Measurements of Consensus in Multi-granular Linguistic Group Decision-Making[*]

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Abstract. The reaching of consensus in group decision-making (GDM) problems is a common task in group decision processes. In this contribution, we consider GDM with linguistic information. Different experts may have different levels of knowledge about a problem and, therefore, different linguistic term sets (multi-granular linguistic information) can be used to express their opinions.

The aim of this paper is to present different ways of measuring consensus in order to assess the level of agreement between the experts in multi-granular linguistic GDM problems. To make the measurement of consensus in multi-granular GDM problems possible and easier, it is necessary to unify the information assessed in different linguistic term sets into a single one. This is done using fuzzy sets defined on a basic linguistic term set (BLTS). Once the information is uniformed, two types of measurement of consensus are carried out: consensus degrees and proximity measures. The first type assesses the agreement among all the experts' opinions, while the second type is used to find out how far the individual opinions are from the group opinion. The proximity measures can be used by a moderator in the consensus process to suggest to the experts the necessary changes to their opinions in order to be able to obtain the highest degree of consensus possible. Both types of measurements are computed in the three different levels of representation of information: pair of alternatives, alternatives and experts.

Keywords: Consensus, multi-granular linguistic information, group decision-making, linguistic modelling, fuzzy preference relation

1   Introduction

A group decision-making (GDM) problem may be defined as a decision situation where: i) there exist two or more experts that are characterized by their own perceptions, attitudes, motivations and knowledge, ii) there exists a problem to be solved, and iii) they try to achieve a common solution.

Fuzzy sets theory has proven successful for handling fuzziness and modelling qualitative information [6, 7, 13]. In this theory, the qualitative aspects of the problem are represented by means of "linguistic variables" [14], i.e., variables whose values are not numbers but words or sentences in a natural or artificial language.

An important parameter to determine in a linguistic context is the "granularity of uncertainty", i.e., the cardinality of the linguistic term set that will be used to express the information. Because experts may come from different research areas, and thus have different levels of knowledge, it is natural to assume that linguistic term sets of different cardinality and/or semantics could be used to express their opinions on the set of alternatives. In these cases, we say that we are working in a multi-granular linguistic context [4, 12], and we will call this type of problem a multi-granular linguistic GDM problem.

In GDM problems there are two processes to carry out before obtaining a final solution [3, 5, 8, 9]: *the consensus process* and *the selection process* (see Figure 1). The first one refers to how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Normally this process is guided by the figure of a moderator [5, 9]. The second one consists of how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. In [4], the selection process for multi-granular linguistic GDM problem was studied. Therefore, in this paper, we focus on the consensus process, and in particular we address the problem of how to measure the consensus in such a type of GDM problem.

Traditionally, the consensus process is defined as a dynamic and iterative group discussion process, coordinated by a moderator, who helps the experts to bring their opinions closer. In each step of this process, the moderator, by means of a consensus measure, knows the actual level of consensus between the experts which establishes the distance to the ideal state of consensus. If the consensus level is not acceptable, i.e., if it is lower than a specified threshold, then the moderator would urge the experts to discuss their opinions further in an effort to bring them closer [2, 15].

The aim of this paper is to present two different measurements to assess the level of agreement between the experts in multi-granular GDM problems. These measurements can be classified into two types:

a) *Consensus degrees* to identify the level of agreement among all experts and to decide when the consensus process should stop.
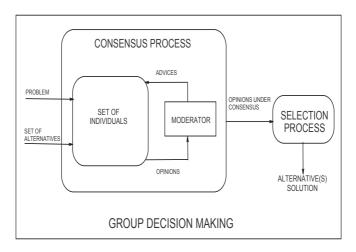
Fig. 1. Resolution process of a group decision-making problem

b) *Proximity measures* to evaluate the distance between the experts' individual opinions and the group or collective opinion. The proximity values are used by the moderator to guide the direction of the changes in the experts' opinions in order to increase the degree of consensus.

For each one of these measurements, it is interesting not only to know the global agreement or proximity amongst experts' but also the partial degrees on a particular alternative or pair of alternatives. To do this, both types of measurements are carried out at three different levels of representation of information:

Level 1 or pair of alternatives level. At this level both the agreement amongst all the experts and the distance between each individual expert's opinion and the group opinion on each pair of alternatives are calculated.

Level 2 or alternatives level. At this level, the consensus degree and the proximity on each alternative are obtained.

