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Energy internet project evaluation in circular economy practices: A novel multi-criteria decision-making framework with flexible linguistic expressions based on multi-granularity cloud-rough set ☆

Han Wang [a,b], Yanbing Ju [a,*], Carlos Porcel [b]

[a] School of Management, Beijing Institute of Technology, Beijing 100081, China
[b] Andalusian Research Institute on Data Science and Computational Intelligence (DaSCI), Department of Computer Science andAI, University of Granada, Granada 18071, Spain

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ABSTRACT

Energy internet (EI) serves as a practical platform for implementing the principles of the circular economy. As the process of realizing the transformation of industrial systems to a sustainable circular economy accelerates, the importance of evaluating and selecting the most effective EI projects becomes increasingly apparent. This paper proposes a new multi-criteria decision-making (MCDM) framework based on the multi-granularity cloud-rough set (MGCRS) and flexible linguistic expressions (FLEs) to evaluate and select different EI projects. Firstly, a comprehensive index system is established from three aspects including grid technology, green energy, and composite benefits. Secondly, FLE is used to express experts' preferences and a transformation method converting discrete FLEs into continuous cloud information is proposed. To select the best EI project, the optimistic MGCRS and pessimistic MGCRS over two universes are presented to deal with the continuous cloud information in the decision-making process. Furthermore, the comprehensive multi-granularity lower approximation and upper approximation based on the cloud model are proposed to rank different EI projects. Finally, a case study of China's Beijing–Tianjin–Hebei region is analyzed to illustrate the proposed model, and the simulation and comparative analyses are provided to demonstrate the effectiveness of the proposed framework.

1. Introduction

Facing the dual challenges of decarbonization and sustainable development, industrial systems are evolving towards a sustainable circular economy (Sassanelli, Garza-Reyes, Liu, de Jesus Pacheco, & Luthra, 2023; Taddei, Sassanelli, Rosa, & Terzi, 2024). The core of circular economy lies in optimizing resource utilization, reducing waste generation, and improving material re-utilization to achieve sustainable development (Tripathy, Bhuyan, Padhy, Mangla, & Roopak, 2023). Therefore, industrial systems must shift from a traditional linear economy to a resource-efficient and waste-minimizing circular system to meet modern sustainability needs by reducing raw material consumption and focusing on material reuse and energy recycling. Energy plays an important role in industrial production, directly impacting the efficiency of resource utilization and waste generation. To achieve circular economy goals, energy flow management is crucial for industrial systems, which involves improving every stage of the energy supply chain to maximize energy use and reduce waste from generation and transmission to consumption and recycling. As a cutting-edge concept in energy research, the energy internet (EI) occupies an important position in the transformation process (Martínez, Dinçer, & Yüksel, 2023). EI connects each generator set through multiple communication platforms and builds an interconnected energy ecosystem with the power system as the core, which is not only a crucial tool for driving the transformation of industrial systems to a circular economy but also a concrete implementation of circular economy concepts at both technical and operational levels. By integrating technologies such as smart grids, real-time data analysis, and distributed energy resources, the EI provides comprehensive energy optimization solutions for industrial systems (Verma, Gope, & Kumar, 2021). These technologies enhance energy efficiency and support resource recycling and waste reduction, helping industrial systems achieve the core goals of a circular economy.

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* Corresponding author.
E-mail addresses: wangh936@163.com, wangh936@correo.ugr.es (H. Wang), juyb@bit.edu.cn (Y. Ju), cporcel@decsai.ugr.es (C. Porcel).

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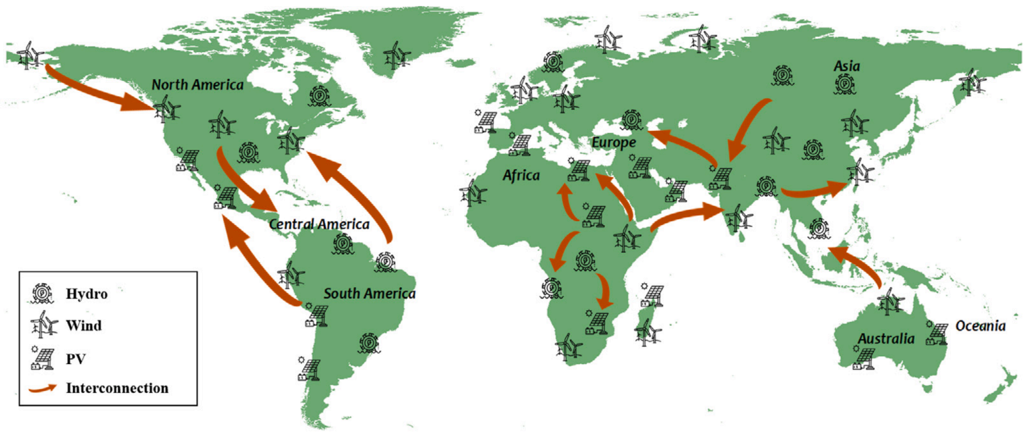


Fig. 1. The blueprint for global EI in 2050.

The GEIDCO (Global Energy Interconnection Development and Cooperation Organization) highlights the active promotion of strategic and technological innovations to expedite the development of a global EI, aiming to achieve intra-continental interconnection by 2030 and intercontinental interconnection by 2050 (Menacho, Rodrigues, & Behrens, 2022). The blueprint of the global EI in 2050 is shown in Fig. 1. With the rapid development of EI, more and more countries and regions have begun to pay attention to the theoretical research and practical application in this field. IEEE (Institute of Electrical and Electronics Engineers) promotes standardization research in the field of EI. The technical council of the IEEE Power and Energy Society approved the establishment of the IEEE-EICC (Energy Internet Coordinating Committee) in July 2020 (Hafez et al., 2023). The NIST (National Institute of Standards and Technology) has released version 4.0 of NIST Framework and Roadmap for Smart Grid Interoperability Standards in 2021. Europe countries and Japan have also developed relevant standards and technical frameworks (Schaffert et al., 2022). To advance the transition of industrial systems towards a circular economy, the NEA (National Energy Administration) in China has approved 55 green energy demonstration projects, covering urban energy, energy, electric cars, flexible energy, and other types of energy (Sassanelli, Rosa, Rocca, & Terzi, 2019) . These efforts have promoted the implementation of circular economy policies and deepened the exploration of technological innovation and operational models of industrial systems.

However, current research about EI has focused on concepts, standards, and technology systems of EI (Song et al., 2022; Yin & Li, 2022). Evaluating different EI projects is essential to understanding the practical effects of driving the transformation of industrial systems to a circular economy, which not only contributes to an in-depth understanding of the actual effect of EI in promoting the transformation of industrial systems to a sustainable circular economy, but also provides data support and decision-making basis for further technology optimization and policy-making. Employing a comprehensive and strategic approach to evaluate EI projects will provide a clear insight into how the EI specifically supports circular economy objectives, thereby more effectively driving industrial systems towards a greener and more sustainable future (Singh Rawat, Komal, Dincer, & Yüksel, 2023). To guarantee the accuracy and comprehensiveness of the evaluation, adopting a scientific and systematic approach is essential. Evaluating EI projects involves a multi-criteria decision-making (MCDM) problem, encompassing the comprehensive analysis of various dimensions including technological, environmental, social, and economic factors. By applying comprehensive criteria and MCDM methods (He, Wang, & Martínez, 2022; Liu, Wang, & Liu, 2022), companies and technology developers can identify and prioritize EI projects with technological innovation and positive environmental and social impact, thereby driving the transformation of industrial systems to a more sustainable and circular economy (Aragonés & Torralvo, 2024). The evaluation of EI projects promotes green innovation and low-carbon development of industrial systems and ensures that these projects play a key role in the sustainable circular transformation of industrial systems, laying a solid foundation for achieving long-term environmental and social benefits (Vinante, Sacco, Orzes, & Borgianni, 2021).

For multiple EI projects, providing evaluation information in an easily understandable format is essential. The linguistic term is commonly used to convey complex information, particularly when dealing with intricate EI systems or multidimensional environmental factors. In the face of complex systems or multi-dimensional factors, EI project evaluation often relies on multiple discrete linguistic terms to accurately convey evaluation information. These linguistic terms typically describe various levels of evaluation and are accompanied by probabilities. For example, the sustainable influence of an EI project might be characterized with a 70% probability as ‘high’ or ‘medium’ and a 30% probability as ‘low’, i.e., $\{(\{high, medium\}, 0.7), (\{low\}, 0.3)\}$. This expression, referred to as flexible linguistic expressions (FLEs) (Wu, Dong, Qin, & Pedrycz, 2019), integrates the intuitive nature of linguistic terms with the quantitative precision of probabilities, which effectively addresses the ambiguity and uncertainty inherent in evaluations and provides a consistent framework for experts and stakeholders, thereby facilitating robust decision-making. However, due to the subjectivity and fuzziness of discrete linguistic terms, the evaluation results are often uncertain and difficult to support accurate decision-making and analysis. Additionally, discrete evaluation information is difficult to compare and integrate, which limits the effective interaction and integration between different evaluation indicators and information. Previous research on FLEs has focused on the normalization method of symbolic proportion and qualitative analysis of linguistic terms, ignoring the quantitative analysis of linguistic terms. To minimize the loss of information while preserving the fuzziness and uncertainty of the original evaluation, how to convert discrete FLEs into continuous information is still a research gap. Aiming at this problem, this paper proposes an optimization method that transforms discrete FLEs into continuous cloud model information, which provides a more precise representation of evaluation continuity while retaining the inherent fuzziness and uncertainty of the original evaluation.

The multi-granularity rough set (MGRS) theory provides an efficient and flexible tool for handling and analyzing complex and uncertain evaluation information within the context of EI project evaluation (Qian, Liang, & Dang, 2009). Evaluating EI systems requires a comprehensive consideration of various factors, including diverse technological and environmental variables, as well as dynamic influences such as policy changes, market demands, and operational conditions. Consequently, these evaluations often encounter significant uncertainty and vagueness. The core advantage of MGRS lies in its ability to process information at different levels of granularity, effectively accommodating the diversity of factors involved. By constructing upper and lower approximation sets, this method captures the inherent vagueness in the data and provides classification and ranking of systems or projects (Qian, Liang, Yao, & Dang, 2010). This structured analytical framework for handling uncertain and fuzzy data supports comprehensive evaluation and comparative analysis of EI projects. However, traditional MGRS methods are primarily designed for handling discrete information, which poses limitations when dealing with continuous data or highly fuzzy situations. These constraints may impede the effectiveness of conventional methods in addressing the diverse and continuous information often encountered in EI evaluations. To overcome these challenges, this paper integrates MGRS with the cloud model and proposes a novel multi-granularity cloud-rough set (MGCRS). This approach not only retains the inherent vagueness and uncertainty of the original data but also allows the final evaluation results to be presented in a continuous format and ranked accordingly. This innovative method significantly enhances the accuracy and practicality of the evaluation process, providing more comprehensive and precise support for the selection and optimization of EI projects.

To solve the above research gaps in the evaluation of EI projects and advance the goal of circular economy effectively, we propose an MCDM framework based on FLEs and MGCRS. The contributions of this paper are as follows:

(1) A novel method to convert discrete FLEs into continuous cloud information is proposed. FLEs are first aggregated into floating clouds using a series of basic clouds. Then an programming model is constructed to minimize the difference to derive a comprehensive cloud to obtain continuous cloud information for each discrete FLE, effectively achieving the conversion from discrete to continuous representation. This approach preserves the uncertainty of evaluation information while making it easier to integrate to support accurate decision-making and analysis for EI projects.

(2) Based on the relevant literature and analyses, we develop an evaluation index system tailored to evaluate various EI projects, which is designed to capture the multifaceted impacts of these projects on sustainable development. To assign appropriate weights to the evaluation criteria, we utilize the Shannon entropy method, leveraging the cloud information derived from our novel transformation technique. This approach ensures that the evaluation framework accurately reflects the EI projects' effectiveness in advancing energy efficiency and circular economy goals.

(3) To select the best EI project, we define the MGCRS and present the optimistic and pessimistic MGCRSs over two universes to deal with the continuous cloud information in the decision-making process. Sequentially, the comprehensive multi-granularity lower approximation and upper approximation based on the cloud model are proposed to rank different EI projects. This approach allows the final evaluation results to be presented in a continuous format and ranked accordingly and significantly enhances the accuracy and practicality of the evaluation process, offering a more precise and reliable basis for optimizing EI projects in the context of the circular economy.

The remainder of this paper is structured as follows. Section 2 reviews the literature on the research of EI evaluation. Section 3 introduces the basic knowledge necessary related to FLE, cloud model, and rough sets. Section 4 proposes an MCDM framework for EI project evaluation in the FLE environment based on the Shannon entropy method and MGCRS. Section 5 provides a numerical analysis of the EI project evaluation in China's Beijing–Tianjin–Hebei region and the relevant analyses including the simulation analysis and comparative analysis to demonstrate the feasibility and effectiveness of the proposal. Section 6 discusses the results. The final section presents the conclusions.

2. Literature review

This section reviews the existing literature on the MCDM methods and evaluation index system for energy systems. The evaluation index system for EI projects is established based on the literature review and analyses.

2.1. Energy internet evaluation approaches

To better characterize the influence of various energy sources in advancing circular economy and sustainable development, numerous MCDM approaches have been applied to the evaluation and selection of different integrated energy systems, including AHP (Analytic Hierarchy Process) (Islam, Aziz, Alauddin, Kader, & Islam, 2024; Kong et al., 2022; Liu, Liu, Ren, Liu, & Liu, 2022), ANP (Analytic Network Process) (Dagtekin, Kaya, & Besli, 2022), TOPSIS (The Technique for Order Preference by Similarity to an Ideal Solution) (Jiang et al., 2022; Otay, Onar, Öztayşi, & Kahraman, 2024; Wang et al., 2022; Zhao et al., 2022), VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) (Shang, 2022; Wang, Xu, Wang, & Ren, 2019), DEMATEL (Decision Making Trail and Evaluation Laboratory) (Bagherian, Gershon, Kumar, & Mishra, 2024; Xu, Gao, Xiao, Liu, & Wu, 2022; Zhao et al., 2022), ELECTRE(ELimination and Choice Expressing REality) (Dagtekin et al., 2022), etc. The summary of evaluation approaches for energy systems is shown in Table 1, in which the research goal, evaluation approaches, and criteria dimensions are listed. The relevant approaches can be divided into two categories: single MCDM method and hybrid MCDM methods.

(1) Single MCDM method: Kong et al. (2022) analyzed the internationalization implementation effect of technical standards using the fuzzy AHP method. Jiang et al. (2022) used the TOPSIS method to evaluate the operational performance of community-integrated energy systems. Wang et al. (2022) established an evaluation model based on the improved TOPSIS method to select an urban integrated energy station. Shang (2022) proposed the fuzzy VIKOR method to select a distributed energy storage system. To explore the difference between these single MCDM methods, Dagtekin et al. (2022) compared the ranking results of different methods including AHP, ANP, TOPSIS, ELECTRE, PROMETHEE and VIKOR for distributed energy systems selection.

(2) Hybrid MCDM methods: Otay et al. (2024) combined the BWM (Best Worst Method) and TOPSIS for multiple experts to evaluate sustainable energy systems in smart cities. Ke, Liu, Meng, Fang, and Zhuang (2022) proposed a hybrid method integrating BWM and CRITIC (Criteria Importance Though Inter-criteria Correlation) to determine the urban integrated energy systems site. Esangbedo, Xue, Bai, and Esangbedo (2024) employed a hybrid method in subjective and objective aspects to determine the weight of criteria in the subcontractor selection of the photothermal power station problem. Zhao et al. (2022) proposed multiple decision-making methods including TOPSIS, anti-entropy weight method, grey-DEMATEL, and quotient grey relation analysis to select the best one from eight building-typed microgrid systems. Bagherian et al. (2024) adopted the ISM-MICMAC and DEMATEL method to analyze the energy sustainability and digitalization. Bac, Alaloosi, and Turhan (2021) developed a hybrid framework integrating modified SWARA (Stepwise Weight evaluation Ratio Analysis) and WASPAS (Weighted Additive Sum Product evaluation) methods to evaluate air-conditioning systems.

Since experts may be irrational, the preferences of experts are characterized by uncertainty and fuzziness. To deal with the judgment uncertainty, many fuzzy discrete expressions like HFS (Hesitant fuzzy set), TFN (Triangular Fuzzy Number), and TrFN (Trapezoid Fuzzy Number) have been used to characterize the preference information of experts. Liu, Liu, et al. (2022) proposed a new decision-making method under interval type-2 fuzzy numbers to evaluate the multi-energy transaction performance, and presented a novel integrated performance evaluation method with flexible fuzzy boundaries to deal with linguistic imprecision and ambiguity of expert judgments. Qin, Zhang, Yan, Xu, and Kammen (2021) extended a fuzzy AHP method based on the cloud model and TFN to evaluate the performance of regional EI, where fuzzy linguistic terms are converted into the cloud model by the aggregated weight method. Tan et al. (2023) proposed a probabilistic hesitant fuzzy MCDM method considering prospect theory to evaluate different rural EI scenarios without the information transformation. Wu, Zhang, and Yi (2021) constructed a fuzzy evaluation framework based on interval type-2 TrFN and applied the Choquet integral fuzzy synthetic model to fuse the evaluation information. Zhou, Chen, Zhao, and Wang (2022) proposed a probabilistic evaluation approach based on the Dirichlet mixture model to evaluate an integrated energy supply system, where expectation and variance values are used to process probability evaluation information. Xu et al. (2022) applied the DEMATEL HFS method to evaluate the risk of integrated energy systems, where a hesitant fuzzy entropy is used to aggregate the HFS information. Otay et al. (2024) developed a novel interval-valued Pythagorean fuzzy method where multi-expert fuzzy BWM and TOPSIS methodology to better handle uncertainty and vagueness in experts' linguistic assessments. Although the existing literature has developed different evaluation approaches for integrated energy systems, few studies have focused on the EI project evaluation considering the transformation between discrete information and continuous information to integrate the information smoothly and prevent information loss. Therefore, this paper proposes an MCDM approach with FLEs based on MGCRS to evaluate and select the best EI project.

2.2. Evaluation index system for EI project

The evaluation criteria of the integrated energy system have been investigated in some previous works, including but not limited to aspects of economy, society, environment, resources, reliability, etc. Economy, society and environment are the most commonly used dimensions when determining evaluation index systems. Otay et al. (2024) considered six criteria to evaluate the energy system in a smart city involving environmental, economic, social, and technical factors. Pamucar, Ecer, Gligorić, Gligorić, and Deveci (2024) claimed that environmental factors were more essential than social and economic factors from the perspective of green energy. Zhao et al. (2022) constructed a performance evaluation index system for the microgrid system from the economy, environment, and energy dimensions. Wang et al. (2022) established a comprehensive index system for nature, economy, and society to select the final optimal urban integrated energy station. Ke et al. (2022) proposed a comprehensive evaluation index system from the economy, energy, environment, and society for urban integrated energy systems selection. Bac et al. (2021) prioritized transformation of the energy market and smart manufacturing technologies based on the critical measurements in Europe's energy domain. To select the subcontractor for the photothermal power station, Esangbedo et al. (2024) included enterprise reputation as a criterion in addition to the commonly used evaluation system mentioned above.

