A Discriminative Dynamic Index Based on Bipolar Aggregation Operators for Supporting Dynamic Multi-criteria Decision Making

Yeleny Zulueta, Juan Martínez-Moreno, Luis Martínez, and Macarena Espinilla

Abstract. While Multi-Criteria Decision Making (MCDM) models are focus on selecting the best alternative from a finite number of feasible solutions according to a set of criteria, in Dynamic Multi-Criteria Decision Making (DMCDM) the selection process also takes into account the temporal performance of such alternatives during different time periods. In this contribution is proposed a new discriminative dynamic index to handling differences in temporal behavior of alternatives, which are not discriminated in preceding dynamic approaches. An example is provided to illustrate the feasibility and effectiveness of the proposed index.

1 Introduction

A Multi-Criteria Decision Making (MCDM) problem consists of selecting the most desirable alternative from a given feasible set according to a set of criteria [12, 16]. As a matter of fact, MCDM problems could involve the current and past performance of alternatives, they are called Dynamic Multi-Criteria Decision Making (DMCDM) problems because the time dimension is considered [4, 8, 14].

DMCDM approaches are commonly focused on problems in which the final decision is performed based on all information collected at multiple time periods [8, 15, 18, 19, 24, 25]. However, they are not effective in handling situations including large sets of alternatives or criteria and changes of such sets over the time. Recently in [4] was introduced a framework for DMCDM that allows to overcome this weakness by means of a dynamic feedback mechanism. The crucial phase in the DMCDM framework is the selection of an appropriate associative aggregation

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operator for the computation of dynamic ratings due to its properties can highly modify the computing cost (e.g.: associativity) and obtain very different results attending to the type of reinforcement supported by the aggregation operator [13, 22]. However using any associative aggregation operator there are situations in which equal dynamic ratings are generated independently from the temporal performance of the alternatives. While the associativity property of the aggregation operator avoids the storing of all past alternatives (dynamic and non-dynamic) rating values, the lack of such information prevents a final decision based on temporal evolution of alternatives.

Therefore, this contribution proposes a novel *discriminative dynamic index* to extend the general approach in [4] such that the use of this index in the framework provides a temporal behavior differentiation of alternatives throughout time. The remaining of this paper is organized as follows. Sect. 2 reviews DMCDM approaches with special attention to the framework presented in [4]. In Sect. 3 it is introduced the new discriminative dynamic index to extend the initial approach. Sect. 4 shows an illustrative example and Sect. 5 concludes the paper.

2 Dynamic Multi-criteria Decision Making Approaches

In [3] are stated three common characteristics for a DMCDM problem: alternatives are not fixed, criteria are not fixed and the temporal profile of an alternative matters for comparison with other ones. To deal with decision making in dynamic environments, some authors have proposed different approaches [8, 15, 18, 19, 24, 25] that commonly model the problem as a three-dimensional decision matrix which is firstly transformed into conventional two-dimensional decision matrix by aggregating the time dimension and next is solving the problem through traditional MCDM models (or viceversa).

As in MCDM, an important issue in DMCDM is the selection of the aggregation operator (see [2] for a formal definition) because it directly impacts output values as well as the final ranking of alternatives. Some proposals have presented time dependent aggregation operators to deal with the information provided at different periods. Xu developed in [18] the concept of dynamic weighted averaging operator, and introduced some methods to obtain the associated weights, while in [19] the dynamic intuitionistic fuzzy weighted averaging operator and the uncertain dynamic intuitionistic fuzzy weighted averaging operator is defined.

Previous studies are focused on decision making problems in which the original decision information is usually collected at different time periods and a final decision is needed. Therefore, they are dynamic because the temporal profile of alternatives is considered for such final decision. However, there are other MCDM problems in which different, separated and interlinked decisions are taken either frequently, or just at the end of the process. In such context, it is remarkable the framework for DMCDM recently introduced in [4].

While most of the revised approaches provide solutions based on specific MCDM techniques oriented to problems dealing with specific types of information and where the final decision is performed using all that information collected at multiple periods; in [4] it is properly formalized the DMCDM, by extending the classic MCDM model, in a general framework operating without the need of storing all past information. Such framework is suitable for any dynamic problem, including consensus problems or situations requiring several steps before reaching a final decision. It is revised in further detail below.

