

## New matching pursuit-based algorithm for SNR improvement in ultrasonic NDT

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### Abstract

In this paper a fast and efficient matching pursuit-based algorithm is proposed for SNR improvement in ultrasonic NDT of highly scattering materials. The proposed algorithm utilizes time-shifted Morlet functions as dictionary elements because they are well matched with the ultrasonic pulse echoes obtained from the transducer used in the experiments. The proposed algorithm is fast enough to be used in the signal processing stage of real time inspection systems. Computer simulation has been performed to verify the SNR improvement for diverse ultrasonic waves embodied in high-level synthetic grain noise. This improvement is also experimentally verified using ultrasonic traces acquired from a carbon fibre reinforced plastic material. Numerical results show meaningful SNR improvements for low input SNR ratios (below 0 dB).

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### 1. Introduction

Flaw detection by ultrasonic Non-Destructive Evaluation or Testing (NDE or NDT) has been proven to be an effective means to assure the quality of materials. In the analysis of back-scattered ultrasonic signals, the microstructure of the tested materials can be considered as an unresolved and randomly distributed set of reflection centres. The back-scattered ultrasonic signal is the result of convoluting the transmitted acoustic pulse with these reflection centres. This noise-like signal of structural origin (ultrasonic grain noise) is time-invariant and, unfortunately, in some cases presents a frequency band very similar to that of the echoes issuing from the flaws to be detected. Therefore, it cannot be cancelled by classical time averaging or matched band-pass filtering techniques.

Many signal processing techniques have been utilized for Signal-to-Noise Ratio (SNR) enhancement in ultrasonic NDE of highly scattering materials. The most popular one is the Split Spectrum Processing (SSP) [1,2], because it makes

possible real time ultrasonic test for industrial applications, providing quite good results. Alternatively to SSP, wavelet transform-based denoising methods have been proposed during the last years [3–5], yielding usually to higher improvements of SNR at the expense of an increase in complexity. Adaptive time-frequency analysis by basis pursuit [6] is a quite recent technique for decomposing a signal into an optimal superposition of elements in an over-complete waveform dictionary. This technique has been successfully applied to denoising ultrasonic signals contaminated with grain noise in highly scattering materials [7] as an alternative to the wavelet transform technique, being the computational cost of the basis pursuit algorithm its main drawback.

In this paper, a novel matching pursuit-based signal processing method is proposed for SNR improvement in ultrasonic NDT of highly scattering materials, such as steel and composites. The proposed method uses matching pursuit instead of basis pursuit to reduce the complexity. Despite of its iterative nature, the method is fast enough to be real time implemented. The performance of the proposed method has been evaluated using both computer simulation and experimental results, even when the input SNR ( $\text{SNR}_{\text{in}}$ )

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is lower than 0 dB (the level of echoes scattered by microstructures is above the level of flaw echoes).

## 2. Matching pursuit

The matching pursuit algorithm was introduced by Mallat and Zhang [8]. Let us suppose an approximation of the ultrasonic back-scattered signals  $x[n]$  as a linear expansion in terms of functions  $g_i[n]$  chosen from an over-complete dictionary. Let  $\mathbf{H}$  be a Hilbert space. We define the over-complete dictionary as a family  $D = \{g_i; i=0,1,\dots,L\}$  of vectors in  $\mathbf{H}$ , such as  $\|g_i\|=1$ .

The problem of choosing functions  $g_i[n] \in D$  that best approximate the analyzed signal  $x[n]$  is computationally very complex. Matching pursuit is an iterative algorithm that offers sub-optimal solutions for decomposing signals in terms of expansion functions chosen from a dictionary, where  $l^2$  norm is used as the approximation metric because of its mathematical convenience. When a well-designed dictionary is used in matching pursuit, the non-linear nature of the algorithm leads to compact adaptive signal models.