In addition, security, technique, and politics are also used to measure the performance of energy systems. Qin et al. (2021) selected sixteen criteria of region EI about technical, economic, social, and engineering dimensions. Zhou et al. (2022) proposed an evaluation system from the economy, efficiency, environment, and security. Xu et al. (2022) summarized sixteen risk factors from the economy, technology, politics, society, and management to evaluate the risk of integrated energy systems. Bac et al. (2021) selected twenty-seven criteria under several categories including ergonomic, environmental, reliability, technical, and economical aspects to evaluate seven air-conditioning systems. Lu and Liu (2024) constructed an MCDM framework using cost, reliability, energy consumption, and environmental factors.

The idea of systems engineering is gradually applied in the establishment of index systems. Wu et al. (2021) constructed a criteria system from internal and external attributes to evaluate the regional EI investment. Berjawi, Walker, Patsios, and Hosseini (2021) summarized six characteristics for evaluating the integrated energy systems multi-dimensional, multivectorial, systemic, applicability, futuristic, and systematic. Liu, Liu, et al. (2022) established a multi-energy transaction index system about suppliers, transaction attributes, consumers, and distributors. Dagtekin et al. (2022) determined five criteria primary energy utilization rate, operating cost, primary energy consumption, carbon emissions, and investment cost for distributed energy systems. Despite the above research, the existing index system often only focuses on a few dimensions, which is a lack of comprehensiveness and objectivity. As the foundation for the evaluation and selection of EI, a comprehensive and reasonable index system is urgently needed to extract objectively and thoroughly.

Considering the power interoperability, environmental protection, and efficiency of the EI, we propose a new evaluation index system according to existing literature and analyses, which is shown in Fig. 2. The evaluation index system includes 10 criteria from grid technology, green energy, and composite benefits aspects. The grid technology mainly indicates the internal and external technical level of power grids in the EI, including grid interconnection, reliability of grid structure, grid informatization, and power transmission capacity 4 sub-criteria. Corresponding to the core part of the EI—energy, green energy measures the clean energy usage, waste utilization, and waste emissions of the EI, which are indicated by the proportion of clean energy, the utilization rate of waste, and the emission of waste gas, respectively. To explore the benefits of EI projects, social, economic, and environmental benefits are used to measure the composite benefits. Details of the evaluation index system are specifically summarized and explained in Table 2.

3. Preliminaries

This section briefly introduces some basic definitions, regarding linguistic scale function (Wang, Peng, Zhang, & Chen, 2014), FLEs (Wu et al., 2019), cloud model (Li, Liu, & Gan, 2009; Liu, Wang, Li, & Hu, 2018; Wang & Feng, 2005), Pawlak rough set (Pawlak, 1982), and MGRS (Qian et al., 2009, 2010).

3.1. Linguistic scale function and flexible linguistic expression

Definition 1. (Wang et al., 2014) Given a linguistic term set $L = \{l_0, l_1, \ldots, l_g\}$, the linguistic scale function H mapping from l_i to δ_i is defined as follows:

$$H : l_i \to \delta_i \ (i = 0, 1, \ldots, g),$$

where $0 < \delta_0 < \delta_2 < \cdots < \delta_g < 1$. The symbol δ_i reflects the preference of experts using the linguistic term l_i. The function H is a strictly monotonically increasing function with respect to l_i, which is denoted as follows:

$$H(l_i) = \delta_i = \begin{cases} \dfrac{a^{\frac{g}{2}} - a^{\frac{g}{2}-i}}{2a^{\frac{g}{2}} - 2} & (i = 0, 1, \ldots, \frac{g}{2}) \\ \dfrac{a^{\frac{g}{2}} + a^{i-\frac{g}{2}} - 2}{2a^{\frac{g}{2}} - 2} & (i = \frac{g}{2} + 1, \frac{g}{2} + 2, \ldots, g) \end{cases}, \tag{1}$$

the value of a can be determined using a subjective method. Assuming the indicator A is far more important than indicator B and the important ratio is m, then $a^k = m$ (k represents the scale level) and $a = \sqrt[k]{m}$. The vast majority of researchers believe that $m = 9$ is the upper limit of the important ratio. If the scale level is 7, then $a = \sqrt[7]{9} \approx 1.37$ can be obtained.

Definition 2. (Wu et al., 2019) Let $L = \{l_0, l_1, \ldots, l_g\}$ be a fixed linguistic term set with odd cardinality. $\hat{S}_L$ is a set composed of the subsets s_L of L and individuals express preferences by providing the

Table 1
Summary of evaluation approaches for energy systems.

References	Research goal	Evaluation approaches	Criteria dimensions
Kong et al. (2022)	Evaluation of the internationalization implementation effect	Fuzzy AHP	Standard adoption, activities, benefit, compilation, and text internationalization
Dagtekin et al. (2022)	Evaluation of distributed energy storage system	AHP, ANP, TOPSIS, ELECTRE, PROMETHEE and VIKOR	Energy, cost, emissions, and investment
Jiang et al. (2022)	Performance evaluation of community integrated energy systems	TOPSIS	System efficiency, renewable energy penetration and operation cost
Shang (2022)	Evaluation of distributed energy storage systems	Fuzzy measure and VIKOR	Environment, society and business
Wang et al. (2022)	Site selection for urban integrated energy station	GIS and improved TOPSIS	Nature, economy and society
Ke et al. (2022)	Site selection for urban integrated energy systems	BWM and CRITIC	Economy, energy, environment and society
Wang et al. (2019)	Evaluation of distributed energy systems	DEMATEL and VIKOR	Technique, economy, environment and society
Zhou et al. (2022)	Evaluation of integrated energy supply system	Probabilistic approach and Dirichlet mixture model	Resource, economy, and environment
Zhao et al. (2022)	Evaluation of building-typed microgrid systems	TOPSIS, anti-entropy weight method, grey-DEMATEL and quotient grey relation analysis	Economy, environment and energy
Bac et al. (2021)	Evaluation of HVAC system	SWARA and WASPAS	Ergonomics, environment, reliability, technique and economy
Liu, Liu, et al. (2022)	Performance evaluation of multi-energy transaction	Fuzzy comprehensive evaluation and AHP	Suppliers, transaction attributes, consumers and distributors
Qin et al. (2021)	Performance evaluation of regional EI	Fuzzy AHP and cloud model	Technique, economy, society and engineering
Tan et al. (2023)	Feasibility evaluation of rural EI	Probability hesitation fuzzy method and prospect theory	Economy
Wu et al. (2021)	Investment evaluation of regional EI	Interval type-2 trapezoid fuzzy number and Choquet integral	Internal and external attributes
Zhou et al. (2019)	Evaluation of park-level integrated energy systems	DEMATEL and the extended TODIM	Economy, environment, energy utilization, reliability and sustainability
Xu et al. (2022)	Risk evaluation of integrated energy systems	DEMATEL and HFS	Economy, technology, politics, society and management

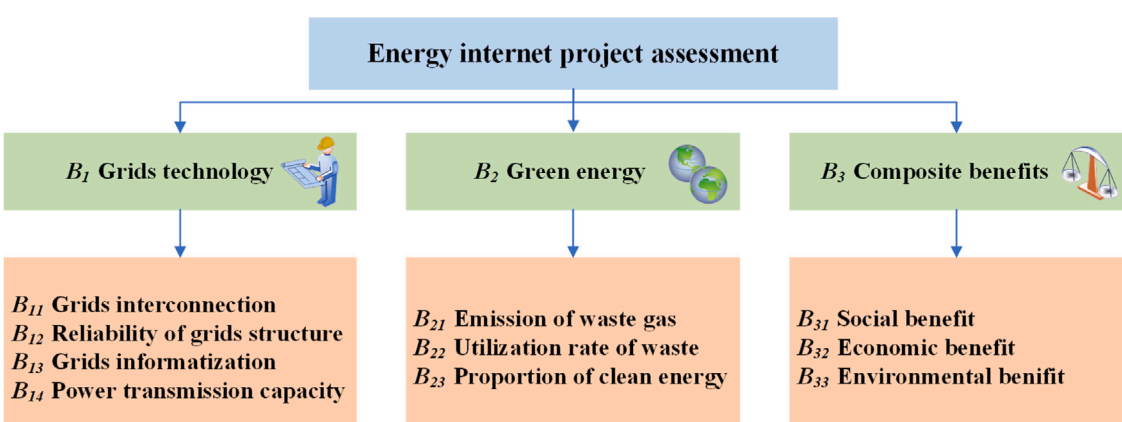


Fig. 2. The evaluation index system for EI project.

Table 2
A detailed description of the evaluation index system.

Dimensions	Criteria	Criteria description
Grid technology (B_1)	Grids interconnection(B_{11})	It indicates the degree of interconnection between internal and external power grids in a region.
	Reliability of grids structure(B_{12})	It measures the resilience and reliability of power grids structure.
	Grids informatization(B_{13})	It refers to the application of smart technologies such as modern information, communication, and control to the power grids.
	Power transmission capacity(B_{14})	It indicates the transmission capacity of the grid, the length of the line, and the amount of power.
Green energy (B_2)	Emission of waste gas(B_{21})	It is the reciprocal of the level of waste emissions per unit of electricity generated.
	Utilization rate of waste(B_{22})	It measures the utilization rate of waste generated per unit of electricity generation.
	Proportion of clean energy(B_{23})	It is the ratio of clean energy to the total energy.
Composite benefits (B_3)	Social benefit(B_{31})	It indicates the benefit that the EI brings to society.
	Economic benefit(B_{32})	It refers to the benefit of the EI to the macroeconomy.
	Environmental benefit(B_{33})	It measures the environmental benefit of an EI.

distribution information of s_L. Then, the individual's preference is FLE, denoted as

$$m_L = \{s_L, p(s_L) | s_L \in \hat{S}_L, p(s_L) \in [0,1]\}, \tag{2}$$

where $p(s_L)$ is the symbolic proportion assigned to the subset s_L. The negation operator of an FLE m_L is $Neg(\{s_L, p(s_L)|s_L \in \hat{S}_L\}) = \{Neg(s_L), p(s_L)|s_L \in \hat{S}_L\}$, where $Neg(s_L) = \{l_{g-t}|l_t \in s_L, t \in \{0,1,\ldots,g\}\}$.

The set L is not fixed because individuals may use different subsets to express preferences for special decision-making problems. Wu et al. (2019) argued that the sum of symbolic proportions should not be restricted to be one or less than one because the constraint is hard to satisfy.

3.2. Cloud model

Definition 3. (Li et al., 2009) Let U be the universe of discourse and $\tilde{A}$ be a qualitative concept in U. If $x \in U$ is a random instantiation of the qualitative concept $\tilde{A}$ that satisfies $x \sim N(Ex, En'^2)$ and $En' \sim N(En, He^2)$, and the certainty degree y of x belonging to concept $\tilde{A}$ is a probability distribution, which satisfies

$$y = e^{-\frac{(x-Ex)^2}{2En'^2}}, \tag{3}$$

then the distribution of x in the universe U is called a normal cloud, and the cloud drop can be denoted as (x, y). The overall quantitative properties of concept $\tilde{A}$ can be perfectly depicted in cloud C with three numerical features: expectation Ex, entropy En, and hyper entropy He. Cloud C can be described as $C = (Ex, En, He)$.

For a cloud $C = (Ex, En, He)$, Ex is the mathematical expectation that the cloud drops belong to a concept in the universe, which can be regarded as the most typical sample of the qualitative concept. En represents the numerical range of qualitative concepts which reflects the uncertainty measurement of the concept. The larger En is, the fuzzier the concept is. He is the second-entropy of entropy En, which represents the uncertainty degree of En. The larger He reflects that cloud drops are more random which means the cloud is thicker. For example, a normal cloud generated by $C(5, 0.38, 0.03)$ with 3000 cloud drops is shown in Fig. 3.

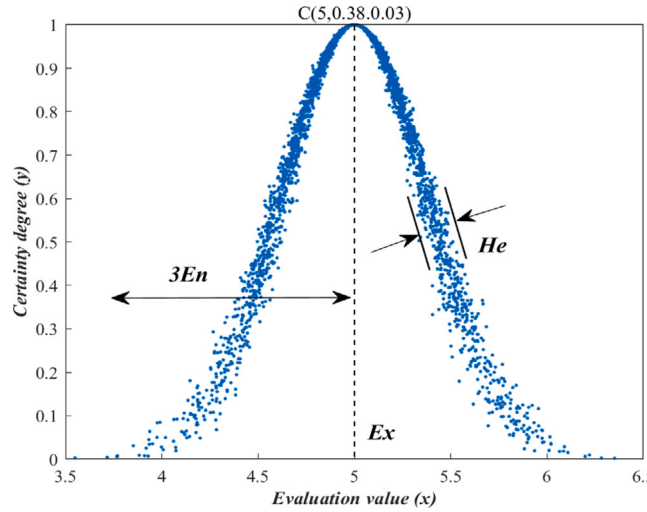


Fig. 3. The normal cloud generated by $C(5, 0.38, 0.03)$ with 3000 cloud drops.

To measure the difference between clouds, the definition of distance between two clouds is proposed, which is essential for decision-making problems.

Definition 4. (Liu et al., 2018) If $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$ are two arbitrary normal clouds, then the Euclid distance $d(C_1, C_2)$ between C_1 and C_2 is denoted as follows:

$$d(C_1, C_2) = \sqrt{\frac{1}{2}((Ex_1 - Ex_2)^2 + (En_1 - En_2)^2 + (He_1 - He_2)^2)}. \tag{4}$$

Definition 5. (Wang & Feng, 2005) If $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$ are two arbitrary basic normal clouds, then the floating cloud $C = (Ex, En, He)$ between C_1 and C_2 is calculated as follows:

$$\begin{cases} Ex = \alpha Ex_1 + (1-\alpha)Ex_2 \\ En = \dfrac{\alpha Ex_1 En_1 + (1-\alpha)Ex_2 En_2}{\alpha Ex_1 + (1-\alpha)Ex_2} \\ He = \sqrt{He_1^2 + He_2^2} \end{cases}, \tag{5}$$

where $\alpha \in [0,1]$ is the adjustment coefficient affecting three numerical features of the floating cloud C.

3.3. Pawlak rough set and multi-granularity rough set

Definition 6. (Pawlak, 1982) Let U be a non-empty finite universe and $R \in U \times U$ be a binary equivalence relation over universe U, then (U, R) is Pawlak approximation space. The lower and upper approximations for $X \in U$ are defined as follows:

$$\begin{aligned} &\underline{R}(X) = \cup\{[x]_R | [x]_R \subseteq X, X \in U\}, \\ &\overline{R}(X) = \cup\{[x]_R | [x]_R \cap X \neq \emptyset, X \in U\}, \end{aligned} \tag{6}$$

where $[x]_R$ is the equivalence class of x under the binary equivalence relation R. $X (X \in U)$ is called Pawlak rough set if $\overline{R}(X) \neq \underline{R}(X)$.

The Pawlak rough set only contains a binary relation over the universe, which is regarded as a single granularity rough set. Qian et al. (2009, 2010) extended the Pawlak rough set concerning multiple binary relations over two universes, which is called multi-granularity rough sets (MGRSs). The MGRSs contain the optimistic MGRS and the pessimistic MGRS corresponding to risk preference decision-making and risk-averse decision-making, respectively.

Definition 7. (Qian et al., 2010) Let U and V be two non-empty finite universes and $\{R_1, R_2, \dots, R_m\} \in U \times V$ be m generalized binary equivalence relations over universe $U \times V$, then $(U, V, \{R_i\}_{i=1,2,\dots,m})$ is multi-granularity approximation space over two universes. The optimistic multi-granularity lower approximation $\underline{R}^O_{\sum_{i=1}^m R_i}(X)$ and upper approximation $\overline{R}^O_{\sum_{i=1}^m R_i}(X)$ for $X \subseteq V$ are defined as follows:

$$\underline{R}^O_{\sum_{i=1}^m R_i}(X) = \{x \in U | R_1(x) \subseteq X \vee R_2(x) \subseteq X \vee \cdots \vee R_m(x) \subseteq X\},$$
$$\overline{R}^O_{\sum_{i=1}^m R_i}(X) = \{x \in U | R_1(x) \cap X \neq \emptyset \wedge R_2(x) \cap X \neq \emptyset \wedge \cdots \wedge R_m(x) \cap X \neq \emptyset\}, \tag{7}$$

where $R_i(x) = \{y \in V | x \in U, (x, y) \in R_i\}$. The interval set $(\underline{R}^O_{\sum_{i=1}^m R_i}(X), \overline{R}^O_{\sum_{i=1}^m R_i}(X))$ is called optimistic MGRS over two universes if $\underline{R}^O_{\sum_{i=1}^m R_i}(X) \neq \overline{R}^O_{\sum_{i=1}^m R_i}(X)$.

Definition 8. (Qian et al., 2009) Let U and V be two non-empty finite universes and $\{R_1, R_2, \dots, R_m\} \in U \times V$ be m generalized binary equivalence relations over universe $U \times V$, then $(U, V, \{R_i\}_{i=1,2,\dots,m})$ is multi-granularity approximation space over two universes. The pessimistic multi-granularity lower approximation $\underline{R}^P_{\sum_{i=1}^m R_i}(X)$ and upper approximation $\overline{R}^P_{\sum_{i=1}^m R_i}(X)$ for $X \subseteq V$ are defined as follows:

$$\underline{R}^P_{\sum_{i=1}^m R_i}(X) = \{x \in U | R_1(x) \subseteq X \wedge R_2(x) \subseteq X \wedge \cdots \wedge R_m(x) \subseteq X\},$$
$$\overline{R}^P_{\sum_{i=1}^m R_i}(X) = \{x \in U | R_1(x) \cap X \neq \emptyset \vee R_2(x) \cap X \neq \emptyset \vee \cdots \vee R_m(x) \cap X \neq \emptyset\}, \tag{8}$$

where $R_i(x) = \{y \in V | x \in U, (x, y) \in R_i\}$. The interval set $(\underline{R}^P_{\sum_{i=1}^m R_i}(X), \overline{R}^P_{\sum_{i=1}^m R_i}(X))$ is called pessimistic MGRS over two universes if $\underline{R}^P_{\sum_{i=1}^m R_i}(X) \neq \overline{R}^P_{\sum_{i=1}^m R_i}(X)$.