Level 3 or experts' level. The global consensus degree amongst all the experts and the distance between each individual expert's opinion and the group opinion on all the alternatives are calculated.

This means that in total six measurements are obtained, a consensus degree and a proximity measure at each one of the three levels. To make the computation of these six measurements in multi-granular GDM problems possible and easier, it is necessary to unify the different linguistic term sets into a single one. To do so, fuzzy sets on a basic linguistic term set (BLTS) are used, and the appropriate transformation functions are defined.

The rest of the paper is set out as follows. The multi-granular linguistic GDM problem is described in Section 2. The different consensus and proximity measures are presented in Section 3. Finally, in Section 4 we draw our conclusions.
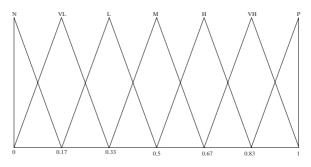
Fig. 2. A set of seven terms with their semantics

2   Multi-granular Linguistic GDM Problems

We focus on GDM problems in which two or more experts express their preferences about a set of alternatives by means of linguistic labels. A classical way to express preferences in GDM problems is by means of preference relations [3]. A GDM problem based on linguistic preference relations may be defined as follows: there are $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$), a finite set of alternatives, and a group of experts, $E = \{e_1, e_2, \ldots, e_m\}$ ($m \geq 2$); each expert e_i provides his/her preferences on X by means of a linguistic preference relation, $\mu_{P_{e_i}} : X \times X \to S$, where $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set characterized by its cardinality or *granularity*, $\#(S) = g + 1$. Additionally, the following properties are assumed:

1. The set S is ordered: $s_i \geq s_j$, if $i \geq$ j.
2. There is the negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.
3. There is the min operator: $Min(s_i, s_j) = s_i$ if $s_i \leq s_j$.
4. There is the max operator: $Max(s_i, s_j) = s_i$ if $s_i \geq s_j$.

The semantics of the terms is represented by fuzzy numbers defined on the [0,1] interval. One way to characterize a fuzzy number is by using a representation based on parameters of its membership function [1]. For example, the following semantics, represented in Figure 2, can be assigned to a set of seven terms via triangular fuzzy numbers:

$$P = Perfect = (0.83, 1, 1) \quad VH = Very\_High = (0.67, 0.83, 1)$$
$$H = High = (0.5, 0.67, 0.83) \; M = Medium = (0.33, 0.5, 0.67)$$
$$L = Low = (0.17, 0.33, 0.5) \quad VL = Very\_Low = (0, 0.17, 0.33)$$
$$N = None = (0, 0, 0.17)$$

The ideal situation in GDM problems in a linguistic context would be one where all the experts use the same linguistic term set S to provide their opinions. However, in some cases, experts may belong to distinct research areas and have different levels of knowledge about the alternatives. A consequence of this is that the expression of preferences will be based on linguistic term sets with

different granularity, which means that adequate tools to manage and model multi-granular linguistic information become essential [4, 7, 12].

In this paper, we deal with multi-granular linguistic GDM problems, i.e., GDM problems where each expert e_i may express his/her opinions on the set of alternatives using different linguistic term sets with different cardinality $S_i = \{s_0^i, \ldots, s_p^i\}$, by means of a linguistic preference relation $\mathbf{P_{e_i}} = (p_i^{jk})$, where $p_i^{jk} \in S_i$ represents the preference of alternative x_j over alternative x_k for that expert.

3   The Measurement of Consensus in Multi-granular Linguistic Context

The measurement of consensus in GDM problems is carried out using two different measures: consensus degrees and proximity measures. However, as we assume multi-granular linguistic context, the first step must be to obtain a uniform representation of the preferences, i.e., experts' preferences must be transformed (using a transformation function) into a single domain or linguistic term set that we call *basic linguistic term set* (BLTS) and is denoted by S_T.