In each step of the iterative procedure, vector  $g_i[n] \in D$  which gives the largest inner product with the analyzed signal is chosen. The contribution of this vector is then subtracted from the signal and the process is repeated on the residual. At the  $m$ th iteration the residue is

$$r^{m+1}[n] = \begin{cases} x[n], & m = 0 \\ r^m[n] - \alpha_{i(m)} g_{i(m)}[n], & m \neq 0 \end{cases} \quad (1)$$

where  $\alpha_{i(m)}$  is the weight associated to optimum vector  $g_{i(m)}[n]$  at the  $m$ th iteration.

The orthogonality principle ( $\langle r^{m+1}[n], g_{i(m)}[n] \rangle = 0$ ) allows us to compute the weight  $\alpha_i^m$  associated to each dictionary element  $g_i[n]$  at the  $m$ th iteration:

$$\alpha_i^m = \frac{\langle g_i[n], r^m[n] \rangle}{\langle g_i[n], g_i[n] \rangle} = \frac{\langle g_i[n], r^m[n] \rangle}{\|g_i[n]\|^2} \quad (2)$$

The optimal function to choose at the  $m$ th iteration can be expressed as:

$$\begin{aligned} g_{i(m)}[n] &= \arg \min_{g_i \in D} |r^{m+1}[n]|^2 = \arg \max_{g_i \in D} |\alpha_i^m|^2 \\ &= \arg \max_{g_i \in D} |\alpha_i^m| \end{aligned} \quad (3)$$

The computation of correlations  $\langle g_i[n], r^m[n] \rangle$  for all vectors  $g_i[n] \in D$  at each iteration implies a high computational effort, which can be substantially reduced using an updating procedure derived from Eq. (1).

The correlations at the  $m+1$ th iteration can be obtained as follows:

$$\langle g_i[n], r^{m+1}[n] \rangle = \langle g_i[n], r^m[n] \rangle - \alpha_{i(m)} \langle g_i[n], g_{i(m)}[n] \rangle \quad (4)$$

Correlations  $\langle g_i[n], g_{i(m)}[n] \rangle$  can be pre-calculated and stored, once set  $D$  has been defined. Therefore, according to

(4), only computing correlations ( $\langle g_i[n], x[n] \rangle$ ) at the first iteration is required.

## 3. The proposed method to improve ultrasonic signals

According to the matching pursuit approach, our method has to define the dictionary elements and the way to obtain an enhanced signal. Since a real time implementation is claimed for usefulness, the complexity of the proposed method must be reduced as much as possible. Because matching pursuit is an iterative algorithm, the goal can be achieved if both the amount of operations at each iteration and the number of iterations to reach an adequate SNR increase are reduced.

If the dictionary is based on a pulsed function that mimics the echoes coming from the flaws to be detected, matching pursuit can efficiently extract flaw echoes in only a few iterations, because grain echoes have random amplitude and phase, while flaw echoes have large intensity and are far more stable in frequency.

A small and simple dictionary is desirable, too. Analysing the impulse response of the transducer used in the experiments, a  $(L=N+M)$  size dictionary composed of discrete-time shifted Morlet functions is defined. Here,  $N$  is the length of the ultrasonic data register and  $M$  the length of the discrete Morlet function. Such a pulsed function is well correlated with the flaw echoes, as depicted in Fig. 1. The size of the dictionary has been selected to extend over all possible flaw echo positions.

The dictionary elements (time-shifted Morlet pulses) can be expressed as

$$\begin{aligned} g_i[n] &= S_i e^{-((n-i)/T_m f_s)} \cos\left(2\pi \frac{f_m}{f_s} (n-i) + \phi_m\right), \\ i &= 1, \dots, L; \quad n = i - \frac{M}{2} + 1, \dots, i + \frac{M}{2} \end{aligned} \quad (5)$$