4. The MCDM framework with FLEs based on MGCRS for EI project evaluation

4.1. Problem description for EI project evaluation

For the EI project evaluation problem, $E = \{e_1, e_2, \dots, e_K\}$ is the set of experts, and $V = \{v_1, v_2, \dots, v_K\}$ is the set of experts' weights. $B = \{b_1, b_2, \dots, b_m\}$ is the criteria set of EI projects, and the criteria weight set is $W = \{W_1, W_2, \dots, W_m\}$. There are n EI projects $X = \{x_1, x_2, \dots, x_n\}$ to be evaluated. Experts express their preference through providing linguistic terms set $L = \{l_0, l_1, \dots, l_g\}$ with symbolic proportions, i.e., FLEs. The evaluation matrix provided by expert e_k using FLEs is $M^k = (m^k_{ij})_{n\times m}$, where m^k_{ij} is an FLE provided by expert e_k over the EI project x_i under criterion b_j.

The proposed MCDM framework for EI project evaluation contains three parts: (1) The transformation between FLE and cloud model. To quantify linguistic in FLEs and obtain a normalized FLE, the FLEs in $M^k = (m^k_{ij})_{n\times m}$ are converted into corresponding clouds according to the cloud model. Therefore, the cloud matrix $R^k = (\tilde{C}^k_{ij})_{n\times m}$ can be obtained. (2) Determine weights of criteria using the Shannon entropy method. Based on the cloud matrix, using the Shannon entropy method to obtain the criteria weights. (3) The ranking method is based on MGCRS over two universes. The multiple decision-making cloud information system over two universes is presented, and the optimistic and pessimistic MGCRSs over two universes are proposed. Furthermore, the comprehensive MGCRS is proposed to rank these EI projects. The MCDM framework for EI project evaluation is shown in Fig. 4.

4.2. Transformation between FLE and cloud model

An FLE is composed of a series of linguistic terms and symbolic proportions. To obtain the corresponding cloud model, the first step is to convert these linguistic terms into the basic clouds. Inspired by Wang et al. (2014), the method transforming linguistic terms into the basic clouds is as follows:

(1) Calculate δ_i. Experts tend to be risk-sensitive when evaluating different EI projects. Therefore, $\delta_i = H(l_i)$ can be obtained using Eq. (1). The absolute deviation between adjacent linguistic terms also increases with the extension from the middle of the linguistic term to both ends.

(2) Calculate Ex_i. According to the effective domain $U = [U^L, U^U]$, $Ex_i = U^L + \delta_i(U^U - U^L)$ can be calculated. Therefore, we can obtain $Ex_0 = U^L$ and $Ex_g = U^U$.

(3) Calculate En_i. For a cloud drop (x, y), $x \sim N(Ex, En'^2)$ means that the '3σ principle' of the normal distribution curve should be satisfied, i.e., $3En'_i = \max\{U^U - Ex_i, Ex_i - U^L\}$. Since $En' \sim N(En, He^2)$, En_i can be regarded as the expectation of En' corresponding to the ith cloud and its adjacent clouds. Then En'_i and En_i can be determined by the following two equations:

$$En'_i = \begin{cases} \dfrac{(1-\delta_i)(U^U - U^L)}{3} & (i = 0, 1, \dots, \frac{g}{2}) \\ \dfrac{\delta_i(U^U - U^L)}{3} & (i = \frac{g}{2}+1, \frac{g}{2}+2, \dots, g) \end{cases}, \text{ and}$$

$$En_i = \begin{cases} \dfrac{En'_{i+1} + En'_i}{2} & (i = 0) \\ \dfrac{En'_{i-1} + En'_i + En'_{i+1}}{3} & (0 < i < g) \\ \dfrac{En'_{i-1} + En'_i}{2} & (i = g) \end{cases}.$$

(4) Calculate He_i. Due to $En' \sim N(En, He^2)$, He_i should obey the '3σ principle' of the normal distribution curve, then $He_i = \frac{\max\{\max_k\{En'_k\} - En_i, En_i - \min_k\{En'_k\}\}}{3}$ $(i = 0, 1, \dots, g)$.

(5) Obtain the basic clouds $C_i = (Ex_i, En_i, He_i)$. The basic cloud C_i corresponding to the linguistic term l_i can be obtained, which is composed of the three numerical features Ex_i, En_i, and He_i.

Based on the above method, all linguistic terms in s_L can be transformed into the corresponding basic clouds. In other words, an FLE $m_L = \{(s^1_L, p^1), (s^2_L, p^2), \dots, (s^T_L, p^T)\}$ can be transformed into a cloud-FLE $m'_L = \{(C^1_L, p^1), (C^2_L, p^2), \dots, (C^T_L, p^T)\}$, where $C^t_L (t = 1, 2, \dots, T)$ is the set of basic clouds corresponding to linguistic term set s^t_L.

Example 1. Given the domain $U = [0, 10]$ and the linguistic evaluation set $L = \{l_0 : very\ poor, l_1 : poor, l_2 : fair, l_3 : good, l_4 : very\ good\}$, then $\delta = (\delta_0, \delta_1, \delta_2, \delta_3, \delta_4) = (0, 0.2890, 0.5, 0.71097, 1)$. These linguistic variables can be converted into asymmetric normal clouds, which are as follows: $C_0 = (0, 2.8518, 0.3950)$, $C_1 = (2.8892, 2.4568, 0.2922)$, $C_2 = (5, 2.1357, 0.3992)$, $C_3 = (7.1108, 2.4568, 0.2922)$, $C_4 = (10, 2.8518, 0.3950)$. Therefore, an FLE $m_L = \{(\{s_0, s_1\}, 0.3), (\{s_2, s_3\}, 0.2), (\{s_4\}, 0.2)\}$ can be transformed into a cloud-FLE $m'_L = \{(C^1_L, 0.3), (C^2_L, 0.2), (C^3_L, 0.2)\}$, i.e., $m'_L = \{(\{C_0, C_1\}, 0.3), (\{C_2, C_3\}, 0.2), (\{C_4\}, 0.2)\}$.

After obtaining the basic clouds, the approximate cloud between these basic clouds can be obtained through the definition of floating cloud using Eq. (5), in which the calculation of float hyper entropy ignored the effect of α on He. In this way, the floating cloud may be thicker than two basic clouds because the float hyper entropy may be larger than the hyper entropies of two clouds. For example, the floating cloud between $C_0 = (0, 2.8518, 0.3950)$ and $C_1 = (2.8892, 2.4568, 0.2922)$ is $C = (1.4446, 2.4568, 0.4913)$ when $\alpha = 0.5$. In this case, the uncertainty of the floating cloud will increase as the number of basic clouds. To obtain the float hyper entropy He closer to the basic normal clouds, the adjustment coefficient α is considered in the process of fusing two hyper entropies. Furthermore, to aggregate more than two basic clouds, the floating cloud is extended into the case of n clouds $C_i (i = 1, 2, \dots, n)$, which is shown in Definition 9.

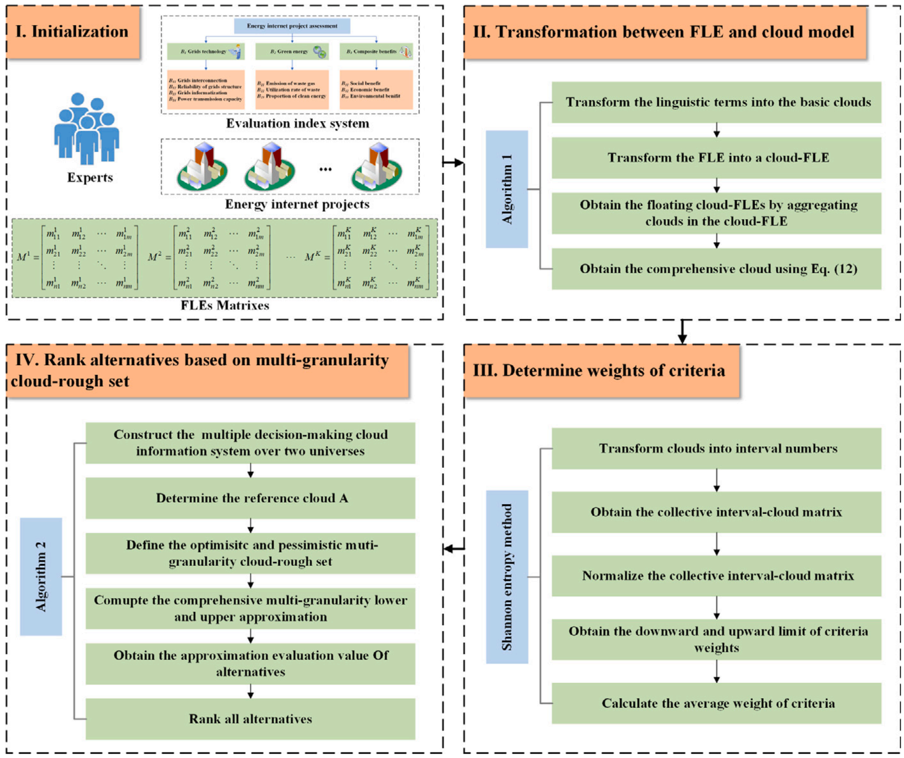


Fig. 4. Flowchart of the proposed MCDM framework for EI project evaluation.

Definition 9. If there are n basic clouds $C_i (i = 1, 2, \ldots, n)$ in the universe, then the floating cloud $C = (Ex, En, He)$ between n clouds is calculated as follows:

$$\begin{cases} Ex = \sum_{i=1}^{n} \alpha_i Ex_i \\ En = \dfrac{\sum_{i=1}^{n} \alpha_i Ex_i En_i}{\sum_{i=1}^{n} \alpha_i Ex_i} \\ He = \sqrt{\sum_{i=1}^{n} \alpha_i He_i^2} \end{cases}, \tag{9}$$

where $\alpha_i \in [0, 1]$ is the adjustment coefficient corresponding to the basic cloud C_i, and $\sum_{i=1}^{n} \alpha_i = 1$.

Based on the concept of floating cloud, all basic clouds in $C_L^t (t = 1, 2, \ldots, T)$ can be formed as a floating cloud which can be regarded as an approximate cloud between these basic clouds. Therefore, a cloud-FLE $m_L' = \{(C_L^1, p^1), (C_L^2, p^2), \ldots, (C_L^T, p^T)\}$ can be converted into a floating-cloud-FLE $m^f = \{(C^1, p^1), (C^2, p^2), \ldots, (C^T, p^T)\}$, where $C^t (t = 1, 2, \ldots, T)$ is a floating cloud element between these clouds in C_L^t.

Example 2. For the cloud-FLE $m_L' = \{(\{C_0, C_1\}, 0.3), (\{C_2, C_3\}, 0.2), (\{C_4\}, 0.2)\}$ in Example 1, the clouds in s_L can be transformed into the floating clouds using Eq. (9). If clouds have the same adjustment coefficients, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \frac{1}{n}$, then the first floating cloud C^1 between two clouds C_0 and C_1 is $C^1 = (1.4446, 2.4568, 0.3474)$ and the second floating cloud C^2 between two clouds C_2 and C_3 is $C^2 = (6.0554, 2.3242, 0.3498)$. Since the third element $(\{C_4\}, 0.2)$ in m_L' contains only one cloud C_4, the third floating cloud is $C^3 = C_4 = (10, 2.8518, 0.3950)$. Therefore, the cloud-FLE m_L' can be transformed into a floating-cloud-FLE, i.e., $m^f = \{(C^1, 0.3), (C^2, 0.2), (C^3, 0.2)\}$.

The floating cloud-FLE can be regarded as the approximate cloud set with symbolic proportions between these basic clouds. However, the sum of these symbolic proportions may not be 1, which should be reassigned to each floating-cloud-FLE element. To obtain a normalized cloud-FLE according to the floating cloud-FLE, a programming model is established to convert linguistic variables in a cloud-FLE into several clouds with a normalized probability distribution.

Since the sum of symbolic proportions $p^t (t = 1, 2, \ldots, T)$ in a floating-cloud-FLE $m^f = \{(C^1, p^1), (C^2, p^2), \ldots, (C^T, p^T)\}$ might not be 1, the programming model in Eq. (10) is proposed to obtain a normalized cloud-FLE $\tilde{m}^f = \{(\tilde{C}^1, \tilde{p}^1), (\tilde{C}^2, \tilde{p}^2), \ldots, (\tilde{C}^T, \tilde{p}^T)\}$, where $\sum_{t=1}^{T} \tilde{p}^t = 1$. The main idea of the programming model is to obtain the normalized cloud-FLE $\tilde{m}^f$ that is as close to the floating-cloud-FLE m^f as possible. Therefore, the minimum objective function includes two parts: (1) The weighted distance between the floating-cloud-FLE m^f and the normalized cloud-FLE $\tilde{m}^f$, denoted as $\sum_{t=1}^{T} d(\tilde{C}^t, C^t) * \tilde{p}^t$. (2) The sum of distances between symbolic proportions, denoted as $\sum_{t=1}^{T} |\tilde{p}^t - p^t|$. Then the programming model is as follows:

$$\min \sum_{t=1}^{T} d(\tilde{C}^t, C^t) * \tilde{p}^t + \sum_{t=1}^{T} |\tilde{p}^t - p^t|$$

$$s.t. \begin{cases} \sum_{t=1}^{T} \tilde{p}^t = 1 \\ 0 \le \tilde{p}^t \le 1 \\ d(\tilde{C}^t, C^t) = \sqrt{\frac{1}{2}((Ex^t - \widetilde{Ex}^t)^2 + (En^t - \widetilde{En}^t)^2 + (He^t - \widetilde{He}^t)^2)} \\ \widetilde{Ex}^t - 3\widetilde{En}^t - 9\widetilde{He}^t > 0 \\ \widetilde{En}^t \ge 3\widetilde{He}^t \\ \widetilde{Ex}^t - 3\widetilde{En}^t \ge U^L \\ \widetilde{Ex}^t + 3\widetilde{En}^t \le U^U \\ \widetilde{En}^t \ge 0, \widetilde{En}^t \ge 0, \widetilde{He}^t \ge 0 \end{cases}, \tag{10}$$

where the tth cloud in the floating-cloud-FLE m^f is $C^t = (Ex^t, En^t, He^t)$, and the tth cloud in the normalized cloud-FLE is $\tilde{C}^t = (\widetilde{Ex}^t, \widetilde{En}^t, \widetilde{He}^t)$.

In the programming model in Eq. (10), the objective function is to obtain the normalized cloud-FLE $\tilde{m}^f = \{(\tilde{C}^1, \tilde{p}^1), (\tilde{C}^2, \tilde{p}^2), \dots, (\tilde{C}^T, \tilde{p}^T)\}$. Some extra constraints should be considered: (1) $\tilde{p}^t (t = 1, 2, \dots, T)$ is the tth normalized symbolic proportion, then $\sum_{t=1}^{T} \tilde{p}^t = 1$. (2) $\widetilde{Ex}^t$, $\widetilde{En}^t$ and $\widetilde{He}^t$ are non-negative numbers. (3) Based on the '3σ principle', it also satisfies that $3\widetilde{En'}_i = \max\{U^U - \widetilde{Ex}_i, \widetilde{Ex}_i - U^L\}$, which can be rewritten as $(\widetilde{Ex}^t \pm 3\widetilde{En}^t) \in [U^L, U^U]$ since En_i can be regarded as an approximate value of En_i'. (4) Based on the '$3En$ principle', $\widetilde{En}^t \geq 3\widetilde{He}^t$ should also be satisfied.

Remark 1. The above programming model in Eq. (10) is guaranteed to have at least one optimal solution under the following conditions: (1) the objective function is continuous, and (2) the feasible region is non-empty, closed, and bounded. Firstly, the objective function consists of terms involving Euclidean distances, absolute values, and linear combinations, which are well-known to be continuous functions. Since continuity is preserved under summation and scalar multiplication, the entire objective function is continuous. Secondly, the constraint $\sum_{t=1}^{T} \tilde{p}^t = 1$ and $0 \leq \tilde{p}^t \leq 1$ define a standard simplex due to $T > 0$, and the constraints related to $\widetilde{Ex}^t, \widetilde{En}^t, \widetilde{He}^t$ are compatible, then the feasible region satisfies the non-emptiness. Thirdly, all constraints are continuous functions of the decision variables. The feasible region is defined by these constraints as the preimage of closed intervals under continuous functions, which ensures it is closed. Finally, boundedness is guaranteed by the simplex constraint $\sum_{t=1}^{T} \tilde{p}^t = 1$ and $0 \leq \tilde{p}^t \leq 1$, as this restricts all $\tilde{p}^t$ to lie within a finite region, and the remaining constraints involving $\widetilde{Ex}^t, \widetilde{En}^t, \widetilde{He}^t$ further confine the feasible region to a bounded subset of the decision space. The Weierstrass Theorem states that a continuous function achieves its minimum on a non-empty, closed, and bounded set. Given that the objective function is continuous and the feasible region is non-empty, closed, and bounded, the optimization problem satisfies the conditions of the Weierstrass Theorem. Therefore, the model is guaranteed to have at least one optimal solution.

Furthermore, a cloud gathering the normalized cloud-FLE element can be obtained by Definition 10.

