A Dual Graph Pyramid Approach to Grid-Based and Topological Maps Integration for Mobile Robotics

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Abstract. A pyramid is a hierarchy of successively reduced graphs which represents the contents of a base graph at multiple levels of abstraction. The efficiency of the pyramid to represent the information is strongly influenced by the graph selected to encode the information within each pyramid level (data structure) and the scheme used to build one graph from the graph below (decimation process). In this paper, the dual graph data structure and the maximal independent edge set (MIES) decimation process are applied in the context of robot navigation. The aim is to integrate the grid-based and the topological paradigms for map building. In this proposal, dual graphs allow to correctly represent the embedding of the topological map into the metric one.

1 Introduction

Autonomous navigation is a fundamental ability for mobile robots which requires the integration of different modules. Among them, self-localization and environment mapping are two essential ones, as they are needed at different levels, from low-level control to higher-level strategic decision making or navigation supervision. It is well known that to guarantee bounded errors on its pose estimates, the robot must rely on sensors which can perceive stable environment features. Thus, if the robot manages a spatially consistent map of the environment, it could apply a map-based localization approach to obtain a correct estimation of its pose [1]. On the other hand, if the robot pose is exactly known, it could build a consistent environment map with the perceived data. The mapping and localization tasks are then *intimately tied together* [2], and they must be concurrently solved. The problem of the simultaneous localization and mapping (SLAM) has been extensively addressed by the robotic community in the last years.

Specifically, two fundamental paradigms have been developed for modeling indoor environments in mobile robotics: the metric and the topological paradigms. Metric approaches are easy to build and they represent accurately the real features of the world [3]. However, they suffer from their huge data load and time

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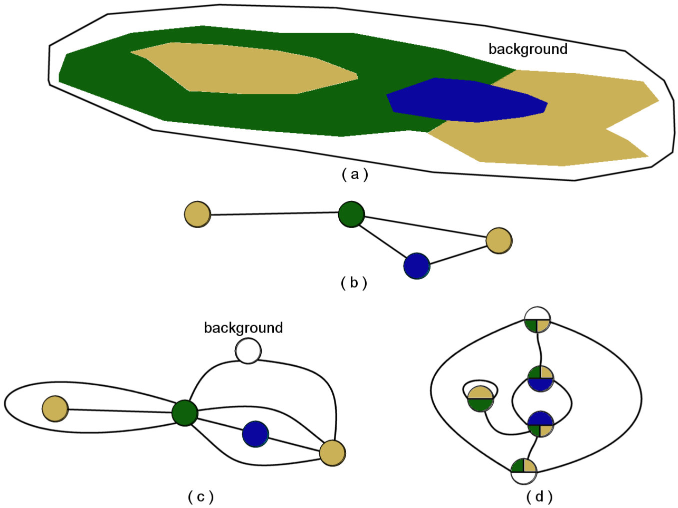

Fig. 1. a) Metric base map (green, blue and brown colored areas represent different surfaces); b) topological representation of a) using a simple graph; and c-d) topological representation of a) using a dual graph

complexity. On the other hand, topological maps are usually more compact, since their resolution is determined by the complexity of the environment. Thus, they allow fast path planning and provide more natural interfaces for human instructions. The principle of topological maps is to split the free-space of the real environment into a small number of regions.

Topological approaches usually represent the environment by using simple graphs [3,4]. Nodes in such graphs correspond to different environment regions and arcs indicate spatial relationships between them. Fig. 1 shows that simple graphs only take into account adjacency relationships, being unable to distinguish from the graph an adjacency relation from an inclusion relation between two regions. Besides, if there is two non-connected boundaries which will allow a robot to cross from one region to another one, the simple graph only joins these nodes by one arc. These limitations can be raised if dual graphs are employed because their structure is adapted to the processed data and they correctly encode the topology in 2D (see Fig. 1). In this paper, the dual graph data structure and the maximal independent edge set (MIES) decimation process are used to integrate the grid-based and the topological paradigms. The use of the dual graph allows to preserve the topology of the metric map and to correctly code the relation of adjacency and inclusion between topological regions.

The rest of this paper is organized as follows: Section 2 briefly reviews the metric and topological strategies for indoor environments mapping. Section 3 presents the proposed method. Experimental results revealing the efficacy of the method are described in Section 4. The paper concludes along with discussions and future work in Section 5.