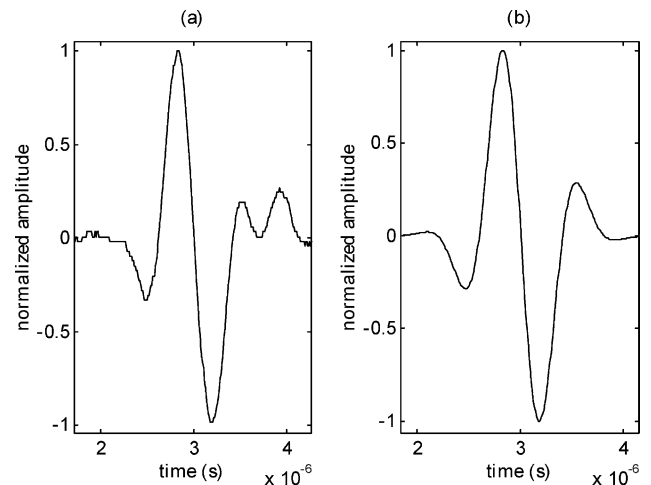


Fig. 1. Comparison: (a) flaw signal, (b) Morlet pulse.

where  $T_m$ ,  $f_m$  and  $\phi_m$  are the Morlet pulse parameters. They are related to the frequency and the bandwidth of the transducer response.  $S_i$  is a constant to achieve  $\|g_i\|=1$  and  $f_s$  is the sampling frequency of the ultrasonic signal.

Note that the method is valid for any kind of transducer if a function adapted to the transducer response is defined.

Next, we focus on describing our matching pursuit-based method to obtain an enhanced ultrasonic signal. Matching pursuit searches for the optimum Morlet function within the dictionary (which gives the higher correlation with the ultrasonic signal) at each iteration. A residual signal is obtained by subtracting the contribution of the optimum Morlet pulse from the previous residue (see Eq. (1)). In line with this iterative approach, an enhanced signal can be defined as follows

$$x_e^{m+1}[n] = \begin{cases} x[n], & m = 0 \\ x_e^m[n] - \alpha_{i(m)} g_{i(m)}[n] + G_m g_{i(m)}[n], & m \neq 0 \end{cases} \quad (6)$$

where  $G_m$  is the gain associated to optimum function  $g_{i(m)}$  at the  $m$ th iteration. In order to enlarge this echo in the enhanced signal, this gain must be higher than optimum weight  $\alpha_{i(m)}$  when the optimum function corresponds to the flaw echo. On the other hand, the gain must be close to zero when the optimum function represents back-scattered noise and, as a result, the method removes it in the enhanced signal. Therefore, the enhanced signal is obtained either by amplifying or attenuating optimum function  $g_{i(m)}$  at the  $m$ th iteration, whether this function corresponds or not to the flaw echo.

Once the enhanced signal  $x_e[n]$  has been defined, which depends on gain  $G_m$ , computing this gain is required. For this purpose, we must consider that if a given ultrasonic back-scattered signal contains a flaw echo, its energy will prevail in a localized part of the ultrasonic signal, because flaw echoes are usually bigger than grain echoes. Therefore, in order to achieve the above stated objective, we compute the following energy ratio in that localized part of the ultrasonic signal:

$$a^m = \frac{E_{(\alpha_{i(m)} g_{i(m)})}}{E_{r^{m+1}}} = \frac{\alpha_{i(m)}^2 E_{g_{i(m)}}}{E_{r^{m+1}}} = \frac{\alpha_{i(m)}^2}{\sum_{n=i-(M/2)+1}^{i+(M/2)} |r^{m+1}[n]|^2} \quad (7)$$

It must be noted that residual energy  $E_{r^{m+1}}$  is computed once the contribution of the weighted optimum function is subtracted. Parameter  $a^m$  informs us about the probability that optimum vector  $g_{i(m)}[n]$  at the  $m$ th iteration corresponds to the flaw signal. This probability, denoted by  $P^m$ , is related to parameter  $a^m$  as follows:

$$P^m \propto e^{-(1/a^m)} \quad (8)$$

The higher parameter  $a^m$  is, the most probable selected vector  $g_{i(m)}[n]$  at the  $m$ th iteration corresponds to a flaw echo and vice-versa.

