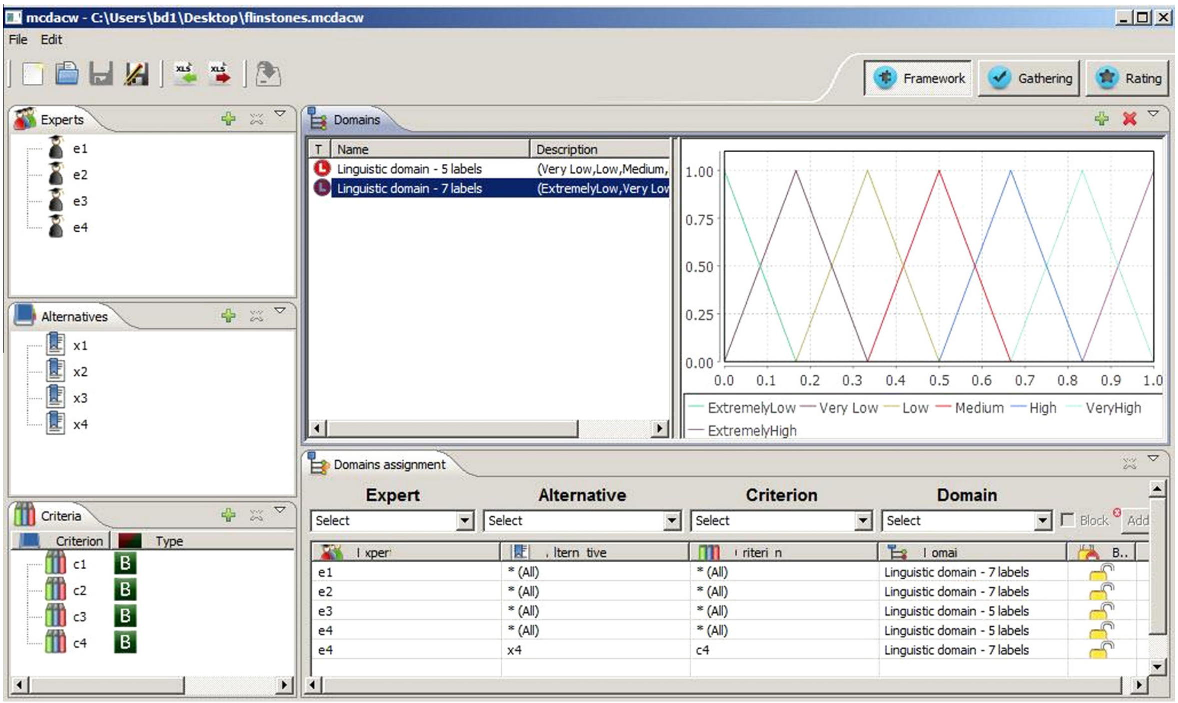


Fig. 15. Framework of ERP case study.