2.1 The General Framework

Some basic notations from the original framework [4] are reviewed in the following.

Let $T = \{1, 2, \ldots\}$ be the (possibly infinite) set of discrete decision moments, and A_t the set of available alternatives at each decision moment $t \in T$.

At each time period $t \in T$, for each available alternative $a \in A_t$, a *non-dynamic rating* $R_t(a) \in [0,1]$ is computed. It is usually obtained by using an aggregation operator $Agg_1 : [0,1]^n \to [0,1]$, that combines the assessments of all criteria, $M_t = \{m_1, ..., m_n\}$ according to their weights $w_t \in [0,1]^n, \sum_{w \in w_t} w = 1, \forall t \in T$.

The information about the set of alternatives over time is carried out from one iteration to another in the historical set. Depending on the specific characteristics of each dynamic problem we may fix a *retention policy* that is the rule for selecting alternatives to be remembered in the H_t, which is defined as:

$$H_0 = \emptyset, \qquad H_t = \bigcup_{t' \leq t} A_{t'}, \quad t, t' \in T. \tag{1}$$

The dynamic nature of the decision process is supported by an evaluation function $E_t(a)$ it is defined for each $t \in T$ as:

$$E_t : A_t \cup H_{t-1} \to [0,1]$$

$$E_t(a) = \begin{cases} R_t(a), & a \in A_t \setminus H_{t-1} \\ Agg_2(E_{t-1}(a), R_t(a)), & a \in A_t \cap H_{t-1} \\ E_{t-1}(a), & a \in H_{t-1} \setminus A_t \end{cases} \tag{2}$$

Being $Agg_2 : [0,1]^n \to [0,1]$ an associative aggregation operator that can apply different types of reinforcements to the alternatives according to the attitudinal character of the decision making problem.

Aggregation operator for scoring alternatives in the non-dynamic part (Agg_1) is completely independent from one used in evaluation function of the dynamic part (Agg_2). It is worth noting that the dynamic rating computation requires the associativity property for the aggregation operator Agg_2, to ensure that repeated application of the aggregation function will generate, at every particular decision moment, the same result as application over the whole set of past non-dynamic ratings. Furthermore it is suggested that Agg_2 should fulfill the reinforcement property [13, 22] in order to strength high or low ratings in the dynamic context.

2.2 Drawbacks on Dynamic Evaluation Function Performance

The associativity property of Agg_2 avoids the storing of all past alternatives (dynamic and non-dynamic) rating values and it is simple to calculate the effect of adding new arguments to the aggregation. As stated in [21] this can be seen as a kind of Markovian property in which the new aggregated value just depends on the previous aggregated value and the new argument. However, this advantage brings out that the original framework outputs equal dynamic ratings for different alternatives without a discrimination about their temporal profile because associativity property does not allow to distinguish the order of such previous and new aggregated values.

Remark: This drawback arises from the associativity property of the aggregation operator therefore it appears using any associative aggregation operator.

Without loss of generality and for the sake of simplicity, this problem is illustrated in the following situation in which a decision maker wants to select the best option from alternatives $a1$ and $a2$ considering the retention policy of accumulating all alternatives in historical set. The dynamic ratings are calculated using the probabilistic sum operator (which exhibits an upward reinforcement) in order to corroborate the tendency of previous high non-dynamic ratings. Table 1 shows the results during five decision periods.

Table 1 Results obtained for alternatives with different temporal profile

Alternative	$R_1 = E_1$	R_2	E_2	R_3	E_3	R_4	E_4	R_5	E_5
a1	0.100	0.800	0.820	0.900	0.982	0.200	0.996	0.910	0.999
a2	0.900	0.800	0.980	0.100	0.982	0.200	0.996	0.910	0.999

At $t = 3$, $a1$ increases its rating while $a2$ decreases it, however both obtain the same dynamic rating $(E_3(a1) = E_3(a2))$. At $t = 4$, the rating of $a1$ decreases and the rating of $a2$ increases, but still both obtain the same dynamic rating $(E_4(a1) = E_4(a2))$. Eventually at $t = 5$ the rating of both alternatives performances the same increment and the dynamic rating is also the same $(E_5(a1) = E_5(a2))$. At independent decision periods $t = 3,4,5$, the decision maker cannot choose the best alternative just based on the dynamic rating because:

1. Alternatives obtain equal dynamic rating although they perform different rating evolution.
2. Alternatives obtain equal dynamic rating despite they perform opposed rating evolution.
3. Alternatives obtain equal dynamic rating though all of them perform an increasing evolution or decreasing rating evolution.