Nevertheless, energy calculations are highly time consuming due to multiplications. To reduce the complexity associated to the computation of parameter  $a^m$ , a new probability indicator is defined:

$$b^m = \frac{\alpha_{i(m)}^2}{\left( \sum_{n=i-(M/2)+1}^{i+(M/2)} |r^{m+1}[n]| \right)^2} \quad (9)$$

Although  $a^m$  and  $b^m$  coefficients give similar results,  $b^m$  is the simplest one.

Then, to observe the above stated gain conditions, gain  $G_m$  in Eq. (6) and probability indicator  $b^m$  in Eq. (9) are related as follows:

$$G_m = b^m \alpha_{i(m)} \quad (10)$$

Therefore, the new signal  $x_e[n]$  to enhance the visibility of flaw echoes is expressed as follows:

$$x_e^{m+1}[n] = \begin{cases} x[n], & m = 0 \\ x_e^m[n] - \alpha_{i(m)} g_{i(m)}[n] + b^m \alpha_{i(m)} g_{i(m)}[n], & m \neq 0 \end{cases} \quad (11)$$

In order to ascertain the complexity of the method, the operations required by the algorithm are next explained. At the first iteration, correlations  $\langle g_i[n], x[n] \rangle$  are calculated by using the FFT algorithm. At the next following iterations, our method performs:

- $N+M-1$  comparisons, corresponding to Eq. (3);
- $2M-1$  multiplications and accumulations, corresponding to Eq. (4);
- $M$  additions, one division and one multiplication, corresponding to Eq. (9);
- one multiplication, one addition and  $M$  multiplications and accumulations, corresponding to Eq. (11).

The above values have been obtained once correlations  $\langle g_i[n], g_{i(m)}[n] \rangle$  have been pre-calculated and stored.

After a few iterations, the enhanced signal  $x_e[n]$  has a higher SNR than the original one  $x[n]$ , as will be shown in computer simulations and experimental results. The criterion chosen to stop the iteration is: when the maximum value of weights  $\alpha_i^m$  computed at the  $m$ th iteration is below a threshold, which depends on the minimum size of the flaws to be detected, the matching pursuit stops. When the number of iterations exceeds the upper bound, the increase in computational cost does not improve the SNR significantly, because the highest SNR improvement is obtained at the iteration in which the flaw signal is selected and amplified. A few iterations are required to achieve a noticeable SNR enhancement, as will be shown in computer and experimental results.