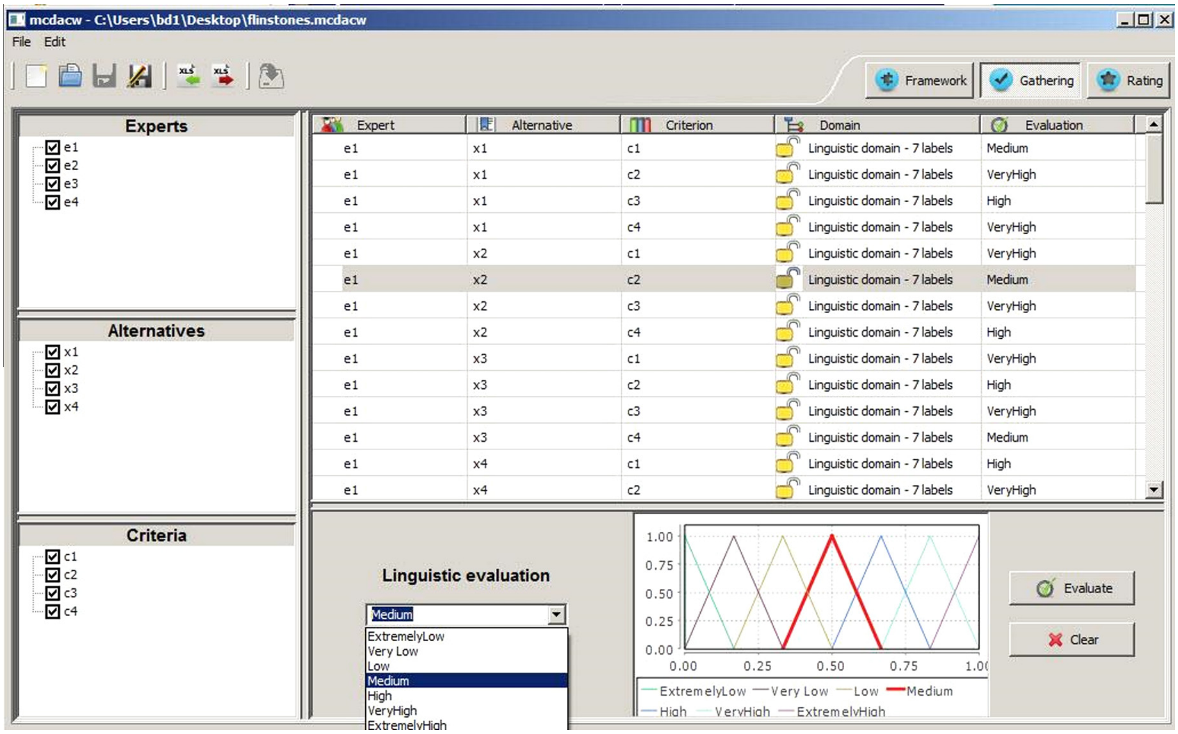


Fig. 16. Gathered information.