Different perspectives to solve the problem can be assumed. From a *static perspective*, the decision maker can select the alternative with highest rating at the current period but this contradictorily implies to *loss the dynamic perspective* of the DM-CDM problem.

To overcome this drawback, our aim in this contribution is to extend the original framework formalizing a new dynamic index that allows the decision maker to discriminate the best alternative according to the rating changes behavior throughout time.

3 A Discriminative Dynamic Index for DMCDM

To keep the *dynamic perspective* of the decision making problem when the situations pointed out in Sect. 2.2 arises, seems logic and suitable to find a solution in which *the temporal profile of an alternative matters for comparison with other alternatives*, as stated in Sect. 2.

To that end, we improve the resolution procedure for DMCDM, as can be seen in Figure 1, by performing a new aggregation process for computing a discriminative dynamic index that allows to distinguish alternatives and consequently obtain rankings for supporting dynamic decisions.

In this general resolution procedure the first step is essentially carried out through MCDM traditional methods. The second step lies on the DMCDM approach previously reviewed. The third step consists of computing the discriminative dynamic index and is performed just if equal dynamic ratings values are generated in the second step. These tree steps will finally enable to obtain a final ranking of alternatives.

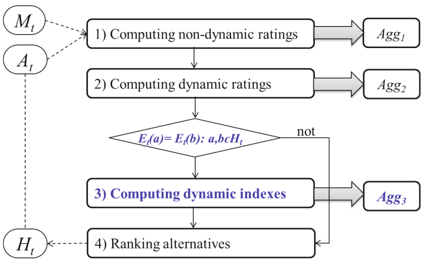

Fig. 1 Improved DMCDM resolution procedure

3.1 Computation of the Discriminative Dynamic Index

In this subsection we present in detail how to compute the discriminative dynamic index to perform steps 3) and 4) from the improved DMCDM resolution procedure.
Definition 1. The change in rating, $D_t(a)$, is the difference between the ratings at the current and previous period and is defined as:

$$D_t(a) = \begin{cases} 0, & t = 1 \\ R_t(a) - R_{t-1}(a), & t > 1. \end{cases} \tag{3}$$

Since $R_t(a), R_{t-1}(a) \in [0,1]$, the rating increment/decrement, $D_t(a)$, at each decision period is assessed in a *bipolar scale* $D_t(a) \in [-1,1]$ [5]. In which 0 is so-called the *neutral* element that represents no change in rating from period $t-1$ to t.

The rating change $D_t(a)$, just encloses the rating behavior from $t-1$ to t, hence it is necessary to formalize a dynamic mechanism that encloses all rating changes during all considered periods.

The benefits of computing final results without storing all previous values (through the associativity) and additionally modulating the importance of these values in such final results (through the reinforcements) are also used in the discriminative dynamic index proposal, $I_t(\cdot)$, due to its features:

- *Dynamic*: it must represent the rating change over time without storing all of them.
- *Customizable*: it should be able to model different behaviors regarding alternative rating decrements or increments over different periods.

Definition 2. Let $D_t(a)$ be the change in rating of an alternative a at a decision period t and $Agg_3 : [-1,1]^2 \to [-1,1]$ be a bipolar aggregation operator, the discriminative dynamic index, which represents the rating behavior of the alternative until t, is defined as:

$$I_t : A_t \cup H_{t-1} \to [-1,1]$$

$$I_t(a) = \begin{cases} D_t(a), & a \in A_t \setminus H_{t-1} \\ Agg_3(I_{t-1}(a), D_t(a)), & a \in A_t \cap H_{t-1} \\ I_{t-1}(a), & a \in H_{t-1} \setminus A_t. \end{cases} \tag{4}$$

The index $I_t(a)$ performance depends on the alternative, a as:

- if $a \in A_t \setminus H_{t-1}$ then its discriminative dynamic index $I_t(a)$ is the rating change $D_t(a)$,
- if $a \in A_t \cap H_{t-1}$, then its discriminative dynamic index is computed by Agg_3 that aggregates the discriminative dynamic index in the previous iteration with the current rating change, $Agg_3(I_{t-1}(a), D_t(a))$,
- if $a \in H_{t-1} \setminus A_t$, then its discriminative dynamic index is obtained from previous iteration, $I_t(a) = I_{t-1}(a)$.