2 Metric and Topological Paradigms for Mapping

Conventional approaches to SLAM rely on a metric, probabilistic representation of the robot pose and map. These metric approaches attempt to reconstruct the spatial distribution of the perceived environment, commonly in the form of a feature map or an occupancy grid. Although they have been successfully employed to map relatively large-sized environments, the main limitation of these techniques is related to the excessive computational complexity associated to these mapping processes. The classical alternative to metric maps is to model the environment using a topological map. Topological maps attempt to capture the spatial connectivity of the environment by representing it as a graph with arcs connecting the nodes that designate distinctive places in the environment [5]. These maps usually require reduced storage requirements, but such a representation usually lacks the necessary information to localize arbitrarily (can only localize to nodes in the topological graph) and to disambiguate similar topological regions. Besides, while probabilistic methods have been extensively investigated for performing inference over the space of metric maps, it is not the same case for topological maps. As a significant exception, the probabilistic topological maps (PTMs) [6] is a sample-based representation that approximates the posterior distribution over topologies given available sensor measurements. However, PTMs assume that the robot can detect whether it is near or on one of the nodes of the topological map. This can be considered a too restrictive assumption where the diversity of metric information is lost [7].

In order to deal with large, complex environments, the internal representation acquired by the robot can be organized as a hierarchy of maps which represent the whole environment at different levels of abstraction. Typically, these hierarchical representations consists of two layers: a metric map and a higher-level topological map. The hybrid approach usually attaches a local metric map to the nodes of a graph-based environment representation, where arcs represent coordinates transformation between nodes. Thus, the complexity can be bounded within each local map.

3 Hybrid Approach for Mapping Indoor Environments

3.1 Simple and Dual Graphs

Basically, a pyramid is a hierarchical structure which represents the contents of an input graph at multiple levels of abstraction. Each level of this hierarchy is at least defined by a set of vertices V_l connected by a set of edges E_l. These edges define the horizontal relationships of the pyramid and represent the neighbourhood of each vertex at the same level (intra-level edges). Another set of edges define the vertical relationships by connecting vertices between adjacent pyramid levels (inter-level edges). These inter-level edges establish a dependency relationship between each vertex of level l+1 and a set of vertices at level l (reduction window). The vertices belonging to one reduction window are the sons of the vertex which defines it. The value of each parent is computed from the set of values of its sons using a reduction function. Using this general framework, the procedure to build the level G_{l+1} from level G_l consists of three steps:

1. Selection of the vertices of G_{l+1} among V_l : This selection step is a decimation procedure and selected vertices V_{l+1} are called the surviving vertices.
2. Inter-level edges definition: Each vertex of G_l is linked to its parent vertex in G_{l+1}. This step defines a partition of V_l.
3. Intra-level edges definition: The set of edges E_{l+1} is obtained by defining the adjacency relationships between the vertices V_{l+1}.

The parent-son relationship defined by the reduction window may be extended by transitivity down to the base level. The set of sons of one vertex in the base level is named its receptive field.

A simple graph is a non-weighted and undirected graph containing no self-loops. In this hierarchy, the graph edges E_l represent adjacency relationships among pyramidal vertices of the level l. Simple graphs encode the adjacency between two vertices by only one edge, although their receptive fields may share several boundary segments. Therefore, a graph edge may thus encode a non-connected set of boundaries between the associated receptive fields. Moreover, the lack of self-loops in simple graphs does not allow to differentiate an adjacency relationship between two receptive fields from an inclusion relationship. These facts are shown in Fig. 1b, which represents the top of a simple graph pyramid encoding the connected components of Fig. 1a. In a dual graph pyramid, a level consists of a dual pair $(G_l, \tilde{G}_l)$ of planar graphs G_l and $\tilde{G}_l$. If level l defines a partition of the image into a connected subsets of pixels, then the vertices of G_l are the representatives of these subsets and the edges of G_l represent their neighborhood relationships. The edges of $\tilde{G}_l$ represent the boundaries of these connected subsets in level l and the vertices of $\tilde{G}_l$ define meeting points of boundary segments of $\tilde{G}_l$. Fig. 1c represents the top of a dual graph pyramid encoding the connected components of Fig. 1a. Fig. 1d shows the dual graph corresponding to 1c.