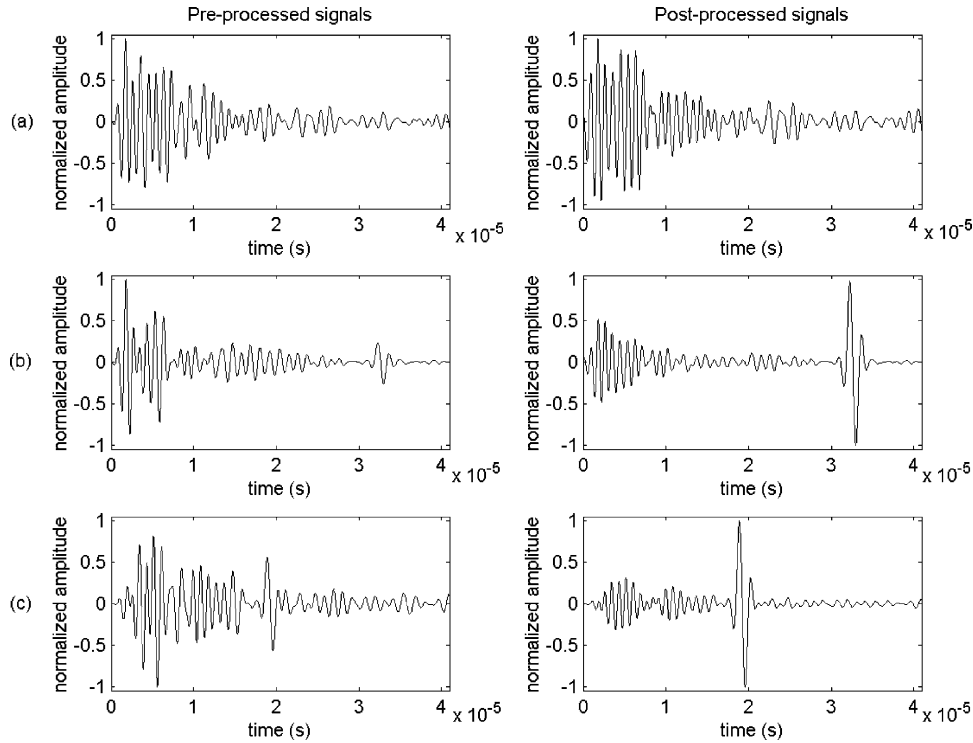


Fig. 2. Processed results using simulated signals: (a) no hole, (b) hole at 9.62 cm depth, (c) hole at 5.62 cm depth.

#### 4. Computer simulation

Grain noise (clutter or speckle) models are frequently used to generate synthetic noise registers for the evaluation of the performance of noise reduction algorithms [9,10]. This grain noise of structural origin is described as the superposition of back-scattered signals from the grains boundaries. If material scattering and attenuation are considered, the noise signal can be described as [10]

$$Y_n(\omega) = H_{\text{trans}}(\omega) H_{\text{trans}}(\omega) \left( \sum_{k=1}^K \frac{\beta_k \omega^2}{z_k} e^{-2\alpha_s z_k \omega^4} e^{-j(2z_k \omega/c_l)} \right) \quad (12)$$

where  $H_{\text{trans}}(\omega)$  is the electromechanical transducer response,  $z_k$  the position of the  $k$ th grain echo,  $\alpha_s$  the material attenuation coefficient,  $c_l$  the propagation velocity,  $K$  the number of grain back-scattered signals and  $\beta_k$  the material scattering coefficients.

The flaw signal was simulated by one single Morlet pulse according to the model defined by Eq. (13)

$$Y_f(\omega) = H_{\text{trans}}(\omega) H_{\text{trans}}(\omega) \left( e^{-2\alpha_s z_f \omega^4} e^{-j(2z_f \omega/c_l)} \right) \quad (13)$$

where  $z_f$  is the flaw position.

Superimposing the normalized grain noise and the flaw echo signal at a known flaw location results in the simulated signal.  $N=4096$  samples-length grain noise signals were generated by superimposing  $K=2000$  grain echoes with Gaussian-distributed amplitudes at

the uniformly distributed positions. The material attenuation coefficient used was  $\alpha_s = 8 \times 10^{-28} \text{ m}^{-1} \text{ s}^4$ . The sampling rate for the simulated signals was  $f_s = 100 \text{ MHz}$  and the propagation velocity  $c_l = 6000 \text{ m/s}$ . The grain noise signals then corresponds to a  $0.5 c_l N/f_s = 12.29 \text{ cm}$  material path. The transducer response has  $5 \text{ MHz}$  centre frequency and  $50\%$  bandwidth. Therefore, the Morlet pulse parameters in Eq. (5) are:  $T_m = 0.025 \mu\text{s}$ ,  $f_m = 5 \text{ MHz}$  and  $\phi_m = \pi/2$ . The Morlet pulse is defined as a  $M=300$  samples-length function, as depicted in Fig. 1. We have considered different flaw echo amplitudes and locations achieving signal-to-noise ratios between

Table 1

SNR improvement and complexity results of the proposed method for 10 different simulated signals

Trial No.	SNR <sub>in</sub> (dB)	SNR <sub>out</sub> (dB)	SNR improvement = SNR <sub>out</sub> – SNR <sub>in</sub> (dB)	Iterations
1	–11.3406	8.3361	19.6768	20
2	–12.6157	5.5979	18.2136	19
3	–10.9950	8.3395	19.3345	19
4	–6.1784	7.8504	14.0288	36
5	–7.1309	5.9683	13.0993	22
6	–9.6825	9.8970	19.5795	22
7	–8.7304	8.5205	17.2508	21
8	–5.3040	14.6559	19.9599	24
9	–7.2302	19.0935	26.3237	27
10	–13.4324	3.6083	17.0407	24
Mean	–8.8667	10.4504	19.1976	23.4
Standard deviation				5.08