Therefore, if different alternatives obtain equal dynamic rating, $E_t(.)$ at a period t, the final ranking will be generated considering the discriminative dynamic index values $I_t(.)$ that will reflect a *dynamic perspective*.

The choice of the aggregation operator Agg_3 will depend on the decision makers' attitude regarding the dynamic rating change but independent of the others aggregation operators as Agg_1 and Agg_2.

Table 2 summarizes the key features of aggregation operators used in the three aggregation processes illustrated in Figure 1 which are applied in the resolution procedure of the DMCDM improved approach. It is noteworthy to point out that

Table 2 Characterization of the aggregation operators to be used in DMCDM

Feature	Agg_1	Agg_2	Agg_3
Definition	$[0,1]^n \to [0,1]$	$[0,1]^2 \to [0,1]$	$[-1,1]^2 \to [-1,1]$
Required Property		Associativity	Associativity, Bipolarity
Desired Property		Reinforcement	Reinforcement

there is a key difference between the characterization of Agg_2 and Agg_3: Agg_3 must deal with values in a *bipolar scale* $[-1,1]$ meanwhile Agg_2 operates in $[0,1]$.

Consequently it is necessary to extend the latter to the bipolar scale [6] in $[-1,1]$ in which a remarkable point, *e*, of the interval plays an specific role as a neutral or an absorbant element. This fact leads to a *bipolar aggregation* in which the key feature is the different effects of arguments above and below *e* on the aggregated value [23].

Uninorms [17] satisfies this characterization but in $[0,1]$. A uninorm U is a commutative, associative and increasing binary operator with a neutral element $e \in [0,1]$.

In [9, 10, 11] the authors development the topic of "pseudo-operations". Pseudo-addition and pseudo-multiplication are examples of them. In [6] was proposed a rescaling to consider $[-1,1]$, such that, given a continuous $S : [0,1]^2 \to [0,1]$ t-conorm , the *symmetric pseudo-addition* $\oplus$ is a binary operation on $[-1,1]$ defined by:

R1 For $x,y \geq 0$: $x \oplus y = S(x,y)$.
R2 For $x,y \leq 0$: $x \oplus y = -S(-x,-y)$.
R3 For $x \in [0,1[, y \in]-1,0]$: $x \oplus y = x \ominus_S (-y)$. Moreover, $1 \oplus (-1) = 1$ or -1.
R4 For $x \leq 0, y \geq 0$: just reverse x and y.

The structure of the binary operation $\oplus$ is closely related to uninorms. From the point of view of bipolar scales, the interval $[-1,1]$ is viewed as the union of two unipolar scales.

Proposition 1. $T = \oplus_{/[-1,0]^2}$ is a t-norm on $[-1,0]$ (i.e., in particular $T(x,0) = x$, for every $x \in [-1,0]$), $S = \oplus_{/[0,1]^2}$ is a t-conorm on $[0,1]$ and H is an average function $H = \oplus_{/[-1,0]\times[0,1]\cup[0,1]\times[-1,0]}$. They have the following properties:

- If $x,y \in [0,1]$, then $x \oplus y = S(x,y) \geq \max\{x,y\}$.
- If $x,y \in [-1,0]$, then $x \oplus y = T(x,y) \leq \min\{x,y\}$.
- If $-1 \leq y \leq 0 \leq x \leq 1$, then $y \leq x \oplus y = H(x,y) \leq x$.

The previous proposition provides a performance that can be interpreted as *attitudes to deal with the ratings changes*:

- Optimistic: when both values are positive the aggregation acts as an upward reinforcement.
- Pessimistic: when both values are negative, it acts as a downward reinforcement.
- Averaging: when one value is negative and the another positive, it acts as an averaging operator.

The aggregation function $\oplus$ exhibits conjunctive behavior on $[-1,0]$ and disjunctive behavior on $[0,1]$. On the rest of the domain the behavior is averaging.