Within the dual graph pyramid framework, the set of edges that define the adjacency relationships among pyramidal vertices of the level $l+1$ is generated in two steps. First, the set of edges that connects each non-surviving vertex to its parent is contracted using a contraction kernel. A contraction kernel of a level l is the set of surviving vertices of l and the edges that connect each non-surviving vertex with its parent. The edge contraction operation collapses two adjacent vertices into one vertex, removing the edge between them. This operation may create redundant edges such as empty self-loops or double edges. The removal of these redundant edges constitutes the second step of the creation of the set of edges E_{l+1}. These redundant edges are characterized in the dual of the graph and removed by a set of edge removal kernels [10]. The key idea of the dual graphs is that a contraction in a graph implies a removal in its dual, and vice versa, in order to maintain the duality between the newly generated graphs. Thus, the generation of the edges in level $l + 1$ can be resumed as follows:

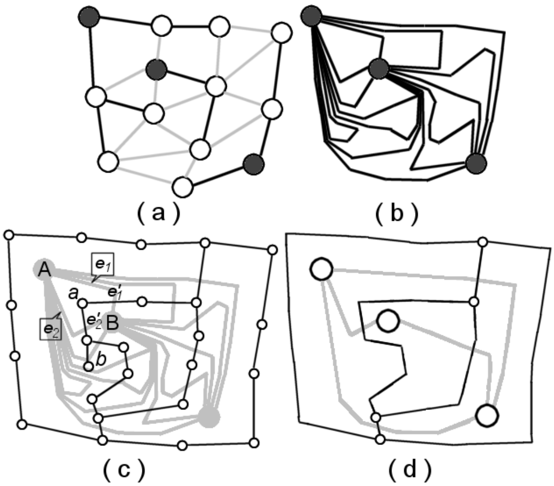

Fig. 2. Contraction and removal kernels: a) contraction kernel composed of three vertices (surviving vertices are marked in black); b) reduction performed by the equivalent contraction kernel in a); c) redundant edges characterization; and d) dual graph pair $(G, \tilde{G})$ after dual decimation step

1. Contraction of edges in G_l which connect non-surviving vertices with their parents. Removal of their corresponding edges in $\tilde{G}_l$. Fig. 2b shows the reduction performed by the contraction kernel in Fig. 2a.
2. Contraction of redundant edges in $\tilde{G}_l$ and removal of their corresponding edges in G_l. In Fig. 2c, the dual vertex a has a face defined by vertices **A** and **B**. The boundary between the regions defined by these vertices is artificially split by this dual vertex. Then, the two dual edges incident to this dual vertex (e'_1 and e'_2) can be contracted. The contraction of these dual edges has to be followed by the removal of one associated edge (e_1 or e_2) in order to maintain the duality between both graphs. In the same way, the dual vertex b encodes an adjacency relationship between two vertices contracted in the same vertex. This relationship can be removed by eliminating this direct self-loop and contracting the associated dual edge.

Using such a reduction scheme each edge in the reduced graph corresponds to one boundary between two regions. Moreover, inclusion relationships may be differentiated from adjacency ones in the dual graph.

3.2 Proposed Mapping Approach

The proposed algorithm is based on a dual graph pyramid that extracts the topological information from the metric map. In this work, the metric map is based on a two-dimensional occupancy grid, as originally proposed by Moravec and Elfes [8]. Thus, each grid cell (x, y) in the map yields the occupancy probability of the corresponding region of the environment. The algorithm works as follows:

1. Metric map thresholding. Each occupancy value in the metric map is thresholded. Cells whose occupancy value is below threshold U_1 are considered free-space. Cells whose occupancy value are above U_1 and below threshold U_2 are considered non-explored. All other cells are considered occupied. Free-space and non-explored cells are the nodes of the base level of the graph pyramid. Two base nodes are related by an arc if the two corresponding cells are neighbors. To achieve planar embedding of the graph in the metric map we use the 4-vicinity. Each node and arc of the base graph is attributed. Nodes are attributed with a discrete name $w_n(n)$ which can only take two different values: free-space or non-explored. Arcs are attributed with the Euclidean distance $w_a(a)$ between the two metric cells they link.
2. Dual graph pyramid generation. To generate the pyramid, the algorithm proposed by Haxhimusa and Kropatsch [9] is used. Two nodes n and m can be contracted if $w_n(n) == w_n(m)$ and $w_a((n, m))$ is below a threshold U_d.
3. Set arc attributes of G_{l+1}. The attributes of those arcs $a_{l+1} \in G_{l+1}$ are updated with the maximum attribute of the arcs $a_l \in G_l$ that are contracted into a_{l+1}.

Using the dual graph pyramid framework, each node is linked to a connected set of nodes in the base level (receptive field). Besides, each arc between two nodes encodes a unique connected boundary between the associated receptive fields. Finally, the use of self-loops within the hierarchy allows to differentiate adjacency relationships between receptive fields from inclusion relations. Threshold U_d sets the maximum distance between two metric map places integrated into the same topological node. Thus, it defines the resolution of the topological representation.