The measurement of consensus in multi-granular linguistic GDM problems is therefore carried out in three steps: (i) *making the linguistic information uniform*, (ii) *computation of consensus degrees* and (iii) *computation of proximity measures*

3.1   Making the Linguistic Information Uniform

In this step, a *basic linguistic term set* (BLTS), S_T, has to be selected. To do this it seems reasonable to impose a granularity high enough to maintain the uncertainty degrees associated to each one of the possible domains to be unified. This means that the granularity of the BLTS has to be as high as possible. Therefore, in a general multi-granular linguistic context, to select S_T we proceed as follows:

1. If there is just one linguistic term set, from the set of different domains to be unified, with maximum granularity, then we choose that one as the BLTS, S_T.
2. If there are two or more linguistic term sets with maximum granularity, then the election of S_T will depend on the semantics associated to them:
   (a) If all of them have the same semantics (with different labels), then any one of them can be selected as S_T.
   (b) If two or more of them have different semantics, then S_T is defined as a generic linguistic term set with a number of terms greater than the number of terms a person is able to discriminate, which is normally 11 or 13 [11], although we can find cases of BLTS with 15 terms symmetrically distributed [4, 10].

Once S_T has been selected, the following multi-granular transformation function is applied to transform every linguistic value into a fuzzy set defined on S_T:

Definition 1 *[4] If $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ are two linguistic term sets, with $g \geq p$, then a multi-granular transformation function, τ_{AS_T}, is defined as*

$$\tau_{AS_T} : A \longrightarrow F(S_T)$$

$$\tau_{AS_T}(l_i) = \{(c_h, \alpha_{ih}) \ /h \in \{0, \ldots, g\}, \ \forall l_i \in A$$

$$\alpha_{ih} = \max_y min\{\mu_{l_i}(y), \mu_{c_h}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined on S_T, and $\mu_{l_i}(y)$ and $\mu_{c_h}(y)$ are the membership functions of the fuzzy sets associated to the linguistic terms l_i and c_h, respectively.

The composition of the linguistic preference relations provided by the experts $\mu_{P_{e_i}}$ with the *multi-granular transformation functions* $\{\tau_{S_i S_T}, \ \forall i\}$ will result in a unification of the preferences for the whole group of experts. In particular, the linguistic preference p_i^{lk} will be transformed into the fuzzy set, defined on $S_T = \{c_0, \ldots, c_g\}$,

$$\tau_{S_i S_T}(p_i^{lk}) = \{(c_h, \alpha_{ih}^{lk}) \ / \ h = 0, \ldots, g\}$$

$$\alpha_{ih}^{lk} = \max_y \min\{\mu_{p_i^{lk}}(y), \mu_{c_h}(y)\}.$$

We will continue to denote $\tau_{S_i S_T}(p_i^{lk})$ by p_i^{lk}, and we will use only the membership degrees to denote the uniformed linguistic preference relation:

$$\mathbf{P_{e_i}} = \begin{pmatrix} p_i^{11} = (\alpha_{i0}^{11}, \ldots, \alpha_{ig}^{11}) & \cdots & p_i^{1n} = (\alpha_{i0}^{1n}, \ldots, \alpha_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ p_i^{n1} = (\alpha_{i0}^{n1}, \ldots, \alpha_{ig}^{n1}) & \cdots & p_i^{nn} = (\alpha_{i0}^{nn}, \ldots, \alpha_{ig}^{nn}) \end{pmatrix}$$

3.2   Computation of Consensus Degrees

In GDM problems, each consensus parameter requires the use of a similarity function to obtain the level of agreement among all the experts. Several similarity functions have been proposed to measure how far each expert is from the remaining ones, including the Euclidean distance, the cosine and sine of the angle between vectors, etc [2, 15].

Initially, we used these traditional distance functions to measure the proximity between the linguistic preferences p_i^{lk}, p_j^{lk} given by experts e_i, e_j, by comparing the membership degrees vectors associated to them. However, after checking the results of some trials, we discovered cases in which unexpected results were obtained, as it is shown in the following example, which implied that these functions were not suitable for our objectives.

Example 1 *Let $p_1^{12} = (1, 0, 0, 0, 0, 0)$, $p_2^{12} = (0, 0, 0, 1, 0, 0)$ and $p_3^{12} = (0, 0, 0, 0, 0, 1)$ be three experts' assessments on the pair of alternatives (x_1, x_2), the following values are obtained using the Euclidean distance:*

$$d(p_1^{12}, p_2^{12}) = \sqrt{\sum_{i=0}^{g}(\alpha_{1i}^{12} - \alpha_{2i}^{12})^2} = \sqrt{2} \quad ; \quad d(p_1^{12}, p_3^{12}) = \sqrt{\sum_{i=0}^{g}(\alpha_{1i}^{12} - \alpha_{3i}^{12})^2} = \sqrt{2}$$