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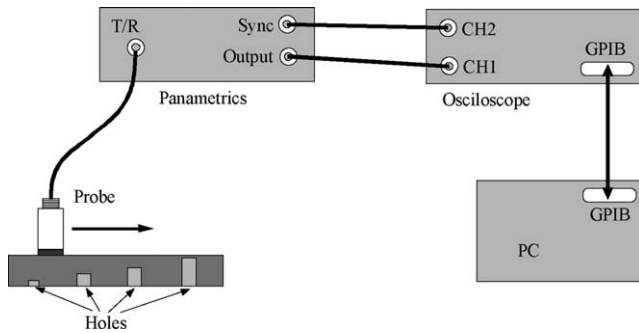


Fig. 3. Figure presenting the experimental setup.

–5 and –15 dB. The minimum depth from where flaw signal starts is 4 cm.

In order to quantify the enhancement in the ultrasonic signal, the SNR of both pre- and post-processed signals ( $SNR_{in}$  and  $SNR_{out}$ , respectively) were evaluated and compared. SNR was computed as the ratio, expressed in dB, of the maximum flaw signal amplitude and the maximum noise amplitude.

Fig. 2 shows three simulated signals, which correspond to (a) no hole, (b) hole at 9.62 cm depth and (c) hole at 5.62 cm depth. Pre- and post-processed signals are displayed in the same figure. Our method clearly enhances the visibility of the flaw echo and reduces the grain noise.

Table 1 presents the SNR improvement and complexity results for 10 different ultrasonic signals (arbitrarily chosen), among 200 simulated, where the flaw echoes were embedded in different locations. The SNR

improvement is about 19.2 dB and the average number of iterations is 24.

## 5. Experimental results

A carbon fibre reinforced parallelepiped plastic (CFRP) block of 120 mm thickness was machined on one of its plain surfaces to perform flat-bottom holes (FBH) at different depths along a straight line. Experimental echo traces were obtained using a circular ultrasonic transducer for longitudinal waves, 6.35 mm in diameter, 5 MHz of nominal frequency and 50% bandwidth. The probe was placed in contact and driven in pulse-echo operation with a Panametrics Ultrasonic Analyser 5052UA. The selected pulser/receiver parameters were: damping resistance 200  $\Omega$ , energy position 2 (810 pF for the HV discharge capacitor), receiver gain 26 dB, cut-off frequency of the receiver high-pass filter 300 kHz. The ultrasonic traces were acquired by means of a digital oscilloscope, Tektronix TDS 744 of 2 GS/s and data length of 4000 samples, which were transferred via GPIB to a computer for further processing. The signals were acquired with a sampling frequency of 100 MHz. The experimental setup is shown in Fig. 3.

Hundreds of experimental signals were recorded scanning with the contact transducer the composite block over the plane opposite to the one where the holes were drilled, in order to obtain input signals with and without hole indications. Fig. 4 shows four experimental signals, which correspond to (a) no hole, (b) hole at 6.75 cm depth, (c) hole at 4.75 cm depth, (d) two holes at 4.75 and 6.75 cm depth.