Definition 10. If there are n elements in a normalized cloud-FLE $\tilde{m}^f = \{(\tilde{C}^1, \tilde{p}^1), (\tilde{C}^2, \tilde{p}^2), \dots, (\tilde{C}^T, \tilde{p}^T)\}$, then a comprehensive cloud $\tilde{C} = (\widetilde{Ex}, \widetilde{En}, \widetilde{He})$ is calculated as follows:

$$\begin{cases} \widetilde{Ex} = \sum_{t=1}^{T} \widetilde{Ex}^t * \tilde{p}^t_{ij} \\ \widetilde{En} = \sqrt{\sum_{t=1}^{T} \left(\widetilde{En^t}\right)^2 * \tilde{p}^t} \\ \widetilde{He} = \sqrt{\sum_{t=1}^{T} \left(\widetilde{He}^t\right)^2 * \tilde{p}^t} \end{cases}. \tag{11}$$

Based on the above methods, an FLE m_L can be converted into a comprehensive cloud $\tilde{C} = (\widetilde{Ex}, \widetilde{En}, \widetilde{He})$ using the transformation model which is shown in Algorithm 1. Furthermore, each FLE matrix $M = (m_{ij})_{n\times m}$ can be converted into a comprehensive cloud matrixes $R = (\tilde{C}_{ij})_{n\times m}$ using the programming model in Eq. (12), where $\tilde{C}_{ij} = (\widetilde{Ex}_{ij}, \widetilde{En}_{ij}, \widetilde{He}_{ij})$. Obviously, the programming model in Eq. (12), which incorporates the aggregation formula Eq. (11), is guaranteed to have at least one optimal solution because the original programming model in Eq. (10) already satisfies the conditions for the existence of an optimal solution. The inclusion of the aggregation formula does not alter the continuity of the objective function or the compactness, i.e., non-emptiness, closedness, and boundedness of the feasible region, as it is formulated in a manner consistent with the original constraints. Thus, the programming model in Eq. (12) inherits the existence of an optimal solution from the original model.

$$\min \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} d(\tilde{C}^t_{ij}, C^t) * \tilde{p}^t_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} \left|\tilde{p}^t_{ij} - p^t_{ij}\right|}{mn}$$
$$s.t. \begin{cases} \sum_{t=1}^{T} \tilde{p}^t_{ij} = 1 \\ 0 \leq \tilde{p}^t_{ij} \leq 1 \\ d(\tilde{C}^t_{ij}, C^t_{ij}) = \sqrt{\frac{1}{2}((Ex^t_{ij} - \widetilde{Ex}^t_{ij})^2 + (En^t_{ij} - \widetilde{En}^t_{ij})^2 + (He^t_{ij} - \widetilde{He}^t_{ij})^2)} \\ \widetilde{Ex}^t_{ij} - 3\widetilde{En}^t_{ij} - 9\widetilde{He}^t_{ij} > 0 \\ \widetilde{En}^t_{ij} \geq 3\widetilde{He}^t_{ij} \\ \widetilde{Ex}^t_{ij} - 3\widetilde{En}^t_{ij} \geq U^L \\ \widetilde{Ex}^t_{ij} + 3\widetilde{En}^t_{ij} \leq U^U \\ \widetilde{En}^t_{ij} \geq 0, \widetilde{En}^t_{ij} \geq 0, \widetilde{He}^t_{ij} \geq 0 \\ \widetilde{Ex}_{ij} = \sum_{t=1}^{T} \widetilde{Ex}^t_{ij} * \tilde{p}^t_{ij} \\ \widetilde{En}_{ij} = \sqrt{\sum_{t=1}^{T} \left(\widetilde{En^t_{ij}}\right)^2 * \tilde{p}^t_{ij}} \\ \widetilde{He}_{ij} = \sqrt{\sum_{t=1}^{T} \left(\widetilde{He}^t_{ij}\right)^2 * \tilde{p}^t_{ij}} \\ i = 1, 2, \dots, n; j = 1, 2, \dots, m. \end{cases} \tag{12}$$

Algorithm 1. The algorithm of transformation between FLE and cloud model

Input: A fixed linguistic term set $L = \{l_0, l_1, \dots, l_g\}$ and the FLE $m_L = \{(s^1_L, p^1), (s^2_L, p^2), \dots, (s^T_L, p^T)\}$.
Output: The comprehensive cloud $\tilde{C} = (\widetilde{Ex}, \widetilde{En}, \widetilde{He})$.
Step 1. Transform the linguistic terms into the basic clouds. Compute $\delta_i = H(l_i)(i = 0, 1, \dots, g)$ using Eq. (1), and then compute expectation Ex_i, entropy En_i and hyper entropy He_i to obtain g asymmetric basic clouds $C_i = (Ex_i, En_i, He_i)$.
Step 2. Transform the FLE into a cloud-FLE. For the FLE $m_L = \{(s^1_L, p^1), (s^2_L, p^2), \dots, (s^T_L, p^T)\}$, linguistic variables in $s^t_L (t = 1,2,\dots,T)$ can be replaced by their corresponding basic clouds. Therefore, a cloud-FLE $m'_L = \{(C^1_L, p^1), (C^2_L, p^2), \dots, (C^T_L, p^T)\}$ can be obtained, where C^t_L is the set of basic clouds corresponding to linguistic term set s^t_L.
Step 3. Obtain the floating cloud-FLEs by aggregating clouds in the cloud-FLE. The cloud-FLE $m'_L = \{(C^1_L, p^1), (C^2_L, p^2), \dots, (C^T_L, p^T)\}$ can be converted into a floating-cloud-FLE $m^f = \{(C^1, p^1), (C^2, p^2), \dots, (C^T, p^T)\}$ using Eq. (9).
Step 4. Compute the normalized cloud-FLE. Based on the floating cloud-FLEs, solve the programming model in Eq. (10) to obtain the normalized cloud-FLE $\tilde{m}^f = \{(\tilde{C}^1, \tilde{p}^1), (\tilde{C}^2, \tilde{p}^2), \dots, (\tilde{C}^T, \tilde{p}^T)\}$.
Step 5. Aggregate all elements in the normalized cloud-FLE. For a normalized cloud-FLE $\tilde{m}^f = \{(\tilde{C}^1, \tilde{p}^1), (\tilde{C}^2, \tilde{p}^2), \dots, (\tilde{C}^T, \tilde{p}^T)\}$, aggregate T elements using the weighted average (WA) operators in Eq. (11) or (12) to obtain a comprehensive cloud $\tilde{C} = (\widetilde{Ex}, \widetilde{En}, \widetilde{He})$.

4.3. Determine weights of criteria using Shannon entropy method

For the sake of calculation, clouds can be converted into other forms like interval numbers. Zhou, Su, and Zeng (2016) proposed a transformation method between cloud model and interval number $[\underline{C}, \overline{C}]$. Therefore, a comprehensive cloud $\tilde{C} = (\widetilde{Ex}, \widetilde{En}, \widetilde{He})$ can be converted into interval number $[\underline{C}, \overline{C}]$, which is calculated as follows:

$$\begin{cases} \overline{C} = \widetilde{Ex} + 3(\widetilde{En} + 3\widetilde{He}) \\ \underline{C} = \widetilde{Ex} - 3(\widetilde{En} + 3\widetilde{He}) \end{cases}. \tag{13}$$

The evaluation matrix provided by expert e_k using FLEs is $M^k = (m^k_{ij})_{n\times m}$, and the comprehensive cloud matrix $R^k = (\tilde{C}^k_{ij})_{n\times m}$ where $\tilde{C}^k_{ij} = (\widetilde{Ex}^k_{ij}, \widetilde{En}^k_{ij}, \widetilde{He}^k_{ij})$ can be obtained by Algorithm 1. The comprehensive cloud matrix $R^k (k = 1, 2, \dots, K)$ can be transformed into interval-cloud matrix $RI^k = (r^k_{ij})_{n\times m}$, where $r^k_{ij} = [\underline{C}^k_{ij}, \overline{C}^k_{ij}]$ can be calculated using Eq. (13). Based on the weighted average (WA) operator, the collective interval-cloud matrix is $RI^c = (r_{ij})_{n\times m}$, where $r_{ij} = [\underline{C}_{ij}, \overline{C}_{ij}]$, $\underline{C}_{ij} = \sum_{k=1}^{K} \upsilon_k \underline{C}_{ij}^k$ and $\overline{C}_{ij} = \sum_{k=1}^{K} \upsilon_k \overline{C}_{ij}^k$. The weights of criteria can be determined by the Shannon entropy method (Shang, Yang, Barnes, & Wu, 2022; Zhao, Li, Wang, & Yuan, 2020) which are as follows:

Step 1. Normalize the collective interval-cloud matrix by calculating $\underline{h}_{ij}$ and $\overline{h}_{ij}$.

$$\underline{h}_{ij} = \frac{\underline{C}_{ij}}{\sum_{i=1}^{n} \underline{C}_{ij}}, \overline{h}_{ij} = \frac{\overline{C}_{ij}}{\sum_{i=1}^{n} \overline{C}_{ij}},\ i = 1,2,\dots,n; j = 1,2,\dots,m. \tag{14}$$

Step 2. Calculate the lower entropy $\underline{g}_j$ of $\underline{h}_{ij}$ and the upper entropy $\overline{g}_j$ of $\overline{h}_{ij}$.

$$\begin{aligned}\underline{g}_j &= -\frac{1}{\ln n}\sum_{i=1}^{n} \underline{h}_{ij} \ln \underline{h}_{ij}, j = 1,2,\dots,m,\\ \overline{g}_j &= -\frac{1}{\ln n}\sum_{i=1}^{n} \overline{h}_{ij} \ln \overline{h}_{ij}, j = 1,2,\dots,m.\end{aligned} \tag{15}$$

Step 3. Obtain the downward limit $\underline{W}_j$ and upward limit $\overline{W}_j$ of criteria weights.

$$\underline{W}_j = \frac{1-\underline{g}_j}{\sum_{j=1}^{m}(1-\underline{g}_j)}, \overline{W}_j = \frac{1-\overline{g}_j}{\sum_{j=1}^{m}(1-\overline{g}_j)}, j = 1,2,\dots,m. \tag{16}$$

Step 4. Calculate the average weight W_j of criterion b_j.

$$W_j = \frac{\overline{W}_j + \underline{W}_j}{\sum_{j=1}^{m}(\overline{W}_j + \underline{W}_j)}, j = 1,2,\dots,m. \tag{17}$$

Based on the above methods, the weight set $W = \{W_1, W_2, \dots, W_m\}$ can be obtained by the Shannon entropy method based on all evaluation matrixes using FLEs.

4.4. The ranking method based on multi-granularity cloud-rough set over two universes for EI project evaluation

We call five-tuple (X, E, F, R, B) a multiple decision-making cloud information system over two universes, where $X = \{x_1, x_2, \dots, x_n\}$ is the set of EI projects, $E = \{e_1, e_2, \dots, e_K\}$ is the expert set and $B = \{b_1, b_2, \dots, b_m\}$ is the criteria set. $F = \{f^1, f^2, \dots, f^m\}$ is a family of mapping set between X and E. For the criterion $b_j \in B$, the comprehensive cloud evaluation value provided by expert $e_k \in E$ over the EI project x_i can be mapped as $f^j\colon X \times E \to \Gamma^j$, where Γ^j is the range of cloud evaluation value. $R = \{R^1, R^2, \dots, R^m\}$ is the comprehensive cloud evaluation matrix set provided by expert set $E = \{e_1, e_2, \dots, e_K\}$, and $R^j = (C_{ik}^j)_{n\times K}$ is the comprehensive cloud evaluation matrix over the EI project x_i under criterion b_j, which can be obtained using Algorithm 1. Based on the definition of multiple decision-making cloud information systems over two universes, the optimistic MGCRS over two universes and pessimistic MGCRS over two universes are defined as follows, respectively.

Definition 11. Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \dots, m)$ is the binary cloud relation between universe X and E. For any $A \in F(E)$, $e \in E$ and $x \in X$, the optimistic multi-granularity lower approximation $\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$ and upper approximation $\overline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$ of A with respect to (X, E, F, R, B) are as follows:

$$\begin{aligned}\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) &= \vee_{j=1}^m \wedge_{e\in E} \max(N(R^j(x,e)), A(e)), x \in X,\\ \overline{R}^O_{\sum_{j=1}^m R^j}(A)(x) &= \wedge_{j=1}^m \vee_{e\in E} \min(R^j(x,e), A(e)), x \in X,\end{aligned} \tag{18}$$

where $\vee$ and $\wedge$ are the maximum operator and minimum operator, respectively. The binary cloud relation under criterion b_j is $R^j(x,e) = (Ex_{R^j}(x,e), En_{R^j}(x,e), He_{R^j}(x,e))$ and $N(R^j(x,e))$ is the negation operator of $R^j(x,e)$. The interval set $(\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x), \overline{R}^O_{\sum_{j=1}^m R^j}(A)(x))$ is called optimistic MGCRS over two universes if $\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) \neq \overline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$.

Remark 2. Let $Ex_{R^j}(x,e)$ and $N(Ex_{R^j}(x,e))$ be symmetric about the middle point in the effective domain $U = [U^L, U^U]$, then $N(Ex_{R^j}(x,e)) = U^U + U^L - Ex_{R^j}(x,e)$. Since entropy En and hyper entropy He reflect the uncertainty degree, let the negations of entropy and hyper entropy be $N(En_{R^j}(x,e)) = En_{R^j}(x,e)$ and $N(He_{R^j}(x,e)) = He_{R^j}(x,e)$. For a cloud $C = (Ex, En, He)$, large Ex, small En, and small He are expected numerical features. Therefore, the negation of $\vee_{j=1}^m \wedge_{e\in E} \max(N(En_{R^j}(x,e)), En_A(e))$ is $\wedge_{j=1}^m \vee_{e\in E} \min(En_{R^j}(x,e), En_A(e))$, and the negation of $\vee_{j=1}^m \wedge_{e\in E} \max(N(He_{R^j}(x,e)), He_A(e))$ is $\wedge_{j=1}^m \vee_{e\in E} \min(He_{R^j}(x,e), He_A(e))$. The upper approximation $\overline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$ can be calculated in the same way.

Therefore, Eq. (18) can be rewritten as follows:

$$\begin{aligned}\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) &= \{x|x\in X, \langle x, \underline{Ex}^O_A(x), \underline{En}^O_A(x), \underline{He}^O_A(x)\rangle\},\\ \overline{R}^O_{\sum_{j=1}^m R^j}(A)(x) &= \{x|x\in X, \langle x, \overline{Ex}^O_A(x), \overline{En}^O_A(x), \overline{He}^O_A(x)\rangle\},\end{aligned} \tag{19}$$

where $\underline{Ex}^O_A(x) = \vee_{j=1}^m \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e))$, $\underline{En}^O_A(x) = \wedge_{j=1}^m \vee_{e\in E} \min(En_{R^j}(x,e), En_A(e))$ and $\underline{He}^O_A(x) = \wedge_{j=1}^m \vee_{e\in E} \min(He_{R^j}(x,e), He_A(e))$ in the lower approximation $\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$. And $\overline{Ex}^O_A(x) = \wedge_{j=1}^m \vee_{e\in E} \min(Ex_{R^j}(x,e), Ex_A(e))$, $\overline{En}^O_A(x) = \vee_{j=1}^m \wedge_{e\in E} \max(En_{R^j}(x,e), En_A(e))$ and $\overline{He}^O_A(x) = \vee_{j=1}^m \wedge_{e\in E} \max(He_{R^j}(x,e), He_A(e))$ in the upper approximation $\overline{R}^O_{\sum_{j=1}^m R^j}(A)(x)$.

Definition 12. Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \dots, m)$ is the binary cloud relation between universe X and E. For any $A \in F(E)$, $e \in E$ and $x \in X$, the pessimistic multi-granularity lower approximation $\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x)$ and upper approximation $\overline{R}^P_{\sum_{j=1}^m R^j}(A)(x)$ of A with respect to (X, E, F, R, B) are as follows:

$$\begin{aligned}\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x) &= \wedge_{j=1}^m \wedge_{e\in E} \max(N(R^j(x,e)), A(e)), x \in X,\\ \overline{R}^P_{\sum_{j=1}^m R^j}(A)(x) &= \vee_{j=1}^m \vee_{e\in E} \min(R^j(x,e), A(e)), x \in X,\end{aligned} \tag{20}$$

where $R^j(x,e) = (Ex_{R^j}(x,e), En_{R^j}(x,e), He_{R^j}(x,e))$ and $N(R^j(x,e))$ is the negation operator of $R^j(x,e)$. The interval set $(\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x), \overline{R}^P_{\sum_{j=1}^m R^j}(A)(x))$ is called pessimistic MGCRS over two universes if $\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x) \neq \overline{R}^P_{\sum_{j=1}^m R^j}(A)(x)$.

Similarly, Eq. (20) can be rewritten as follows:

$$\begin{aligned}\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x) &= \{x|x\in X, \langle x, \underline{Ex}^P_A(x), \underline{En}^P_A(x), \underline{He}^P_A(x)\rangle\},\\ \overline{R}^P_{\sum_{j=1}^m R^j}(A)(x) &= \{x|x\in X, \langle x, \overline{Ex}^P_A(x), \overline{En}^P_A(x), \overline{He}^P_A(x)\rangle\},\end{aligned} \tag{21}$$

where $\underline{Ex}^P_A(x) = \wedge_{j=1}^m \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e))$, $\underline{En}^P_A(x) = \vee_{j=1}^m \vee_{e\in E} \min(En_{R^j}(x,e), En_A(e))$ and $\underline{He}^P_A(x) = \vee_{j=1}^m \vee_{e\in E} \min((He_{R^j}(x,e), He_A(e))$ in the lower approximation $\underline{R}^P_{\sum_{j=1}^m R^j}(A)(x)$. And $\overline{Ex}^P_A(x) = \vee_{j=1}^m \vee_{e\in E} \min(Ex_{R^j}(x,e), Ex_A(e))$, $\overline{En}^P_A(x) = \wedge_{j=1}^m \wedge_{e\in E} \max(En_{R^j}(x,e), En_A(e))$ and $\overline{He}^P_A(x) = \wedge_{j=1}^m \wedge_{e\in E} \max(He_{R^j}(x,e), He_A(e))$ in the upper approximation $\overline{R}^P_{\sum_{j=1}^m R^j}(A)(x)$.

Due to space constraints, the relevant theorems and proofs for the optimistic and pessimistic MGCRSs can be found in Appendix A. Based on the optimistic and pessimistic MGCRSs, the comprehensive MGCRS is defined as follows.

Definition 13. Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \ldots, m)$ is the binary cloud relation between universe X and E. For any $A \in F(E)$, $e \in E$ and $x \in X$, the comprehensive multi-granularity lower approximation $\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ and upper approximation $\overline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ of A with respect to (X, E, F, R, B) are as follows:

$$\begin{aligned}&\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i) = \sum_{j=1}^{m} W_j \wedge_{e\in E} \max(N(R^j(x_i, e_k)), A(e_k)), x_i \in X,\\&\overline{R}_{\sum_{j=1}^m R^j}(A)(x_i) = \sum_{j=1}^{m} W_j \vee_{e\in E} \min(R^j(x_i, e_k), A(e_k)), x_i \in X.\end{aligned} \tag{22}$$

Eq. (22) can be rewritten as follows:

$$\begin{aligned}&\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i) = \{x_i | x_i \in X, \langle x_i, \underline{Ex}_A(x_i), \underline{En}_A(x_i), \underline{He}_A(x_i)\rangle\},\\&\overline{R}_{\sum_{j=1}^m R^j}(A)(x_i) = \{x_i | x_i \in X, \langle x_i, \overline{Ex}_A(x_i), \overline{En}_A(x_i), \overline{He}_A(x_i)\rangle\},\end{aligned} \tag{23}$$

where $\underline{Ex}_A(x_i) = \sum_{j=1}^m W_j \wedge_{e_k\in E} \max((U^U + U^L - Ex_{R^j}(x_i, e_k)), Ex_A(e_k))$, $\underline{En}_A(x_i) = \sqrt{\sum_{j=1}^m W_j(\vee_{e_k\in E} \min(Ex_{R^j}(x_i, e_k), En_A(e_k)))^2}$ and $\underline{He}_A(x_i) = \sqrt{\sum_{j=1}^m W_j(\vee_{e_k\in E} \min(He_{R^j}(x_i, e_k), He_A(e_k)))^2}$ in the lower approximation $\underline{R}_{\sum_{j=1}^m R^j}(A)(x)$. And $\overline{Ex}_A(x_i) = \sum_{j=1}^m W_j \vee_{e_k\in E} \min(Ex_{R^j}(x_i, e_k), Ex_A(e_k))$, $\overline{En}_A(x_i) = \sqrt{\sum_{j=1}^m W_j(\wedge_{e_k\in E} \max(En_{R^j}(x_i, e_k), En_A(e_k)))^2}$ and $\overline{He}_A(x_i) = \sqrt{\sum_{j=1}^m W_j(\wedge_{e_k\in E} \max(He_{R^j}(x_i, e_k), He_A(e_k)))^2}$ in the upper approximation $\overline{R}_{\sum_{j=1}^m R^j}(A)(x)$.