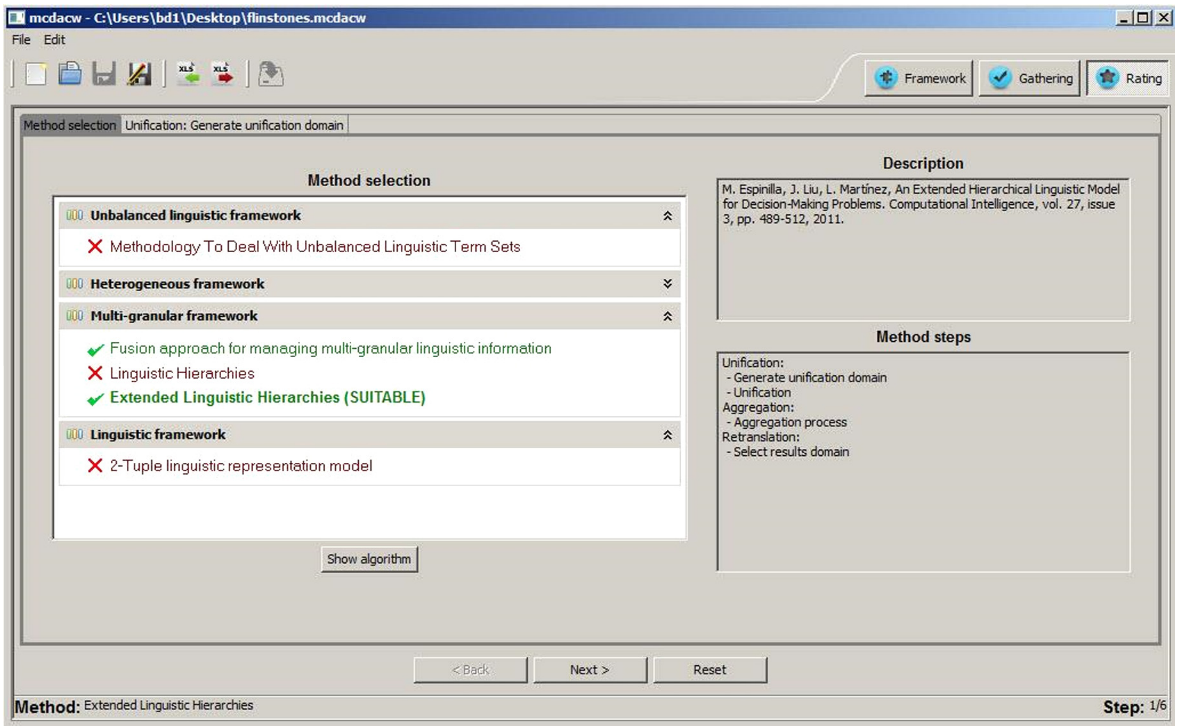


Fig. 17. Selection the suitable solving process.

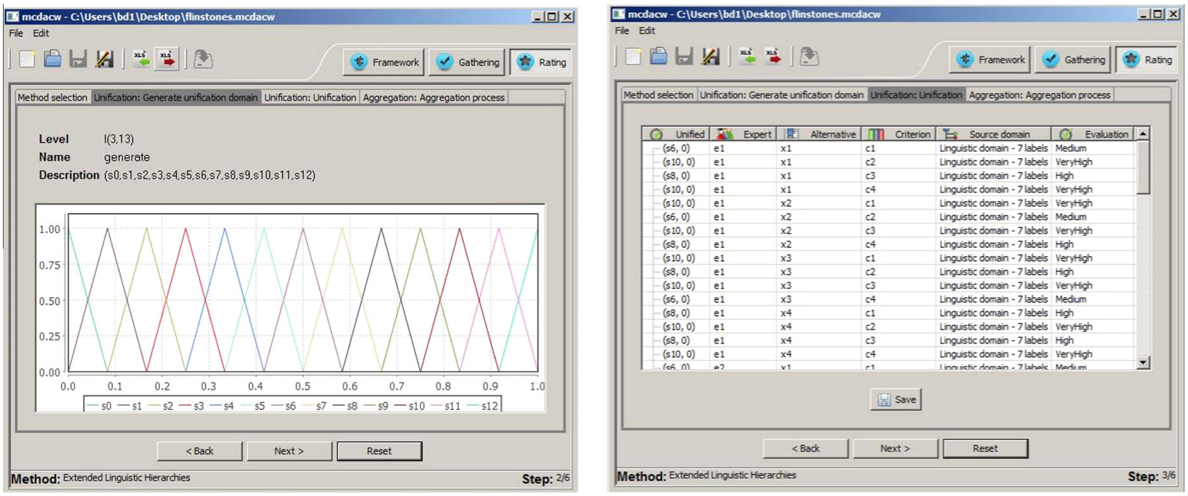


Fig. 18. (a) New generated level and (b) unified information.

4.2.4. Rating alternatives

In this phase, the rating process with the selected solving process based on *extended linguistic hierarchies* is carried out. The processes of the rating alternatives to compute the linguistic assessment for each alternative are described below:

1. *Unification process*. According to the granularities of the initial linguistic term sets defined in the framework, 5 labels and 7 labels, a new linguistic term set with 13 labels is generated (see Fig. 18a), according to Eq. (7): $n(t^*) = lcm(n(1) - 1, n(2) - 1) + 1 = lcm(4, 6) + 1 = 12 + 1 = 13$. The gathered information is then unified into the new generated level (see Fig. 18b).
2. *Aggregation process*. In this process, the information is aggregated selecting the set of aggregation operators for 2-tuple linguistic values (see Fig. 19a).
   In this case study, on the one hand, the *2-tuple linguistic weighted average operator* is selected to aggregate the preferences provided by experts for each criterion with the following weighting vector $w_e = (0.2, 0.3, 0.3, 0.2)$. On the other hand, the *2-tuple linguistic arithmetic mean operator* is used to compute a collective value for each alternative, aggregating its collective assessments. The computed global assessment for each alternative is expressed in the unified level t^* that corresponds to $\langle S^{13} \rangle$.
3. *Retranslation process*. In order to provide 2-tuple linguistic results that are easily understandable for experts, the computed linguistic results are expressed on each initial linguistic term set defined in the decision framework. In this case study, the computed results expressed in S^{13} can be transformed into the two initial scales, S^5 and S^7 (see Fig. 19b). It is noteworthy that these transformations are carried out without loss of information.