With the Euclidean distance, both preference values p_3^{12} and p_1^{12} are at the same distance from preference p_2^{12}, although, it is clear, however, that the first one is further from p_2^{12} than the second one. The problem in this case is the way the information of these fuzzy sets is interpreted, as a vector of membership values without having taking into account their positions in it. To take into account both the values and positions, a different similarity function able to represent the distribution of the information in the fuzzy set p_i^{lk} is necessary. The use of the central value of the fuzzy set, cv_i^{lk}, is suggested:

$$cv_i^{lk} = \frac{\sum_{h=0}^{g} index(s_h^i) \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}}, \quad index(s_h^i) = h \tag{1}$$

This value represents the central position or centre of gravity of the information contained in the fuzzy set $p_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$. The range of the central value function is the closed interval $[0, g]$.

Example 2 *The application of (1) to the assessments of example 1 gives the following central values:*

$$cv_1^{12} = 0, \quad cv_2^{12} = 3, \quad cv_3^{12} = 5.$$

Other experts' assessments as $p_1^{14} = (0.3, 0.8, 0.6, 0, 0, 0)$, $p_1^{24} = (0, 0.3, 0.8, 0.6, 0, 0)$, and $p_1^{34} = (0, 0, 0, 0.3, 0.8, 0.6)$, their central values are:

$$cv_1^{14} = 1.18, \quad cv_1^{24} = 2.18, \text{ and } cv_1^{34} = 4.18.$$

As expected, when the information (membership values) moves from the left part of the fuzzy set to the right part, the central value increases.

The value $|cv_i^{lk} - cv_j^{lk}|$ can be used as a measure of distance between the the preference values p_i^{lk} and p_j^{lk}, and, therefore, a measure of similarity or proximity between these two preference values, measured in the unit interval $[0, 1]$, is defined as:

$$s(p_i^{lk}, p_j^{lk}) = 1 - \left| \frac{cv_i^{lk} - cv_j^{lk}}{g} \right| \tag{2}$$

Clearly, the closer $s(p_i^{lk}, p_j^{lk})$ to 1 the more similar p_i^{lk} and p_j^{lk} are, while the closer $s(p_i^{lk}, p_j^{lk})$ to 0 the more distant p_i^{lk} and p_j^{lk} are.

Example 3 *The values of similarity between the assessments of example* 1 *are:*
$s(p_1^{12}, p_2^{12}) = 0.4$, $s(p_1^{12}, p_3^{12}) = 0$.

Using the above similarity function (2), the computation of the consensus degrees is carried out in several steps:

1. After the experts' preferences are uniformed, the *central values* are calculated:

$$cv_i^{lk}; \ \forall \ i = 1, \ldots, m; \ \ l, k = 1, \ldots, n \ \wedge \ l \neq k \tag{3}$$

2. For each pair of experts e_i, e_j $(i < j)$, a *similarity matrix* $SM_{ij} = \left(sm_{ij}^{lk}\right)$ is calculated, where

$$sm_{ij}^{lk} = s(p_i^{lk}, p_j^{lk}) \tag{4}$$

3. A *consensus matrix*, CM, is obtained by aggregating all the similarity matrices. This aggregation is carried out at the level of pairs of alternatives:

$$cm^{lk} = \phi(sm_{ij}^{lk}); \ i, j = 1, \ldots, m \ \wedge \forall \ l, k = 1, \ldots, n \ \wedge \ i < j$$

In our case, we propose the use of the arithmetic mean as the aggregation function ϕ, although, different aggregation operators could be used according to the particular properties we want to implement [8].

4. Computation of consensus degrees. As we said in Section 1, the consensus degrees are computed at the three different levels: pairs of alternatives, alternatives and experts.

Level 1. *Consensus on pairs of alternatives,* cp^{lk}, to measure the consensus degree amongst all the experts on each pair of alternatives. In our case, this is expressed by the element (l, k) of the consensus matrix CM, i.e.,

$$cp^{lk} = cm^{lk}, \ \ \forall l, k = 1, \ldots, n \ \wedge \ l \neq k$$

The closer cp^{lk} to 1, the greater the agreement amongst all the experts on the pair of alternatives x_l, x_k. This measure will allow the identification of those pairs of alternatives with a poor level of consensus.