Therefore, the approximation evaluation value $R_{\sum_{j=1}^m R^j}(A)(x_i)$ of A for x_i using MGCRS over two universes is as follows:

$$R_{\sum_{j=1}^m R^j}(A)(x_i) = \theta \overline{R}_{\sum_{j=1}^m R^j}(A)(x_i) + (1-\theta)\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i), \tag{24}$$

where θ is the preference coefficient and $\theta \in [0, 1]$.

The reference cloud $A = (Ex_A(E), En_A(E), He_A(E))$ where $E = \{e_1, e_2, \ldots, e_K\}$ can be determined by aggregating all cloud matrixes $R^j(j = 1, 2, \ldots, m)$ under criterion b_j. The positive ideal solution (PIS) and the negative ideal solution (NIS) under criterion b_j are defined as follows:

$$\begin{aligned}&C^{j+} = \{C_1^{j+}, C_2^{j+}, \ldots, C_K^{j+}\}, j = 1, 2, \ldots, m,\\&C^{j-} = \{C_1^{j-}, C_2^{j-}, \ldots, C_K^{j-}\}, j = 1, 2, \ldots, m,\end{aligned} \tag{25}$$

where $C_k^{j+} = \max\{C_k^j | k = 1, 2, \ldots, K\}$ and $C_k^{j-} = \min\{C_k^j | k = 1, 2, \ldots, K\}$. Elements in the jth reference cloud $C^j = \{C_1^j, C_2^j, \ldots, C_K^j\}$ under criterion $b_j(j = 1, 2, \ldots, m)$ can be calculated as follows:

$$C_k^j = \gamma C_k^{j+} + (1-\gamma)C_k^{j-}, j = 1, 2, \ldots, m; k = 1, 2, \ldots, K, \tag{26}$$

where γ is the risk preference coefficient and $\gamma \in [0, 1]$.

Therefore, the reference cloud $A = (Ex_A(E), En_A(E), He_A(E))$ is obtained by $A = \sum_{j=1}^m W_j C^j$, which is as follows:

$$\begin{cases} Ex_A(E) = \sum_{j=1}^{m} W_j Ex^j \\ En_A(E) = \sqrt{\sum_{j=1}^{m} W_j (En^j)^2} \\ He_A(E) = \sqrt{\sum_{j=1}^{m} W_j (He^j)^2} \end{cases}. \tag{27}$$

The ranking method for EI projects using MGCRS over two universes is shown in Algorithm 2.

Algorithm 2. The ranking method for EI projects using MGCRS over two universes

Input: The cloud evaluation matrixes $R^j = (C_{ik}^j)_{n\times K}$ where $r_{ik}^j = (Ex_{ik}^j, En_{ik}^j, He_{ik}^j)$, weight set $W = \{W_1, W_2, \ldots, W_m\}$, preference coefficient θ, risk preference coefficient γ, and the effective domain $U = [U^L, U^U]$.

Output: The approximation evaluation value $R_{\sum_{j=1}^m R^j}(A)(x_i)$ with respect to x_i and the ranking of EI projects.

Step 1. Determine the reference cloud A. For $R^j = (C_{ik}^j)_{n\times K}$, determine the PIS C^{j+} and NIS C^{j-} using Eq. (25) and obtain the jth reference cloud $C^j = \{C_1^j, C_2^j, \ldots, C_K^j\}$ using Eq. (26). The reference cloud A can be calculated by aggregating $C^j(j = 1, 2, \ldots, m)$ using Eq. (27).

Step 2. Obtain the multi-granularity lower and upper approximation of EI projects. Compute the comprehensive multi-granularity lower approximation $\overline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ and upper approximation $\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ of with respect to x_i by Eqs. (22) and (23).

Step 3. Obtain the approximation evaluation value of EI projects. Compute the approximation evaluation value $R_{\sum_{j=1}^m R^j}(A)(x_i)$ of A with respect to x_i using Eq. (24).

Step 4. Obtain the ranking of EI projects. Determine the final ranking by ranking the approximation evaluation values $R_{\sum_{j=1}^m R^j}(A)(x_i)$ of all EI projects in descending order.

5. Case study: the EI project evaluation in the Beijing–Tianjin–Hebei region

5.1. Problem description for EI project evaluation in the Beijing–Tianjin–Hebei region

China's Beijing–Tianjin–Hebei (BTH) region is actively promoting the construction of the green EI based on the principles of the circular economy, aiming to facilitate the circular energy transformation of the traditional power grid industrial system. This transformation focuses on the recycling of resources and sustainable development, enabling the efficient and green operation of energy systems. As a key economic development zone and a renewable energy demonstration area in China, the BTH region faces the dual challenges of upgrading its industrial system and protecting the environment. In June 2024, the BTH Energy Collaboration Task Force was officially established, and a key work plan was formulated. The objectives include promoting the interconnection of energy infrastructure across provinces and regions, cultivating a green, low-carbon energy consumption model, constructing a diversified energy supply system, and driving the innovation and application of key energy technologies. To support this transition, four representative EI projects were selected for evaluation. These projects cover solar power generation, wind power generation, multi-energy integration system optimization, and smart microgrid management, which are the Beijing Haidian North EI Project, the Zhangbei "Internet+ Smart Energy" Wind Power Demonstration Project, the Tianjin Binhai Smart Energy Demonstration Project, and the Xiong'an New Area Green Smart Microgrid Demonstration Project. The aim of evaluating these EI projects is to provide important references for the green energy transition in the BTH region and help achieve its green and low-carbon development goals. Against this backdrop, four experts from universities, the Power Grid Federation, State Grid Corporation, and other institutions will conduct a comprehensive evaluation and analysis of these four EI projects based on ten evaluation criteria. The four EI demonstration projects are represented by the EI project set $X = \{x_1, x_2, x_3, x_4\}$. The four experts are proficient in the fields of technology, management, environmental protection, and policy in the field of EI, and have strong strategic thinking and decision-making consulting ability for the development of EI. Let $E = \{e_1, e_2, e_3, e_4\}$ be the set of

Table 3
The collective interval-cloud matrix RI^c.

RI^c	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}
x_1	[1.4836,11.5958]	[1.9801,12.9266]	[1.8983,11.7933]	[1.6833,10.9297]	[0.6927,10.7101]
x_2	[1.9044,11.0021]	[1.0387,12.7268]	[0.8694,10.6341]	[1.7942,10.9980]	[1.0403,10.9977]
x_3	[1.8147,10.4929]	[0.8492,11.1752]	[1.8525,11.5137]	[1.5980,12.2486]	[1.7216,10.9731]
x_4	[0.8004,11.1575]	[0.6544,12.1694]	[0.5135,10.7098]	[1.3608,11.7684]	[0.3003,12.0447]
	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
x_1	[0.0702,11.4337]	[0.8755,11.0035]	[1.3222,11.1971]	[1.5835,11.3499]	[0.4760,10.9631]
x_2	[1.8908,11.5570]	[1.8207,11.7525]	[0.3386,11.2949]	[0.9073,11.0291]	[0.8684,11.5762]
x_3	[1.5049,11.4905]	[1.2635,10.7625]	[1.3292,12.0433]	[1.2775,11.1359]	[1.5842,11.3049]
x_4	[0.8082,11.1398]	[1.0702,11.2476]	[1.4845,10.8348]	[0.4088,11.3638]	[0.5489,11.2885]

Table 4
The weights of criteria.

Criteria	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
Weight	0.0673	0.1551	0.1284	0.0837	0.1488	0.1375	0.0560	0.1024	0.0517	0.0691

experts, and $V = \{v_1, v_2, v_3, v_4\}$ is the set of experts' weights, where four experts are given exactly equal weight. $B = \{B_1, B_2, B_3\}$ is the set of criteria where $B_1 = \{b_{11}, b_{12}, b_{13}, b_{14}\}$, $B_2 = \{b_{21}, b_{22}, b_{23}\}$ and $B_3 = \{b_{31}, b_{32}, b_{33}\}$ in Table 2. The evaluation matrixes provided by expert e_k using FLEs are $M^k = (m_{ij}^k)_{n\times m}$, which are shown in Table B.1 (See Appendix B). Given the domain $U = [0, 10]$, the linguistic evaluation set is $L = \{l_0 : very\ poor, l_1 : poor, l_2 : fair, l_3 : good, l_4 : very\ good\}$. The relative parameters are assumed that θ=0.5 and γ=0.5.

5.2. The decision process

In this section, the proposed MCDM framework is used to evaluate several EI demonstration projects. The decision steps are demonstrated as follows.

Step 1. For the domain $U = [0, 10]$ and the fixed linguistic set $L = \{l_0 : very\ poor, l_1 : poor, l_2 : fair, l_3 : good, l_4 : very\ good\}$, the basic clouds C_i $(i = 0, 1, \ldots, 4)$ can be calculated using the transforming method shown in Example 1: $C_0 = (0, 2.8518, 0.3950)$, $C_1 = (2.8892, 2.4568, 0.2922)$, $C_2 = (5, 2.1357, 0.3992)$, $C_3 = (7.1108, 2.4568, 0.2922)$, $C_4 = (10, 2.8518, 0.3950)$. Therefore, the FLEs evaluation matrixes $M^k = (m_{ij}^k)_{n\times m}$ in Table B.1 (See Appendix B) can be transformed into the cloud-FlE matrixes $M'^k = (m'^k_{ij})_{n\times m}$ by replacing linguistic term l_i with C_i.

Step 2. Each cloud-FLE m'^k_{ij} in cloud-FLE matrixes M'^k can be transformed into a floating-cloud-FlE $m_{ij}^f = \{(C_{ij}^1, p_{ij}^1), (C_{ij}^2, p_{ij}^2), \ldots, (C_{ij}^T, p_{ij}^T)\}$, Therefore, the floating-cloud-FLE matrixes $M^{fk} = (m_{ij}^{fk})_{n\times m}$ are obtained.

Step 3. The cloud evaluation matrixes can be obtained using the programming model in Eq. (12)$R^k = (\tilde{C}_{ij}^k)_{n\times m}$ are shown in (See Appendix B).

Step 4. To determine the weights of 10 criteria, the cloud evaluation matrixes R^k are aggregating into a collective interval-cloud matrix $RI^c = (r_{ij})_{n\times m}$ in Table 3, and the weights of criteria are obtained by Shannon entropy method in Table 4.

Step 5. To rank these EI demonstration projects, we rewrite the cloud evaluation matrixes $R^k = (\tilde{C}_{ij}^k)_{n\times m}$ as $R^j = (\tilde{C}_{ik}^j)_{n\times K}$. We can determine the PIS and NIS to obtain the jth reference cloud $C^j = \{C_1^j, C_2^j, \ldots, C_K^j\}$ in Table 5. The reference cloud A is shown in Table 6.

Step 6. The comprehensive multi-granularity lower approximation $\underline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ and upper approximation $\overline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ of A can be calculated as follows:

$\underline{R}_{\sum_{j=1}^m R^j}(A)(x_1) = (6.1895, 1.1633, 0.2106)$,
$\underline{R}_{\sum_{j=1}^m R^j}(A)(x_2) = (6.1895, 1.1775, 0.2099)$,
$\underline{R}_{\sum_{j=1}^m R^j}(A)(x_3) = (6.1936, 1.1398, 0.2092)$,
$\underline{R}_{\sum_{j=1}^m R^j}(A)(x_4) = (6.1895, 1.1902, 0.2111)$;
$\overline{R}_{\sum_{j=1}^m R^j}(A)(x_1) = (6.1903, 1.1008, 0.2070)$,
$\overline{R}_{\sum_{j=1}^m R^j}(A)(x_2) = (6.2601, 1.0932, 0.2071)$,
$\overline{R}_{\sum_{j=1}^m R^j}(A)(x_3) = (6.2682, 1.1021, 0.2056)$,
$\overline{R}_{\sum_{j=1}^m R^j}(A)(x_4) = (6.2520, 1.1249, 0.2040)$.

Step 7. The approximation evaluation value $R_{\sum_{j=1}^m R^j}(A)(x_i)$ of A with respect to x_i are calculated using Eq. (24) as follows:

$R_{\sum_{j=1}^m R^j}(A)(x_1) = (6.1899, 1.1325, 0.2088)$,
$R_{\sum_{j=1}^m R^j}(A)(x_2) = (6.2248, 1.1362, 0.2085)$,
$R_{\sum_{j=1}^m R^j}(A)(x_3) = (6.2309, 1.1211, 0.2074)$,
$R_{\sum_{j=1}^m R^j}(A)(x_4) = (6.2207, 1.1580, 0.2076)$.

The clouds generated by four EI projects with 3000 cloud drops are shown in Fig. 5. Therefore, the ranking of four EI projects is: $x_3 \succ x_2 \succ x_4 \succ x_1$. Based on the decision results, the following policy recommendations are served as a reference for the future energy integration of the BTH region. The government can prioritize supporting multi-energy integration projects, such as the Tianjin Binhai Smart Energy Demonstration Project, which demonstrates the great potential of integrating wind energy, solar energy, and energy storage systems with smart grids, aligning with the principles of resource efficiency in the circular economy. This outcome is highly consistent with the strategic goal of promoting cross-provincial and cross-regional energy infrastructure interconnection in the BTH region, further validating the feasibility of multi-energy integration and connectivity. The government can promote the research, development, and application of energy storage and smart grid technologies through financial support and tax incentives, thereby improving system efficiency and flexibility and promoting the recycling of energy resources. Additionally, the success of the Zhangbei "Internet+ Smart Energy" wind power project indicates that policy should strengthen support for the wind energy industry and the integration of smart grid technologies, promoting the efficient integration of wind power into the grid and increasing utilization rates. The abundant wind energy resources in Hebei should be fully utilized to promote the sustainable development of the wind power industry and strengthen coordination between the smart grid and wind power for improved energy efficiency and system stability. From the experience of the Xiong'an New Area green smart microgrid project, policy should encourage the integration of green energy solutions with microgrid technologies, promoting the green and low-carbon development of the New Area and remote regions. Due to its unique geographical and policy advantages, Xiong'an New Area can serve as a demonstration area for green smart microgrids, utilizing solar and wind energy to promote zero-carbon energy supply systems, further advancing regional energy circular economy development. Although the Beijing Energy Haidian project ranks lower, it still provides valuable experience in the application of smart grid technologies. The government should continue to support the construction of smart grid infrastructure, especially in the renovation of old grids, to enhance renewable energy integration

Table 5
The reference cloud $C^j = \{C_1^j, C_2^j, \dots, C_K^j\}$.

C^j	e_1	e_2	e_3	e_4
C^1	(6.8995,0.8928,0.2233)	(6.4304,0.8788,0.1922)	(6.9237,0.8935,0.2247)	(4.6237,1.1250,0.1773)
C^2	(6.1120,1.9820,0.2606)	(6.8995,0.8928,0.2233)	(6.3618,1.0629,0.179)	(6.3880,1.8607,0.2694)
C^3	(6.2483,0.9755,0.2240)	(5.4142,1.0856,0.1468)	(5.8581,0.8637,0.1467)	(6.6236,0.9644,0.2127)
C^4	(5.4142,1.0856,0.1468)	(6.6236,0.9644,0.2127)	(6.2483,0.9755,0.2240)	(6.8057,0.8734,0.1789)
C^5	(5.9653,1.1751,0.1589)	(5.5752,1.3359,0.2278)	(6.9237,0.8935,0.2247)	(5.7710,0.9768,0.1692)
C^6	(6.9237,0.8935,0.2247)	(6.5242,0.8992,0.2341)	(5.5888,1.0880,0.2046)	(6.8995,0.8928,0.2233)
C^7	(6.2483,0.9755,0.2240)	(6.9237,0.8935,0.2247)	(5.9653,1.1751,0.1589)	(6.2483,0.9755,0.2240)
C^8	(5.7710,0.9768,0.1692)	(6.6236,0.9644,0.2127)	(6.9237,0.8935,0.2247)	(5.7832,1.2888,0.1961)
C^9	(6.6236,0.9644,0.2127)	(6.4304,0.8788,0.1922)	(6.2483,0.9755,0.2240)	(5.5888,1.0880,0.2046)
C^{10}	(5.7710,0.9768,0.1692)	(5.7832,1.2888,0.1961)	(5.7832,1.2888,0.1961)	(6.9237,0.8935,0.2247)

Table 6
The reference cloud A.

	e_1	e_2	e_3	e_4
A	(6.1895,1.2100,0.2071)	(6.2772,1.0374,0.2099)	(6.2922,1.0045,0.2011)	(6.2389,1.1818,0.2130)

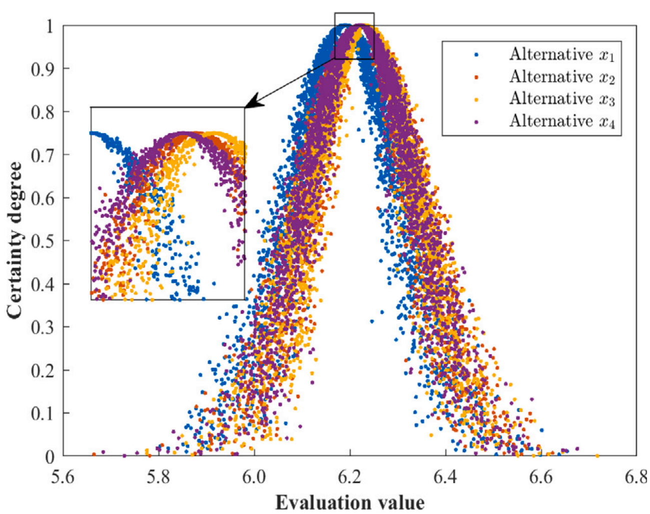


Fig. 5. The clouds generated by four EI projects with 3000 cloud drops.

capabilities and promote the transformation of energy systems toward a circular economy model, achieving efficient use of resources.

5.3. Sensitivity analysis

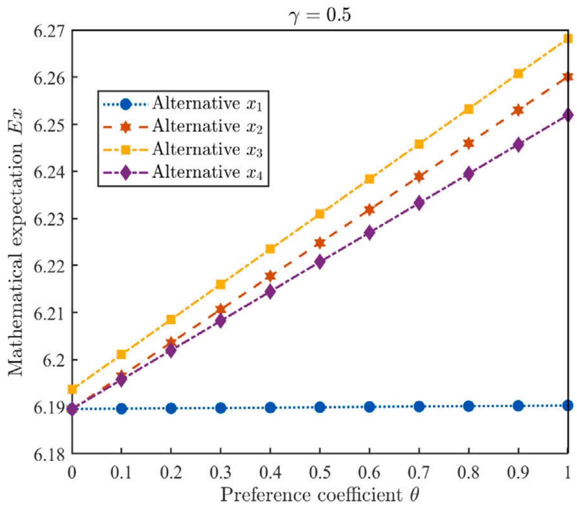
The expectation Ex in the cloud model is the most representative numerical feature, which can reflect the average level of cloud drops. Therefore, the impact of preference coefficient θ and risk preference coefficient γ on the mathematical expectation Ex should be considered. For the EI projects evaluation, the mathematical expectation Ex of four EI projects varies with different preference coefficients and with different risk preference coefficients, which are as shown in Fig. 6.