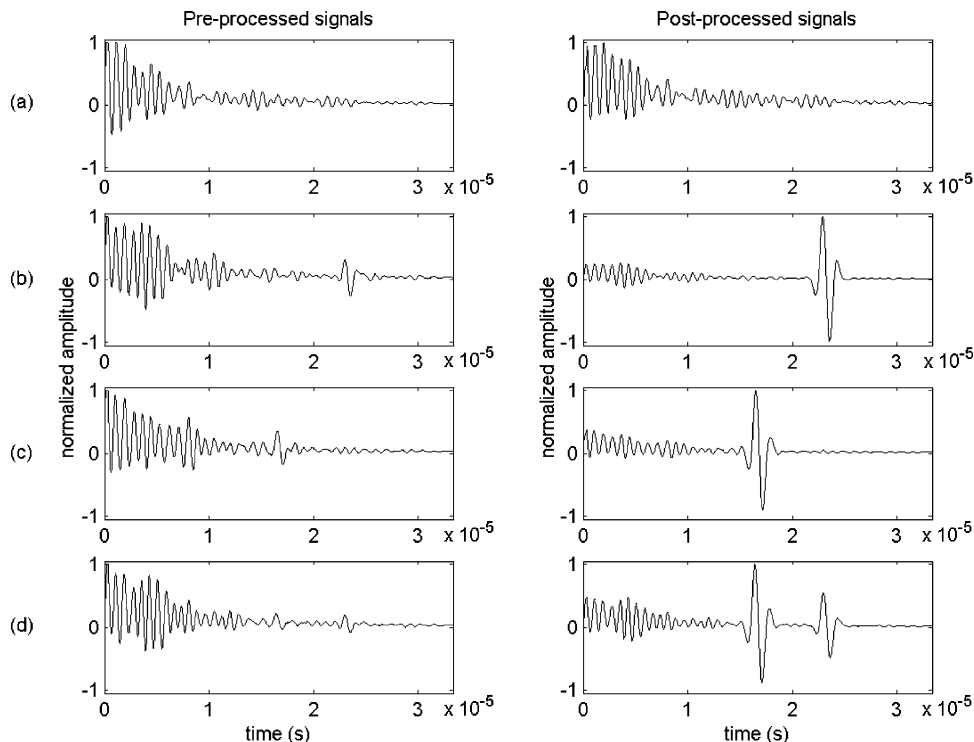


Fig. 4. Processed results using experimental signals: (a) no hole, (b) hole at 6.75 cm depth, (c) hole at 4.75 cm depth, (d) two holes at 4.75 and 6.75 cm depth.

Table 2  
SNR improvement and complexity results of the proposed method for 10 different experimental signals

Trial No.	SNR <sub>in</sub> (dB)	SNR <sub>out</sub> (dB)	SNR improvement = SNR <sub>out</sub> – SNR <sub>in</sub> (dB)	Iterations
1	–15.2391	1.3637	16.6028	22
2	–10.0338	11.0046	21.0384	34
3	–15.0897	2.4770	17.5668	23
4	–14.4708	3.8066	18.2774	20
5	–15.6503	0.4321	16.0824	23
6	–12.5786	5.4832	18.0618	19
7	–13.4324	4.3497	17.7821	27
8	–10.8424	9.6860	20.5284	16
9	–14.3340	4.7609	19.0949	21
10	–14.7510	3.9180	18.6690	22
Mean	–13.4406	5.3580	18.4984	22.7
Standard deviation				4.90

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at 4.75 cm depth and (d) two holes at 4.75 and 6.75 cm depth. Pre- and post-processed signals are displayed in the same figure.

Table 2 shows the SNR improvement and complexity results obtained for 10 experimental ultrasonic signals, all of them with SNR<sub>in</sub> below 0 dB. The SNR improvement in these experiments is about 18.5 dB and the average number of iterations is 23. It is important to underline that even when two flaw echoes appear in the same temporal record the method performance is not degraded. In fact, the case of Fig. 4(d) corresponds to the trial 7 in Table 2.

## 6. Conclusions

A novel matching pursuit-based method has been presented to improve the SNR in ultrasonic NDT of highly scattering materials. The method decomposes the ultrasonic signal into a sub-optimal superposition of dictionary elements within an over-complete dictionary. A dictionary composed of discrete-time shifted Morlet functions have been defined and adapted to the frequency and bandwidth of the ultrasonic transducer used in the work.

The method is very efficient eliminating noise and increasing the visibility of ultrasonic flaw signals. Numerical results show SNR improvements of about 19 dBs with low computational cost. It has been possible to enhance flaw echoes in highly scattering materials (SNR<sub>in</sub> < 0 dB), even when two adjacent flaw echoes appear in the same ultrasonic signal. The number of iterations needed to attain a noticeable SNR improvement is low, which permits its use in practical real-time ultrasonic NDT systems for industrial applications. Only 20 iterations are needed, on average, as has been demonstrated.

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