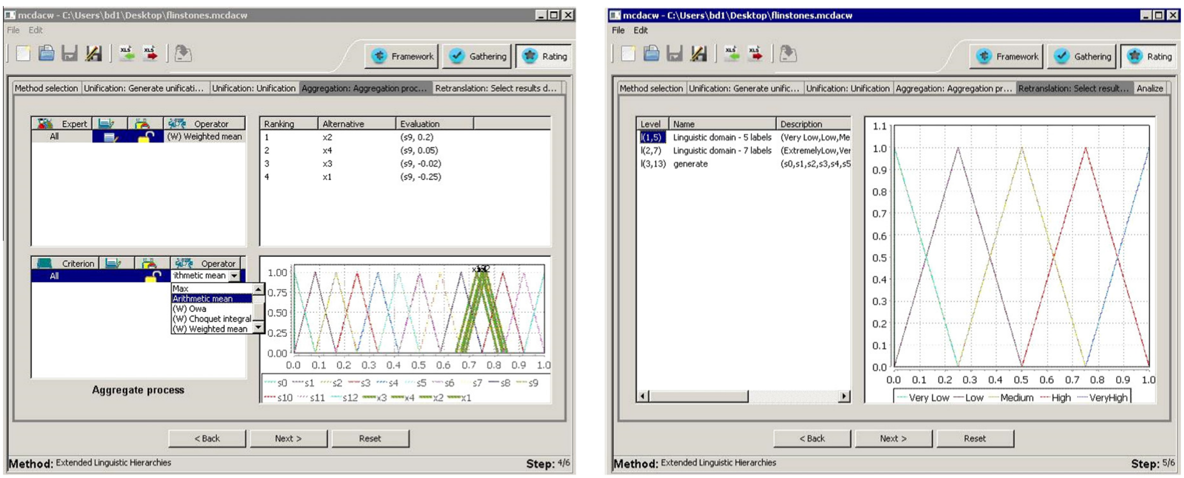


Fig. 19. (a) Aggregation process and (b) retranslation process.

5. Conclusions and future works

There is a perceived need of software tools for solving linguistic decision making problems. The use of the 2-tuple linguistic model and its extensions shows a clear gap to fulfill. Despite the wide range of successful applications in different fields, there is not yet developed any software tool in order to deal with this linguistic model. This paper has presented a software suite so-called *Flintstones* that implements the 2-tuple linguistic model to solve linguistic decision making problems under uncertainty and its extensions to deal with linguistic complex frameworks such as multi-granular linguistic frameworks, heterogeneous frameworks and unbalanced linguistic frameworks. Furthermore, this paper has presented the *Flintstones* website that includes a repository of case studies and datasets for different linguistic decision making problems in order to validate the performance of the software suite with real datasets and to make comparisons with either other proposals or problems. Finally, the description to solve a decision making problem for installing an ERP by *Flintstones* is illustrated.

Our future works are addressed to extend *Flintstones* and its website in different ways. The first way is the development of new extensions based on the 2-tuple linguistic representation model that could be proposed in the future as well as other different methodologies under uncertainty as hesitant linguistic fuzzy sets [53] or type-2 fuzzy sets [43] within the CWW paradigm. The second way is to increase the repository of case studies on the website with the wide range of applications in different fields in which the 2-tuple linguistic representation model and its extensions have provided satisfactory results. Finally, to point out the diffusion of *Flintstones* and its website in order to increase the number of aggregation operators developed by others authors in order to analyze and test the results with them.

Acknowledgment

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