In Fig. 6(a), it can be seen that the mathematical expectation Ex of four EI projects grows in a straight line with the preference coefficient θ when $\gamma = 0.5$. The expectation gap between the four EI projects becomes wider as the preference coefficient increases. The reason is that the increase of θ makes the proportion of the upper approximation bigger in the approximate evaluation value. The gap of the upper approximation is more obvious than that of the lower approximation, which leads to the difference of the expectations Ex under four EI projects increasing with θ.

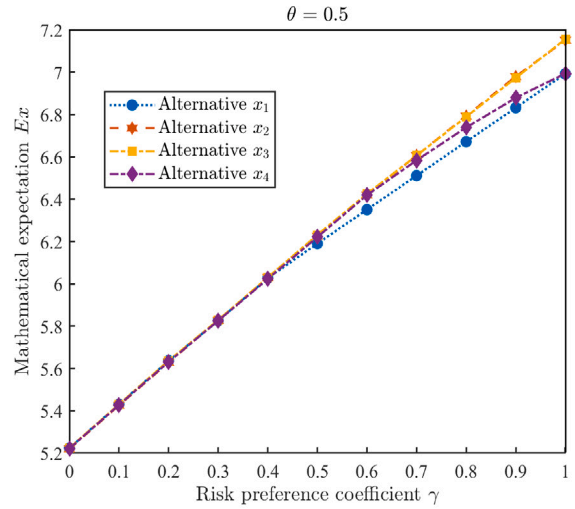
In Fig. 6(b), when the preference coefficient $\theta = 0.5$, the mathematical expectation Ex of four EI projects grows in a straight line with the risk preference coefficient γ. The gap of the four EI projects is not obvious when $\gamma \leq 0.4$ and begins to widen when $\gamma > 0.4$. The expectation curves of EI projects x_2 and x_3 coincide, which means that the difference between the two expectations is small. The gap between EI project x_1 and x_4 gradually increases and then decreases until the expectations of the two basically coincide when $\gamma = 1$.

The ranking results of four EI projects with different coefficients are shown in Fig. 7. Fig. 7(a) shows the ranking results with different preference coefficients when $\gamma = 0.5$, and it can be seen that the ranking of EI projects is always $x_3 \succ x_2 \succ x_4 \succ x_1$ when $\theta \neq 0$, and the ranking is $x_3 \succ x_1 \succ x_2 \succ x_4$ when $\theta = 0$. This is because the approximation evaluation value $R_{\sum_{j=1}^m R^j}(A)(x_i) = \underline{R}_{\sum_{j=1}^m R^j}(A)(x_i)$ when $\theta = 0$. Therefore, the ranking of EI project when $\theta = 0$ is consistent with the ranking of the comprehensive multi-granularity lower approximation. For different preference coefficients, the ranking of EI projects is always $x_3 \succ x_2 \succ x_4 \succ x_1$, which reflects that the change of preference coefficient has little effect on the ranking of EI projects. Fig. 7(b) shows the ranking results with different risk preference coefficients when $\theta = 0.5$. The ranking of EI projects is changing dynamically with the risk preference coefficient. EI project x_3 basically ranks first or second and EI project x_4 ranks third or fourth when $\theta \in [0, 1]$. Meanwhile, the ranking of EI project x_2 is gradually higher and that of EI project x_1 is gradually lower with the increase of the risk preference coefficient.

Based on the above analyses, the risk preference coefficient has a significant effect on the ranking of EI projects. To explore the impact under different preference coefficients, the ranking results when $\theta \in [0, 1]$ are shown in Fig. 8. It can be seen from Fig. 8(a) that there are two inflection points $\gamma_1 = 0.22$ and $\gamma_2 = 0.62$ affecting the ranking of EI projects when $\theta = 0$. For $\gamma < \gamma_1$, $\gamma_1 \leq \gamma \leq \gamma_2$ and $\gamma > \gamma_2$, the best EI project is x_2, x_3 and x_1, respectively. From Fig. 8(b)–(f), the ranking of EI projects is different when selecting different risk preference coefficients, in which $\gamma_3 = 0.43$ is a critical point affecting the ranking of x_1 and x_4, i.e., x_1 ranks fourth and x_4 ranks third when $\gamma > 0.43$. When $\gamma \leq 0.43$, the ranking of EI projects changes fluctuate greatly. When $\theta > 0.5$, a special point $\gamma_4 = 0.24$ appears when x_4 ranks

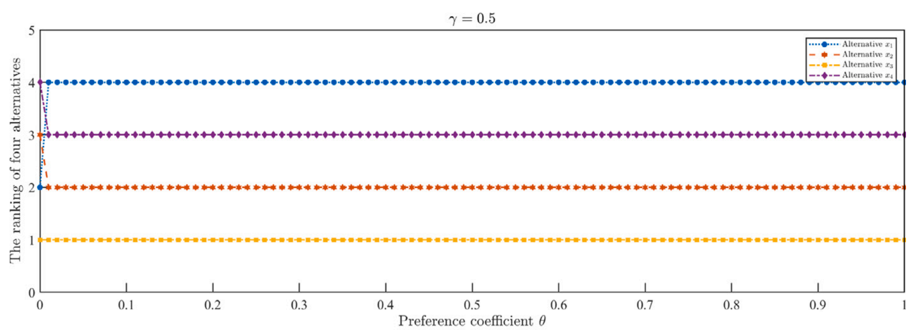


(a) The results with different preference coefficients θ

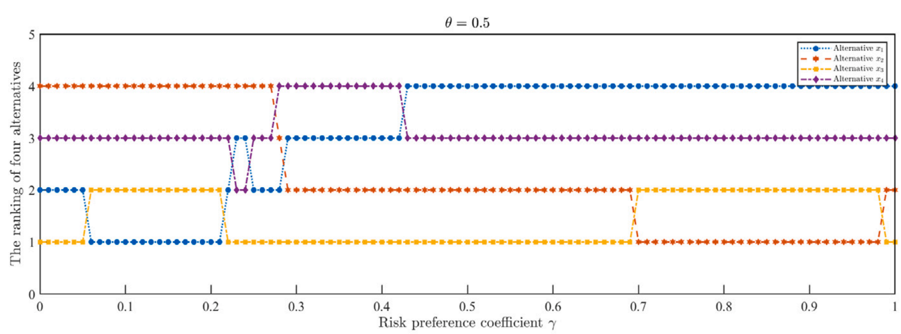


(b) The results with different risk preference coefficients γ

Fig. 6. The expectation Ex results with different coefficients.



(a) The results with different preference coefficients θ



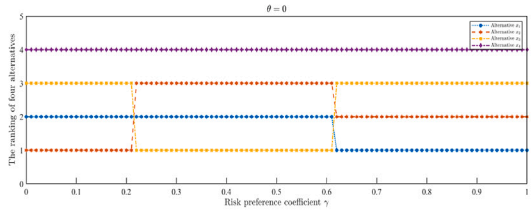
(b) The results with different risk preference coefficients γ

Fig. 7. The ranking results with different coefficients.

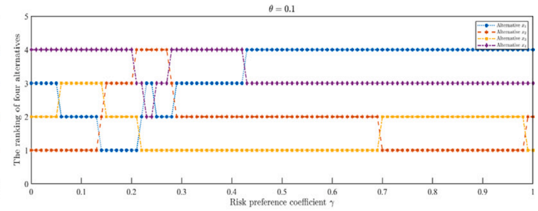
first. In total, Fig. 8(a)–(f) differ from each other to some extent, which reflects that the preference coefficient has an impact on the ranking of EI projects and some inflection points should be concerned. Therefore, determining an appropriate risk preference coefficient and preference coefficient is critical for ranking these EI projects. In practical applications, decision-makers can determine the appropriate range for the risk preference coefficient through extensive discussions or surveys, or estimate the range of risk preference coefficient using existing case data. Simultaneously, through multiple simulations and experiments, the risk preference coefficient can be gradually adjusted based on the results under different scenarios. This process can involve expert evaluations, historical data analysis, and practical experience related to the project, providing decision-making with more contextually relevant parameter guidance.

5.4. Comparative analyses

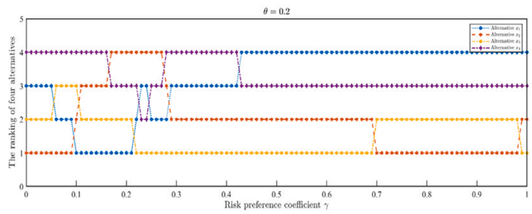
To further demonstrate the effectiveness of the proposed method, we conduct a comparison between our method and the other four MCDM methods for energy system evaluation, including Jiang et al. (2022)'s method, Shang (2022)'s method, Zhou et al. (2019)'s method and Wu et al. (2019)'s method. The ranking results of five methods for energy system evaluation are shown in Fig. 9 and Table 7.



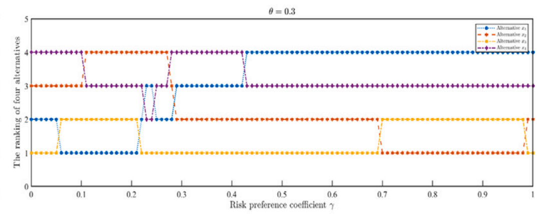
(a) The ranking results when $\theta = 0$



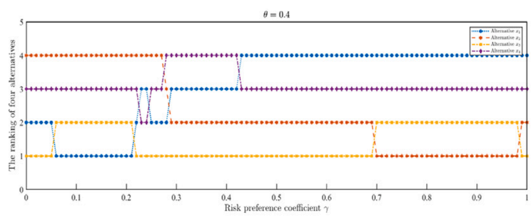
(b) The ranking results when $\theta = 0.1$



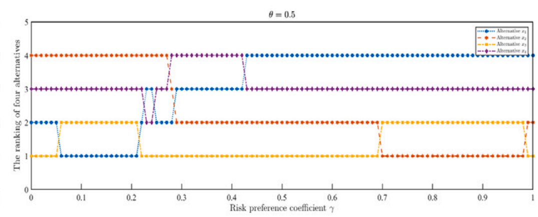
(c) The ranking results when $\theta = 0.2$



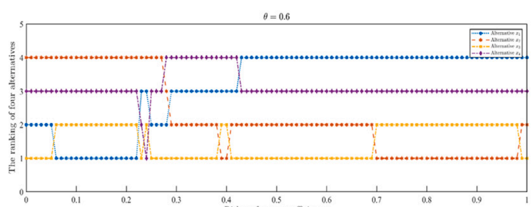
(d) The ranking results when $\theta = 0.3$



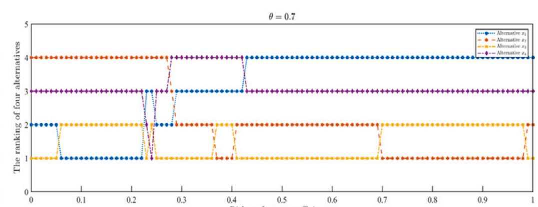
(e) The ranking results when $\theta = 0.4$



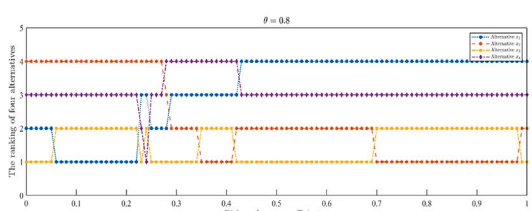
(f) The ranking results when $\theta = 0.5$



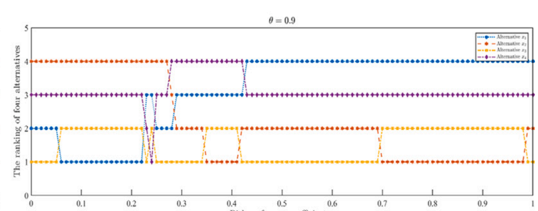
(g) The ranking results when $\theta = 0.6$



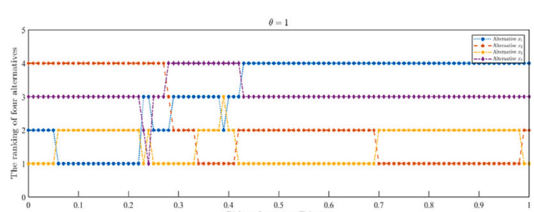
(h) The ranking results when $\theta = 0.7$



(i) The ranking results when $\theta = 0.8$



(j) The ranking results when $\theta = 0.9$



(k) The ranking results when $\theta = 1$

Fig. 8. The ranking results with different θ and γ.

Table 7
The ranking results of five MCDM methods for energy system evaluation.

EI projects	Our proposal	Jiang et al. (2022)'s method	Shang (2022)'s method	Zhou et al. (2019)'s method	Wu et al. (2019)'s method
x_1	3	3	2	2	2
x_2	2	2	3	3	3
x_3	4	1	4	4	1
x_4	1	4	1	1	4

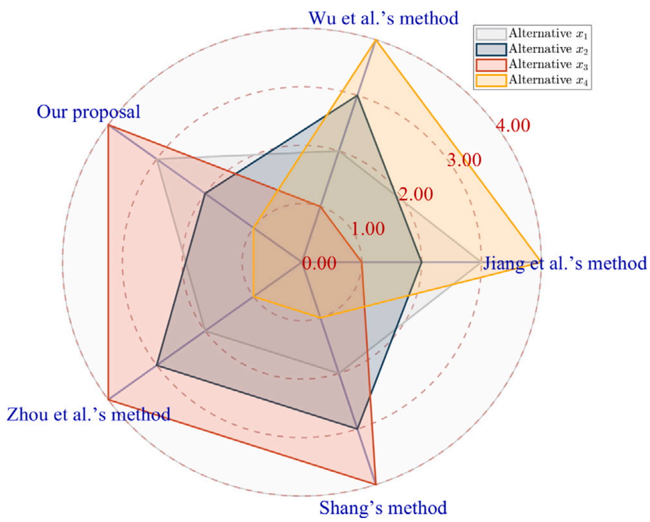


Fig. 9. The comparative results of five MCDM methods for energy system evaluation.

It can be seen from Fig. 9 that the ranking of x_1 and x_2 is more stable than that of x_3 and x_4. The ranking of x_3 and x_4 is sensitive to parameters and MCDM methods, that is, using different parameters and MCDM methods can lead to two extreme states of the best or worst EI project. From Table 7, the ranking result of our proposal is not completely consistent with that of the other four methods. However, x_1 and x_2 rank second or third under five methods, and the ranking of x_3 and x_4 are always first or fourth. The ranking of x_1 and x_2 under our proposed method is consistent with that of Jiang et al.'s method, and the ranking of x_3 and x_4 under our proposed method is the same as that of Shang's method and Zhou et al.'s method. This reflects that there is basically no difference between our method and the other four methods, which demonstrates the effectiveness of our proposed method when determining extreme or intermediate EI projects. Based on the above analyses of preference coefficient θ, the ranking can also be exactly the same as the four methods by selecting the appropriate preference coefficient. Therefore, our proposed method not only has the effectiveness but also has a strong flexibility as the parameters change.

6. Discussion

This section examines the theoretical and management implications of the EI project evaluation framework in the context of circular economy practice, while also discussing the limitations of the proposal.

6.1. Theoretical implications

The proposed MCDM framework provides a substantial theoretical advance in the circular economy practice. First, a new method for transforming discrete FLEs into continuous cloud information enhances the management of uncertainty by transforming discrete data into a continuous format, allowing for a more nuanced and precise integration of assessment criteria. This theoretical advance contributes to a deeper understanding of how various EI projects impact sustainability, leading to a more accurate representation of their contribution to circular economy. Secondly, with the support of the Shannon entropy method, a tailored evaluation index system is developed to expand the theoretical framework for evaluating the different impacts of EI projects. By using this approach to assign appropriate weight to assessment criteria, the framework ensures that assessments reflect the multifaceted nature of green innovation and sustainability in the circular economy. This theoretical refinement improves the ability to capture the effectiveness of projects in improving energy efficiency and achieving sustainable development goals. Finally, the application of MGCRS and integrated multi-granularity approximations has introduced significant theoretical advances to the decision-making process. This approach provides a powerful mechanism for ranking and optimizing EI projects by managing ongoing cloud information. The use of optimistic and pessimistic MGCRSs in both areas can improve the accuracy and reliability of project evaluations. In theory, this innovation provides a more precise and practical basis for optimizing energy grid connections, thereby supporting the circular economy and advancing energy integration. Totally, these theoretical contributions collectively strengthen the understanding and evaluation of green innovation and circular economy by improving uncertainty management, refining evaluation frameworks, and advancing decision-making methods.