4 Experimental Results

In order to evaluate the suitableness of the topological representation, Thrun proposes the assessing of three criteria: consistency, losing and efficiency [3]. In our case, the proposed topological map is always consistent with the metric map. In fact, each topological node is linked to a connected set of nodes in the base level. Hence, there exists a direct correspondence between the topological graph and the metric map. However, topological maps lack details and, therefore, paths found in the topological map may not be as optimal as paths found using the metric representation. Finally, when using topological maps, efficiency is traded off with consistency and performance loss.

The algorithm was tested on a Nomad 200 robot as part of a complete system for mapping unknown office environments. In this system, the robot attempts to space the place nodes in the map at equal intervals of 4 meters. Therefore, threshold U_d has been set to this value. Fig. 3a shows the thresholded metric map acquired by the robot. The highest level of the dual graph based topological representation in Fig. 3b shows the position of the places in global coordinates. It can be noted that the structure correctly represents the inclusion of a non-explored node into a free-space one. Besides, there are two situations where two connected nodes are joined by two different arcs. The map generation process

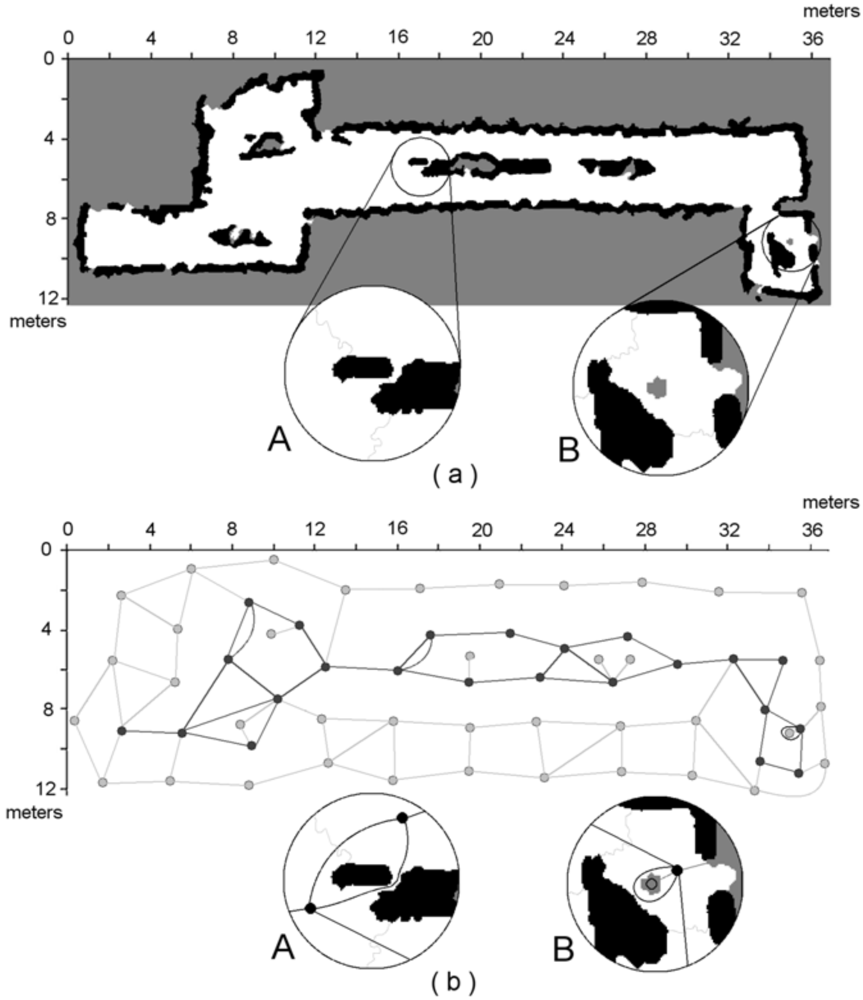

Fig. 3. a) Thresholded global grid-map of an office-like indoor environment (white-free space; black- obstacles and grey- non-explored); and b) highest level of the hierarchy of dual graph which encodes the topological representation (black nodes- free space and grey nodes- non-explored)

takes 200 msec. on a 844 MHz Pentium processor. The low computational time allows the on-line generation of the proposed map.

5 Conclusions and Future Work

This paper proposes an integrated method for indoor robot environment mapping. It combines the metric and topological paradigms. Topological maps are generated using a hierarchy of dual graphs that divides the metric map into homogeneous compact regions. The use of the dual graph allows to preserve the topology of the metric map and to correctly code the relation of adjacency and inclusion between topological regions.

Acknowledgments

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