Level 2. *Consensus on alternatives,* ca^l, to measure the consensus degree amongst all the experts on each alternative. For this, we take the average of each row of the consensus matrix CM.

$$ca^l = \frac{\sum_{k=1}^{n} cm^{lk}}{n} \tag{5}$$

These values can be used to propose modification of preferences associated to those alternatives with a consensus degree lower than a minimal consensus threshold γ, i.e, $ca^l < \gamma$.

Level 3. *Consensus amongst the experts,* ce, to measure the global consensus degree amongst the experts' opinions. It is computed as the average of all consensus on alternative values, i.e,

$$ce = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{6}$$

If the consensus value ce is low then there exists a great discrepancy between the experts' opinions, and therefore they are far from reaching consensus. In this case, the moderator would urge the experts to discuss their opinions further in an effort to bring them closer. However, when the consensus value is high enough, the moderator would finish the consensus process and the selection process would be applied to obtain the final consensus solution to the GDM problem [2, 15].

3.3   Computation of Proximity Measures

Proximity measures evaluate the agreement between the individual experts' opinions and the group opinion. Thus, to calculate them, a collective preference relation, $\mathbf{P_{e_c}} = (p_c^{lk})$, has to be obtained by means of the aggregation of the set of (uniformed) individual preference relations $\{\mathbf{P_{e_i}} = (p_i^{lk}); i = 1, \ldots, m\}$:

$$p_c^{lk} = \psi(p_1^{lk}, \ldots, p_m^{lk})$$

with ψ an "aggregation operator". As $p_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$ then $p_c^{lk} = (\alpha_{c0}^{lk}, \ldots, \alpha_{cg}^{lk})$ with

$$\alpha_{cj}^{lk} = \psi(\alpha_{1j}^{lk}, \ldots, \alpha_{mj}^{lk}).$$

which means that p_c^{lk} is also a fuzzy set defined on S_T.

Clearly, the expression (2) can be used to evaluate the agreement between each individual expert's preferences, $\mathbf{P_{e_i}}$, and the collective preferences, $\mathbf{P_{e_c}}$. Therefore, the measurement of proximity is carried out in two steps:

1. A *proximity matrix*, $PM_i = (pm_i^{lk})$, for each expert e_i, is obtained where $pm_i^{lk} = s(p_i^{lk}, p_c^{lk})$.
2. Computation of proximity measures. Again, we calculate proximity measures at three different levels.

    Level 1. *Proximity on pairs of alternatives, pp_i^{lk},* to measure the proximity between the preferences, on each pair of alternatives, of each individual expert, e_i, and the group's ones. In our case, this is expressed by the element (l, k) of the proximity matrix PM_i, i.e.,

    $$pp_i^{lk} = pm_i^{lk}, \quad \forall l, k = 1, \ldots, n \ \wedge \ l \neq k$$

    Level 2. *Proximity on alternatives, pa^l,* to measure the proximity between the preferences, on each alternative, of each individual expert, e_i, and the group's ones. For this, we take the average of each row of the proximity matrix PM_i.

    $$pa_i^l = \frac{\sum_{k=1}^{n} pp_i^{lk}}{n} \tag{7}$$

    Level 3. *Experts's proximity, pe_i,* to measure the global proximity between the preferences of each individual expert, e_i, and the group's ones. It is computed as the average of all proximity on alternative values, i.e,

    $$pe_i = \frac{\sum_{l=1}^{n} pa_i^l}{n} \tag{8}$$

If the above values are close to 1 then they have a positive contribution for the consensus to be high, while if they are close to 0 then they have a negative contribution to consensus. As a consequence, these proximity measures can be used to build a feedback mechanism, based on simple rules or recommendations to support the experts in changing their opinions and thus obtain the highest degree of consensus possible, as was done in [8].

4   Conclusions

The reaching of consensus in GDM problems needs measurements to assess the consensus between the experts. In this paper, two types of measurements were proposed: consensus degrees and proximity measures. The first one is used to assess the agreement amongst all the experts' opinions, while the second one is used to find out how far the individual opinions are from the group opinion. Both types of measurements are computed at three different levels of representation of information: pair of alternatives, alternatives and experts.

We have also shown that to make the measurement of consensus possible in multi-granular linguistic GDM problems, it was necessary to unify the different linguistic term sets into a single linguistic term set. To do this, fuzzy sets defined on a basic linguistic term set (BLTS) were used.

Finally, for future research, the proximity measures will be used to design a consensus support system able to generate advice on the necessary changes in the experts' opinions in order to reach consensus, which would make the figure of the moderator unnecessary in the consensus reaching process.

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