6.2. Management implications

The proposed MCDM framework highlights several key managerial implications for the circular economy. (1) Enhanced decision-making accuracy and adaptability for the circular economy: Uncertainty is a key challenge in circular economy projects, particularly when assessing feasibility and long-term sustainability. Factors like fluctuating market conditions, technological variability, and unpredictable availability of recyclable materials add complexity to evaluations. The proposed MCDM framework addresses these uncertainties by converting FLEs into continuous cloud information, offering a more reliable foundation for decision-making. For example, uncertainty about the availability of recyclable materials can lead to over-optimistic projections in material recycling projects. The framework allows managers to assess projects under various scenarios, identify potential risks, and make informed adjustments. By modeling supply chain variations, it ensures resources are allocated to projects with higher chances of success and sustainability, helping meet circular economy goals like waste reduction and material recycling. (2) Consistent evaluation of circular economy impact: The tailored evaluation system provides a comprehensive assessment across three key dimensions: grid technology, green energy, and composite benefits. This multi-dimensional evaluation index system ensures managers can holistically evaluate projects, addressing both technical performance and broader sustainability impacts. For example, when assessing a project that incorporates renewable energy, the framework does not only focus on energy efficiency but also includes factors like carbon emission reductions, the use of recycled materials, and socio-economic benefits, such as local job creation. This integrated evaluation system enables managers to understand the full range of a project's impact, ensuring that decisions are based on a well-rounded assessment of both short-term feasibility and long-term sustainability. By incorporating these diverse dimensions, the framework helps identify projects that offer the most balanced and impactful contributions to the circular economy, aligning technical, environmental, and social goals. (3) Optimized selection of high-impact circular economy projects: The

MGCRS method enables precise ranking of circular economy projects based on key factors such as energy efficiency, carbon reduction, and resource utilization. This method allows managers to identify and prioritize projects with the highest potential for advancing circular economy objectives, ensuring that resources are allocated where they will have the greatest impact. For example, the ranking system helps identify initiatives that reduce energy consumption while also utilizing renewable or recycled materials in projects aimed at improving energy efficiency, maximizing the project's environmental and economic benefits. By applying this approach, managers can select projects that not only contribute to energy savings but also promote the circularity of materials, ensuring that investments are directed toward projects that deliver the most significant and sustainable long-term outcomes. (4) Strategic resource allocation for the circular economy: The framework provides clear and actionable insights into which EI projects align best with circular economy goals, enabling managers to allocate resources efficiently. By focusing on projects with the highest potential for carbon reduction and resource efficiency, the framework ensures that resources are directed to initiatives that yield the greatest long-term impact. For example, in projects focused on energy recovery from waste, the framework helps managers prioritize initiatives that maximize the reuse of materials and reduce carbon emissions, ensuring that available resources are not wasted on less effective projects. This approach not only minimizes investment risks but also accelerates the transition towards a sustainable circular economy by ensuring that resources are efficiently allocated to projects that support both environmental and economic sustainability goals. (5) Practical application in circular economy initiatives: The framework enables the translation of complex evaluation data into practical, actionable strategies by systematically processing and analyzing key metrics. This allows managers to make informed decisions and implement carbon-neutral initiatives based on real-world performance. For example, when evaluating a project that uses recycled materials for product manufacturing, the framework helps managers track actual material inputs and outputs, enabling adjustments to improve material efficiency or reduce energy consumption over time. This adaptability ensures that managers can optimize projects in response to performance feedback, enhancing their long-term sustainability and impact. Ultimately, the framework supports the continuous improvement of circular economy initiatives by providing managers with the tools to adjust strategies as needed, driving more effective and sustained progress towards circular economy goals. In summary, this framework equips managers with the necessary tools to make informed decisions, standardize project evaluations, optimize resource allocation, and ultimately, drive significant progress toward achieving circular economy through the effective management of EI projects.

6.3. Limitations of the proposal

The proposed framework faces some notable limitations that need to be addressed. Firstly, the dynamic nature of the circular economy, driven by the rapid advancement of technologies, policies, and market conditions, presents a challenge. The framework may need continuous updates to stay relevant and effective in accommodating new trends and changes in sustainability goals. Secondly, scalability issues may arise when applying the framework to larger and more complex EI projects or managing a large number of evaluation criteria. As project scale and criteria expand, the computational demands and complexity of the evaluation process may increase, potentially affecting the framework's feasibility and efficiency.

7. Conclusions

EI represents an advanced stage in the development of sustainable industrial systems, focusing on enhancing resource efficiency and supporting circular economy principles through an interconnected energy network centered around electricity. We propose an MCDM framework with FLEs based on MGCRS to evaluate multiple EI projects. Firstly, converting discrete and continuous information effectively handles uncertainty and ambiguity in evaluating EI projects, mitigating risks associated with inefficiencies, and ensuring the reliable promotion of sustainable industrial practices. Secondly, by incorporating relevant sustainability and circular economy factors, a tailored evaluation index system helps managers identify and prioritize resources for projects that most effectively contribute to sustainable industrial transformation, enhancing circular economy practices and accelerating progress toward sustainability goals. The MGCRS method ranks EI projects in detail, ensuring that resources are allocated to the most impactful initiatives for improving resource efficiency and supporting circular economy principles. This approach prevents resource waste and maximizes the potential of sustainable innovation. By systematically processing complex evaluation data, the framework helps managers translate theoretical evaluations into practical strategies, ensuring effective implementation and adaptation of plans based on realistic performance, thus significantly advancing the goals of sustainable industrial transformation and circular economy. This data-driven management approach supports decision-making and investment strategies, facilitating the transition to a more sustainable and circular industrial system.

Future development will focus on overcoming the limitations of the proposal to better align with circular economy principles. Specifically, the evaluation index system could be extended to different EI sub-networks, allowing for tailored assessments that address the specific service characteristics and demands of various energy types, in line with the circular economy's emphasis on resource efficiency and minimizing waste. This extension would support the upgrading of EI systems to meet evolving needs, ensuring that energy flows and resources are utilized optimally across sectors, promoting material recycling and reducing waste in the process. Additionally, further research should explore the network and layout planning of energy systems based on the ranking of different alternatives. This includes considering investment costs and the energy supply range of sub-networks to enhance the synergistic interaction between different energy sub-networks. Such optimization can facilitate closed-loop energy systems, where energy, materials, and by-products are reused and recycled, ultimately improving the overall efficiency and sustainability of energy integration in line with circular economy goals.

CRediT authorship contribution statement

Han Wang: Writing – review & editing, Writing – original draft, Software, Methodology, Data curation, Conceptualization. **Yanbing Ju:** Methodology. **Carlos Porcel:** Investigation, Formal analysis.

Appendix A. Proof of Theorems

Theorem 1. *Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \ldots, m)$ is the binary cloud relation between universe X and E. For any $A, B \in F(E)$, $e \in E$ and $x \in X$, the optimistic MGCRS over two universes satisfies the following theorems:*

(1) $\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) = \left(\overline{R}^O_{\sum_{j=1}^m R^j}(A^C)(x)\right)^C$ *and* $\overline{R}^O_{\sum_{j=1}^m R^j}(A)(x) = \left(\underline{R}^O_{\sum_{j=1}^m R^j}(A^C)(x)\right)^C$ *when A is the set of cloud model.*

(2) $\overline{R}^O_{\sum_{j=1}^m R^j}(\emptyset_E)(x) = \emptyset_X$, $\underline{R}^O_{\sum_{j=1}^m R^j}(E)(x) = X$.

(3) If $A \subseteq B$, then $\underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) \subseteq \underline{R}^O_{\sum_{j=1}^m R^j}(B)(x)$ *and* $\overline{R}^O_{\sum_{j=1}^m R^j}(A)(x) \subseteq \overline{R}^O_{\sum_{j=1}^m R^j}(B)(x)$.

(4) $\underline{R}^O_{\sum_{j=1}^m R^j}(A \cup B)(x) = \underline{R}^O_{\sum_{j=1}^m R^j}(A)(x) \cup \underline{R}^O_{\sum_{j=1}^m R^j}(B)(x)$;
$\overline{R}^O_{\sum_{j=1}^m R^j}(A \cup B)(x) = \overline{R}^O_{\sum_{j=1}^m R^j}(A)(x) \cup \overline{R}^O_{\sum_{j=1}^m R^j}(B)(x)$.

(5) $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A \cap B)(x) = \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \cap \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$;
$\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A \cap B)(x) = \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \cap \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$.

Proof. (1) $\left(\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A^C)(x)\right)^C = \left(\wedge_{j=1}^{m} \vee_{e\in E} \min(R^j(x,e), A^C(e))\right)^C = \left\langle x, (\overline{Ex}^{O}_{A^C}(x))^C, (\overline{En}^{O}_{A^C}(x))^C, (\overline{He}^{O}_{A^C}(x))^C \right\rangle$, where $(\overline{Ex}^{O}_{A^C}(x))^C = \left(\wedge_{j=1}^{m} \vee_{e\in E} \min(Ex_{R^j}(x,e), Ex_{A^C}(e))\right)^C = \vee_{j=1}^{m} \wedge_{e\in E} \max(\left(Ex_{R^j}(x,e)\right)^C, \left(Ex_{A^C}(e)\right)^C) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e)) = \underline{Ex}^{O}_{A}(x)$. Similarly, $(\overline{En}^{O}_{A^C}(x))^C = \underline{En}^{O}_{A}(x)$ and $(\overline{He}^{O}_{A^C}(x))^C = \underline{He}^{O}_{A}(x)$ can be proved. Therefore, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) = \left(\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A^C)(x)\right)^C$ is proved. Similarly, we have $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) = \left(\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A^C)(x)\right)^C$.

(2) $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\emptyset_E)(x) = \wedge_{j=1}^{m} \vee_{e\in E} \min(R^j(x,e), \emptyset_E(e))$, where $\overline{Ex}^{O}_{\emptyset_E}(x) = \wedge_{j=1}^{m} \vee_{e\in E} \min(Ex_{R^j}(x,e), Ex_{\emptyset_E}(e)) = Ex_{\emptyset_E}(e)$, $\overline{En}^{O}_{\emptyset_E}(x) = En_{\emptyset_E}(e)$ and $\overline{He}^{O}_{\emptyset_E}(x) = He_{\emptyset_E}(e)$. Therefore, $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\emptyset_E)(x) = \left(Ex_{\emptyset_E}(e), En_{\emptyset_E}(e), He_{\emptyset_E}(e)\right) = \emptyset_X$. Similarly, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(E)(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max(N(R^j(x,e)), E(e)) = \left(Ex_E(e), En_E(e), He_E(e)\right) = X$, therefore $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(E)(x) = X$ can be proved.

(3) Due to $A \subseteq B$, then $Ex_A(x) \le Ex_B(x)$, $En_A(x) \ge En_B(x)$ and $He_A(x) \ge He_B(x)$. We can obtain $\vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e)) \le \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_B(e))$, then $\underline{Ex}^{O}_{A}(x) \le \underline{Ex}^{O}_{B}(x)$. Similarly, $\underline{En}^{O}_{A}(x) \le \underline{En}^{O}_{B}(x)$ and $\underline{He}^{O}_{A}(x) \le \underline{He}^{O}_{B}(x)$. Therefore, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \subseteq \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$. Similarly, $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \subseteq \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$.

(4) For $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A \cup B)(x)$, $\underline{Ex}^{O}_{A\cup B}(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), max(Ex_A(e), Ex_B(e))) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e) \vee Ex_B(e))$. Meanwhile, $\underline{Ex}^{O}_{A}(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e))$, $\underline{Ex}^{O}_{B}(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_B(e))$, then $\underline{Ex}^{O}_{A}(x) \cup \underline{Ex}^{O}_{B}(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_A(e) \vee Ex_B(e)) = \underline{Ex}^{O}_{A\cup B}(x)$. Similarly, $\underline{En}^{O}_{A}(x) \cup \underline{En}^{O}_{B}(x) = \underline{En}^{O}_{A\cup B}(x)$ and $\underline{He}^{O}_{A}(x) \cup \underline{He}^{O}_{B}(x) = \underline{He}^{O}_{A\cup B}(x)$. Therefore, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A \cup B)(x) = \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \cup \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$. Similarly, $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A \cup B)(x) = \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A)(x) \cup \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(B)(x)$ can be proved.

(5) The proof is similar to that of (4) in Theorem 1, so it is omitted.

Theorem 2. *Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \dots, m)$ is the binary cloud relation between universe X and E. For any $A_k \in F(E)(k = 1, 2, \dots, K)$, $e \in E$ and $x \in X$, the optimistic MGCRS over two universes satisfy the following theorems:*

(1) $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$, *and* $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$.

(2) $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cup_{k=1}^{K} \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$, *and* $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cup_{k=1}^{K} \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$.

(3) $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cup_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{R}^j(A_k)(x)\right)$ *and* $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cap_{j=1}^{m} \left(\cup_{k=1}^{K} \overline{R}^j(A_k)(x)\right)$.

Proof. (1) $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max\left(N(R^j(x,e)), min\{A_1(e), A_2(e), \dots, A_K(e)\}\right)$, where $\underline{Ex}^{O}_{\cap_{k=1}^{K} A_k}(x) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_1}(e) \wedge Ex_{A_2}(e) \wedge \cdots \wedge Ex_{A_K}(e))$. For $\cap_{k=1}^{K} \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$, $\cap_{k=1}^{K} \underline{Ex}^{O}_{A_k}(x) = \cap_{k=1}^{K} \left(\vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_k}(e))\right) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_1}(e) \wedge Ex_{A_2}(e) \wedge \cdots \wedge Ex_{A_K}(e))$, i.e., $\underline{Ex}^{O}_{\cap_{k=1}^{K} A_k}(x) = \cap_{k=1}^{K} \underline{Ex}^{O}_{A_k}(x)$. Similarly, $\underline{En}^{O}_{\cap_{k=1}^{K} A_k}(x) = \cap_{k=1}^{K} \underline{En}^{O}_{A_k}(x)$ and $\underline{He}^{O}_{\cap_{k=1}^{K} A_k}(x) = \cap_{k=1}^{K} \underline{He}^{O}_{A_k}(x)$. Therefore, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \underline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$. Similarly, $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \overline{R}^{O}_{\sum_{j=1}^{m} R^j}(A_k)(x)$ can be proved.

(2) The proof is similar to that of (1) in Theorem 2, so it is omitted.

(3) For $\underline{R}^j(A_k)(x)$, $\underline{Ex}_{R^j(A_k)}(x) = \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_k}(e))$, then $\cap_{k=1}^{K} \underline{Ex}_{R^j(A_k)}(x) = \wedge_{e\in E} \max\left((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_1}(e) \wedge Ex_{A_2}(e) \wedge \cdots \wedge Ex_{A_K}(e)\right)$. Afterwards, we have $\cup_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{Ex}_{R^j(A_k)}(x)\right) = \vee_{j=1}^{m} \wedge_{e\in E} \max((U^U + U^L - Ex_{R^j}(x,e)), Ex_{A_1}(e) \wedge Ex_{A_2}(e) \wedge \cdots \wedge Ex_{A_K}(e)) = \underline{Ex}^{O}_{\cap_{k=1}^{K} A_k}(x)$. Similarly, $\cup_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{En}_{R^j(A_k)}(x)\right) = \underline{En}^{O}_{\cap_{k=1}^{K} A_k}(x)$ and $\cup_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{He}_{R^j(A_k)}(x)\right) = \underline{He}^{O}_{\cap_{k=1}^{K} A_k}(x)$. Therefore, $\underline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cup_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{R}^j(A_k)(x)\right)$. Similarly, $\overline{R}^{O}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cap_{j=1}^{m} \left(\cup_{k=1}^{K} \overline{R}^j(A_k)(x)\right)$ can be proved.

Theorem 3. *Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \dots, m)$ is the binary cloud relation between universe X and E. For any $A, B \in F(E)$, $e \in E$ and $x \in X$, the pessimistic MGCRS over two universes satisfies the following theorems:*

(1) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) = \left(\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A^C)(x)\right)^C$ *and* $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) = \left(\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A^C)(x)\right)^C$ *when A is the set of cloud model.*

(2) $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(\emptyset_E)(x) = \emptyset_X$, $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(E)(x) = X$.

(3) If $A \subseteq B$, then $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \subseteq \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$ *and* $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \subseteq \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$.

(4) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A \cup B)(x) = \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \cup \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$;
$\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A \cup B)(x) = \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \cup \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$.

(5) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A \cap B)(x) = \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \cap \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$;
$\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A \cap B)(x) = \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A)(x) \cap \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(B)(x)$.

Proof. The proof is similar to that of Theorem 1, so it is omitted.

Theorem 4. *Let (X, E, F, R, B) be a multiple decision-making cloud information system over two universes and $R^j \in F(X \times E)(j = 1, 2, \dots, m)$ is the binary cloud relation between universe X and E. For any $A_k \in F(E)(k = 1, 2, \dots, K)$, $e \in E$ and $x \in X$, the pessimistic MGCRS over two universes satisfy the following theorems:*

(1) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A_k)(x)$, *and* $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{k=1}^{K} \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A_k)(x)$.

(2) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cup_{k=1}^{K} \underline{R}^{P}_{\sum_{j=1}^{m} R^j}(A_k)(x)$, *and* $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cup_{k=1}^{K} \overline{R}^{P}_{\sum_{j=1}^{m} R^j}(A_k)(x)$.

(3) $\underline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cap_{k=1}^{K} A_k)(x) = \cap_{j=1}^{m} \left(\cap_{k=1}^{K} \underline{R}^j(A_k)(x)\right)$ *and* $\overline{R}^{P}_{\sum_{j=1}^{m} R^j}(\cup_{k=1}^{K} A_k)(x) = \cup_{j=1}^{m} \left(\cup_{k=1}^{K} \overline{R}^j(A_k)(x)\right)$.

Proof. The proof is similar to that of Theorem 2, so it is omitted.

Appendix B. The evaluation information of experts

See Tables B.1–B.4.

Table B.1
The FLEs evaluation matrixes of four experts on 10 criteria.

e_1	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
x_1	$\{(l_2,0.6),(\{l_3,l_4\},0.3)\}$	$\{(l_3,1)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$
x_2	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_2,0.3),(l_3,0.5)\}$
x_3	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.9),(\{l_2,l_3\},0.1)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$
x_4	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.3),(l_3,0.7)\}$
e_2	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
x_1	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$
x_2	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_1,0.9),(\{l_2,l_3\},0.1)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$
x_3	$\{(l_0,0.1),(l_1,l_2,0.5),(\{l_3,l_4\},0.3)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.2),(l_3,0.5)\}$
x_4	$\{(l_2,0.2),(\{l_3,l_4\},0.5)\}$	$\{(l_3,1)\}$	$\{(l_2,0.2),(\{l_3,l_4\},0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$
e_3	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
x_1	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_2,0.2),(\{l_3,l_4\},0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$
x_2	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.2),(l_3,0.5)\}$
x_3	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_3,1)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$
x_4	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.9),(\{l_2,l_3\},0.1)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$
e_4	b_{11}	b_{12}	b_{13}	b_{14}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
x_1	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$
x_2	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_3,1)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$
x_3	$\{(l_1,0.9),(\{l_2,l_3\},0.1)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(l_2,0.2),(l_3,0.3),(l_4,0.5)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_3,0.4),(l_4,0.6)\}$	$\{(\{l_0,l_2\},0.5),(\{l_3,l_4\},0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.3),(l_3,0.7)\}$	$\{(l_3,0.4),(l_4,0.6)\}$
x_4	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_1,0.3),(l_2,0.2),(l_3,0.5)\}$	$\{(l_0,l_2,0.5),(\{l_3,l_4\},0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$	$\{(\{l_2,l_3\},0.8),(l_4,0.2)\}$	$\{(l_1,0.4),(\{l_3,l_4\},0.6)\}$	$\{(l_0,0.1),(\{l_1,l_2\},0.5),(\{l_3,l_4\},0.3)\}$	$\{(l_1,0.2),(l_3,0.5)\}$	$\{(l_2,0.3),(l_3,0.5)\}$

Table B.2
The cloud evaluations of experts under B_1.

e_1	b_{11}	b_{12}	b_{13}	b_{14}
x_1	(5.8647,1.4470,0.1289)	(7.1108,2.4568,0.2922)	(5.1133,1.3217,0.2247)	(5.4776,1.0717,0.1551)
x_2	(8.1339,0.7398,0.1999)	(6.0644,0.8910,0.1823)	(6.4530,1.2628,0.1626)	(7.2460,1.0563,0.2001)
x_3	(7.2460,1.0563,0.2001)	(5.4776,1.0717,0.1551)	(7.3832,0.7357,0.2232)	(3.5823,1.1450,0.0553)
x_4	(5.6651,1.1124,0.2445)	(5.1133,1.3217,0.2247)	(6.0371,1.3480,0.2308)	(7.2460,1.0563,0.2001)
e_2	b_{11}	b_{12}	b_{13}	b_{14}
x_1	(7.3832,0.7357,0.2232)	(8.1339,0.7398,0.1999)	(7.2460,1.0563,0.2001)	(5.1133,1.3217,0.2247)
x_2	(6.0644,0.8910,0.1823)	(5.6651,1.1124,0.2445)	(3.5823,1.1450,0.0553)	(5.4776,1.0717,0.1551)
x_3	(5.4776,1.0717,0.1551)	(6.4530,1.2628,0.1626)	(5.1133,1.3217,0.2247)	(8.1339,0.7398,0.1999)
x_4	(7.0596,1.0719,0.1989)	(7.1108,2.4568,0.2922)	(7.0596,1.0719,0.1989)	(6.0371,1.3480,0.2308)
e_3	b_{11}	b_{12}	b_{13}	b_{14}
x_1	(7.2460,1.0563,0.2001)	(7.0596,1.0719,0.1989)	(8.1339,0.7398,0.1999)	(6.0644,0.8910,0.1823)
x_2	(6.0644,0.8910,0.1823)	(7.2460,1.0563,0.2001)	(5.1133,1.3217,0.2247)	(7.3832,0.7357,0.2232)
x_3	(8.1339,0.7398,0.1999)	(6.4530,1.2628,0.1626)	(5.6651,1.1124,0.2445)	(7.1108,2.4568,0.2922)
x_4	(5.7135,1.1124,0.2470)	(5.4776,1.0717,0.1551)	(3.5823,1.1450,0.0553)	(5.1133,1.3217,0.2247)

(continued on next page)

Table B.2 (*continued*).

e_4	b_{11}	b_{12}	b_{13}	b_{14}
x_1	(5.6651,1.1124,0.2445)	(6.0644,0.8910,0.1823)	(6.4530,1.2628,0.1626)	(8.1339,0.7398,0.1999)
x_2	(5.1133,1.3217,0.2247)	(7.1108,2.4568,0.2922)	(7.2460,1.0563,0.2001)	(5.4776,1.0717,0.1551)
x_3	(3.5823,1.1450,0.0553)	(5.6651,1.1124,0.2445)	(8.1339,0.7398,0.1999)	(7.2460,1.0563,0.2001)
x_4	(5.4776,1.0717,0.1551)	(6.0644,0.8910,0.1823)	(5.1133,1.3217,0.2247)	(7.3832,0.7357,0.2232)

Table B.3
The cloud evaluations of experts under B_2.

e_1	b_{21}	b_{22}	b_{23}
x_1	(5.4776,1.0717,0.1551)	(5.7135,1.1124,0.2470)	(5.1133,1.3217,0.2247)
x_2	(5.6651,1.1124,0.2445)	(8.1339,0.7398,0.1999)	(7.3832,0.7357,0.2232)
x_3	(6.0644,0.8910,0.1823)	(6.0371,1.3480,0.2308)	(5.4776,1.0717,0.1551)
x_4	(6.4530,1.2628,0.1626)	(5.7135,1.1124,0.2470)	(5.6651,1.1124,0.2445)
e_2	b_{21}	b_{22}	b_{23}
x_1	(5.1133,1.3217,0.2247)	(6.0371,1.3480,0.2308)	(6.0644,0.8910,0.1823)
x_2	(5.1133,1.3217,0.2247)	(7.3832,0.7357,0.2232)	(8.1339,0.7398,0.1999)
x_3	(5.4776,1.0717,0.1551)	(6.0644,0.8910,0.1823)	(5.7135,1.1124,0.2470)
x_4	(6.0371,1.3480,0.2308)	(5.6651,1.1124,0.2445)	(6.4530,1.2628,0.1626)
e_3	b_{21}	b_{22}	b_{23}
x_1	(5.7135,1.1124,0.2470)	(5.1133,1.3217,0.2247)	(6.0644,0.8910,0.1823)
x_2	(7.3832,0.7357,0.2232)	(5.6651,1.1124,0.2445)	(6.0371,1.3480,0.2308)
x_3	(8.1339,0.7398,0.1999)	(5.7135,1.1124,0.2470)	(5.4776,1.0717,0.1551)
x_4	(6.0371,1.3480,0.2308)	(6.0644,0.8910,0.1823)	(6.4530,1.2628,0.1626)
e_4	b_{21}	b_{22}	b_{23}
x_1	(6.0644,0.8910,0.1823)	(5.6651,1.1124,0.2445)	(6.0371,1.3480,0.2308)
x_2	(5.4776,1.0717,0.1551)	(5.7135,1.1124,0.2470)	(5.1133,1.3217,0.2247)
x_3	(5.7135,1.1124,0.2470)	(8.1339,0.7398,0.1999)	(7.3832,0.7357,0.2232)
x_4	(6.0371,1.3480,0.2308)	(6.4530,1.2628,0.1626)	(6.0644,0.8910,0.1823)

Table B.4
The cloud evaluations of experts under B_3.

e_1	b_{31}	b_{32}	b_{33}
x_1	(5.4776,1.0717,0.1551)	(5.1133,1.3217,0.2247)	(6.0644,0.8910,0.1823)
x_2	(5.7135,1.1124,0.2470)	(5.6651,1.1124,0.2445)	(6.0371,1.3480,0.2308)
x_3	(6.0371,1.3480,0.2308)	(8.1339,0.7398,0.1999)	(5.4776,1.0717,0.1551)
x_4	(6.0644,0.8910,0.1823)	(6.4530,1.2628,0.1626)	(5.6651,1.1124,0.2445)
e_2	b_{31}	b_{32}	b_{33}
x_1	(5.7135,1.1124,0.2470)	(7.3832,0.7357,0.2232)	(5.1133,1.3217,0.2247)
x_2	(5.1133,1.3217,0.2247)	(6.0371,1.3480,0.2308)	(5.7135,1.1124,0.2470)
x_3	(8.1339,0.7398,0.1999)	(5.4776,1.0717,0.1551)	(5.7135,1.1124,0.2470)
x_4	(7.3832,0.7357,0.2232)	(5.7135,1.1124,0.2470)	(6.4530,1.2628,0.1626)
e_3	b_{31}	b_{32}	b_{33}
x_1	(8.1339,0.7398,0.1999)	(7.3832,0.7357,0.2232)	(5.1133,1.3217,0.2247)
x_2	(6.4530,1.2628,0.1626)	(6.0644,0.8910,0.1823)	(5.7135,1.1124,0.2470)
x_3	(6.0371,1.3480,0.2308)	(5.1133,1.3217,0.2247)	(6.4530,1.2628,0.1626)
x_4	(5.7135,1.1124,0.2470)	(5.6651,1.1124,0.2445)	(5.4776,1.0717,0.1551)
e_4	b_{31}	b_{32}	b_{33}
x_1	(5.7135,1.1124,0.2470)	(5.1133,1.3217,0.2247)	(5.7135,1.1124,0.2470)
x_2	(5.1133,1.3217,0.2247)	(6.0644,0.8910,0.1823)	(7.3832,0.7357,0.2232)
x_3	(6.4530,1.2628,0.1626)	(5.6651,1.1124,0.2445)	(8.1339,0.7398,0.1999)
x_4	(5.4776,1.0717,0.1551)	(5.7135,1.1124,0.2470)	(6.0371,1.3480,0.2308)

Data availability

No data was used for the research described in the article.

References

Aragonés, M. M., & Torralvo, F. A. (2024). How to select the best approach for circular economy assessment? 3D positioning framework, decision support tool and critical analysis for bio-based systems. *Environmental Impact Assessment Review*, *106*, Article 107493.

Bac, U., Alaloosi, K. A. M. S., & Turhan, C. (2021). A comprehensive evaluation of the most suitable HVAC system for an industrial building by using a hybrid building energy simulation and multi criteria decision making framework. *Journal of Building Engineering*, *37*, Article 102153.

Bagherian, A., Gershon, M., Kumar, S., & Mishra, M. K. (2024). Analyzing the relationship between digitalization and energy sustainability: A comprehensive ISM-MICMAC and DEMATEL approach. *Expert Systems with Applications*, *236*, Article 121193.

Berjawi, A., Walker, S., Patsios, C., & Hosseini, S. (2021). An evaluation framework for future integrated energy systems: A whole energy systems approach. *Renewable and Sustainable Energy Reviews*, *145*, Article 111163.

Dagtekin, Y., Kaya, S., & Besli, N. (2022). Distributed energy system selection for a commercial building by using multi-criteria decision making methods. *International Journal of Hydrogen Energy*, *47*(86), 36672–36692.

Esangbedo, M. O., Xue, J., Bai, S., & Esangbedo, C. O. (2024). Relaxed rank order centroid weighting MCDM method with improved grey relational analysis for subcontractor selection: Photothermal power station construction. *IEEE Transactions on Engineering Management*, *71*, 3044–3061.

Hafez, F. S., Sa'di, B., Safa-Gamal, M., Taufiq-Yap, Y., Alrifaey, M., Seyedmahmoudian, M., et al. (2023). Energy efficiency in sustainable buildings: A systematic review with taxonomy, challenges, motivations, methodological aspects, recommendations, and pathways for future research. *Energy Strategy Reviews*, *45*, Article 101013.

He, S. F., Wang, Y. M., & Martínez, L. (2022). Gaussian IT2FSs-based prospect theory method with application to the evaluation of renewable energy sources. *Computers & Industrial Engineering*, *169*, Article 108266.

Islam, M. R., Aziz, M. T., Alauddin, M., Kader, Z., & Islam, M. R. (2024). Site suitability assessment for solar power plants in Bangladesh: A GIS-based analytical hierarchy process (AHP) and multi-criteria decision analysis (MCDA) approach. *Renewable Energy*, *220*, Article 119595.

Jiang, J., Yu, H., Song, G., Zhao, J., Zhao, K., Ji, H., et al. (2022). Surrogate model assisted multi-criteria operation evaluation of community integrated energy systems. *Sustainable Energy Technologies and Assessments*, *53*, Article 102656.

Ke, Y., Liu, J., Meng, J., Fang, S., & Zhuang, S. (2022). Comprehensive evaluation for plan selection of urban integrated energy systems: A novel multi-criteria decision-making framework. *Sustainable Cities and Society*, *81*, Article 103837.

Kong, X., Zhao, X., Wang, C., Duan, Q., Sha, G., & Liu, L. (2022). Promote the international development of energy internet technology standards based on key competition mode. *Sustainable Cities and Society*, *86*, Article 104151.

Li, D., Liu, C., & Gan, W. (2009). A new cognitive model: Cloud model. *International Journal of Intelligent Systems*, *24*(3), 357–375.

Liu, X., Liu, J., Ren, K., Liu, X., & Liu, J. (2022). An integrated fuzzy multi-energy transaction evaluation approach for energy internet markets considering judgement credibility and variable rough precision. *Energy*, *261*, Article 125327.

Liu, H., Wang, L., Li, Z., & Hu, Y. P. (2018). Improving risk evaluation in FMEA with cloud model and hierarchical TOPSIS method. *IEEE Transactions on Fuzzy Systems*, *27*(1), 84–95.

Liu, Z., Wang, W., & Liu, P. (2022). Dynamic consensus of large group emergency decision-making under dual-trust relationship-based social network. *Information Sciences*, *615*, 58–89.

Lu, Q., & Liu, M. (2024). A multi-criteria compromise ranking decision-making approach for analysis and evaluation of community-integrated energy service system. *Energy*, *306*, Article 132439.

Martínez, L., Dinçer, H., & Yüksel, S. (2023). A hybrid decision making approach for new service development process of renewable energy investment. *Applied Soft Computing*, *133*, Article 109897.

Menacho, A. H., Rodrigues, J., & Behrens, P. (2022). A triple bottom line assessment of concentrated solar power generation in China and Europe 2020–2050. *Renewable and Sustainable Energy Reviews*, *167*, Article 112677.

Otay, İ., Onar, S. Ç., Öztayşi, B., & Kahraman, C. (2024). Evaluation of sustainable energy systems in smart cities using a Multi-Expert Pythagorean fuzzy BWM & TOPSIS methodology. *Expert Systems with Applications*, *250*, Article 123874.

Pamucar, D., Ecer, F., Gligorić, Z., Gligorić, M., & Deveci, M. (2024). A novel WENSLO and ALWAS multicriteria methodology and its application to green growth performance evaluation. *IEEE Transactions on Engineering Management*, *71*, 9510–9525.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer and Information Sciences*, *11*(5), 341–356.

Qian, Y., Liang, J., & Dang, C. (2009). Incomplete multigranulation rough set. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, *40*(2), 420–431.

Qian, Y., Liang, J., Yao, Y., & Dang, C. (2010). MGRS: A multi-granulation rough set. *Information Sciences*, *180*(6), 949–970.

Qin, G., Zhang, M., Yan, Q., Xu, C., & Kammen, D. M. (2021). Comprehensive evaluation of regional energy internet using a fuzzy analytic hierarchy process based on cloud model: A case in China. *Energy*, *228*, Article 120569.

Sassanelli, C., Garza-Reyes, J. A., Liu, Y., de Jesus Pacheco, D. A., & Luthra, S. (2023). The disruptive action of Industry 4.0 technologies cross-fertilizing circular economy throughout society. Article 109548,

Sassanelli, C., Rosa, P., Rocca, R., & Terzi, S. (2019). Circular economy performance assessment methods: A systematic literature review. *Journal of Cleaner Production*, *229*, 440–453.

Schaffert, J., Gils, H. C., Fette, M., Gardian, H., Brandstätt, C., Pregger, T., et al. (2022). Integrating system and operator perspectives for the evaluation of power-to-gas plants in the future German energy system. *Energies*, *15*(3), 1174.

Shang, K. (2022). The probabilistic linguistic decision framework of distributed energy storage system project plan based on the sustainability perspective. *Energy Reports*, *8*, 15311–15325.

Shang, Z., Yang, X., Barnes, D., & Wu, C. (2022). Supplier selection in sustainable supply chains: Using the integrated BWM, fuzzy Shannon entropy, and fuzzy MULTIMOORA methods. *Expert Systems with Applications*, *195*, Article 116567.

Singh Rawat, S., Komal, Dincer, H., & Yüksel, S. (2023). A hybrid weighting method with a new score function for analyzing investment priorities in renewable energy. *Computers & Industrial Engineering*, *185*, Article 109692.

Song, D., Yuan, X., Li, Q., Zhang, J., Sun, M., Fu, X., et al. (2022). Intrusion detection model using gene expression programming to optimize parameters of convolutional neural network for energy internet. *Applied Soft Computing*, *134*, Article 109960.

Taddei, E., Sassanelli, C., Rosa, P., & Terzi, S. (2024). Circular supply chains theoretical gaps and practical barriers: A model to support approaching firms in the era of industry 4.0. *Computers & Industrial Engineering*, *190*, Article 110049.

Tan, C., Tan, Z., Du, Y., He, Z., Geng, S., & Jiang, Z. (2023). Feasibility evaluation of virtual power plants participating in rural energy internet under zoning and stratification using prospect theory. *International Journal of Electrical Power & Energy Systems*, *144*, Article 108560.

Tripathy, A., Bhuyan, A., Padhy, R., Mangla, S. K., & Roopak, R. (2023). Drivers of lithium-ion batteries recycling industry toward circular economy in industry 4.0. *Computers & Industrial Engineering*, *179*, Article 109157.

Verma, G. K., Gope, P., & Kumar, N. (2021). PF-DA: Pairing free and secure data aggregation for Energy Internet-based smart meter-to-grid communication. *IEEE Transactions on Smart Grid*, *13*(3), 2294–2304.

Vinante, C., Sacco, P., Orzes, G., & Borgianni, Y. (2021). Circular economy metrics: Literature review and company-level classification framework. *Journal of Cleaner Production*, *288*, Article 125090.

Wang, H., & Feng, Y. (2005). On multiple attribute group decision making with linguistic assessment information based on cloud model. *Control and Decision*, *20*(6), 679.

Wang, J., Peng, L., Zhang, H., & Chen, X. (2014). Method of multi-criteria group decision-making based on cloud aggregation operators with linguistic information. *Information Sciences*, *274*, 177–191.

Wang, Y., Tao, S., Chen, X., Huang, F., Xu, X., Liu, X., et al. (2022). Method multi-criteria decision-making method for site selection analysis and evaluation of urban integrated energy stations based on geographic information system. *Renewable Energy*, *194*, 273–292.

Wang, Z., Xu, G., Wang, H., & Ren, J. (2019). Distributed energy system for sustainability transition: A comprehensive assessment under uncertainties based on interval multi-criteria decision making method by coupling interval DEMATEL and interval VIKOR. *Energy*, *169*, 750–761.

Wu, Y., Dong, Y., Qin, J., & Pedrycz, W. (2019). Flexible linguistic expressions and consensus reaching with accurate constraints in group decision-making. *IEEE Transactions on Cybernetics*, *50*(6), 2488–2501.

Wu, Y., Zhang, T., & Yi, L. (2021). Regional energy internet project investment decision making framework through interval type-2 fuzzy number based Choquet integral fuzzy synthetic model. *Applied Soft Computing*, *111*, Article 107718.

Xu, F., Gao, K., Xiao, B., Liu, J., & Wu, Z. (2022). Risk assessment for the integrated energy system using a hesitant fuzzy multi-criteria decision-making framework. *Energy Reports*, *8*, 7892–7907.

Yin, L., & Li, Y. (2022). Fuzzy vector reinforcement learning algorithm for generation control of power systems considering flywheel energy storage. *Applied Soft Computing*, *125*, Article 109149.

Zhao, H., Li, B., Lu, H., Wang, X., Li, H., Guo, S., et al. (2022). Economy-environment-energy performance evaluation of CCHP microgrid system: A hybrid multi-criteria decision-making method. *Energy*, *240*, Article 122830.

Zhao, D., Li, C., Wang, Q., & Yuan, J. (2020). Comprehensive evaluation of national electric power development based on cloud model and entropy method and TOPSIS: A case study in 11 countries. *Journal of Cleaner Production*, *277*, Article 123190.

Zhou, F., Chen, L., Zhao, J., & Wang, W. (2022). A probabilistic multi-criteria evaluation framework for integrated energy system planning. *Information Sciences*, *610*, 156–170.

Zhou, J., Su, W., & Zeng, S. (2016). Research on multi-attribute decision making approach with interval numbers and linguistic variables based on the cloud model. *Mathematics in Practice and Theory*, *46*(20), 156–164.

Zhou, J., Wu, Y., Wu, C., Deng, Z., Xu, C., & Hu, Y. (2019). A hybrid fuzzy multi-criteria decision-making approach for performance analysis and evaluation of park-level integrated energy system. *Energy Conversion and Management*, *201*, Article 112134.