Let S be a strict t-conorm S with additive generator $s : [0,1] \to [0,\infty]$ and $g : [-1,1] \to [0,\infty]$ the symmetric extension of s, i.e.,

$$g(x) = \begin{cases} s(x), & x \geq 0 \\ -s(-x), & x < 0 \end{cases} \tag{5}$$

It is possible to rescal $\oplus$ to a binary operator U on $[0,1]$ such that U is a generated uninorm operator. Then, $x \oplus y = g^{-1}(g(x) + g(y))$ for any $x,y \in [-1,1]$. We also introduce another function $u : [0,1] \to [-\infty,\infty]$ defined by $u(x) = g(2x-1)$ that is strictly increasing and satisfies $u(\frac{1}{2}) = 0$. Then $U(z,t) = u^{-1}(u(z) + u(t))$ for any $z,t \in [0,1]$. U is an uninorm that is continuous (except in $(0,1)$ and $(1,0)$), is strictly increasing on $]0,1[^2$ and has neutral element $\frac{1}{2}$. Moreover, the induced t-norm T_U is the dual of S.

Such an operator should be used if the decision maker's attitude is influenced by the number of increment or decrement ratings received. Particularly, when all the attributes' ratings are positive, the more these there are, the more positive the agent becomes in its aggregation. That is similar for negative values and when conflict occurs, the ratings are aggregated in a risk-neutral way.

4 Illustrative Example

A high-technology manufacturing company desires to select at five different periods, suitable material supplier to purchase the key components of products. There are six candidates for initial evaluation but at successive periods, there will be additional suppliers while others will be unavailable due to market conditions. The company is interested about supplier's evolution and considers the following elements:

- *Criteria*: quality (m_1), delivery performance (m_2), price (m_3) and technological capability (m_4).
- *Retention policy* keeps all alternatives from A_t to H_t.
- *Non-dynamic rating* is computed with the weighted sum operator, using the weighting vector $w_t = (0.15, 0.20, 0.25, 0.40), \forall t \in T$.
- *Dynamic rating* is computed with the probabilistic sum operator .
- *Discriminative dynamic index* is computed with the Van Melle's combining function $C : [-1,1]^2 \to [-1,1]$ modified in [17] as:

$$C(x,y) = \begin{cases} S(x,y) = x + y - xy, & \text{if} \min\{x,y\} \geq 0 \\ T(x,y) = x + y + xy, & \text{if} \max\{x,y\} \leq 0 \\ H(x,y) = \frac{x+y}{1-\min\{|x|,|y|\}} & \text{otherwise} \end{cases} \tag{6}$$

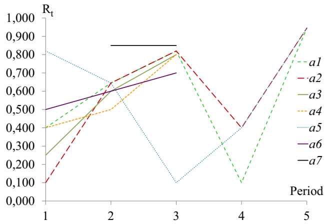

Fig. 2 Rating behavior of suppliers

4.1 *The Resolution Procedure and Index Performance*

In order to clarify the suppliers behavior Figure 2 depicts their ratings over the five periods. The difference between time events is not a variable neither for the index nor for the original approach.

As illustrated in Figure 1, at each period the non-dynamic and dynamic ratings are computed. These results are shown in Table 3, columns "R_t" and "E_t" respectively. To better understand the resolution procedure, following we focus on the discriminative dynamic index computations and performance. Index values are shown in Table 3, column "I_t".

Period t=1: Here it is not necessary to compute the dynamic ratings.

Period t=2: Dynamic ratings for a_3 and a_4 are equal therefore we have to compute their discriminative dynamic index, for instance:
$I_2(a_3) = C(D_2(a_3), I_1(a_3))$
$D_2(a_3) = R_2(a_3) - R_1(a_3) = 0.3500$ and $I_1(a_3) = D_1(a_3) = 0$ then
$I_2(a_3) = 0.35000$.
Note that for both alternatives the index evidences an *optimistic attitude*.

Period t=3: The discriminative dynamic index is computed for: a_2 and a_5 that present opposed rating evolution; and for a_3, a_4 and a_6 that present increasing rating evolution. The index attitude for a_2 is *optimistic* while for a_5 is *pessimistic*.

Period t=4: All suppliers obtain same dynamic ratings. The discriminative dynamic index shows an *averaging attitude* because a_1 and a_2 present increasing ratings in temporal profile but decreasing one at the current period while a_5 presents the inverse situation. This *averaging attitude* compensates both values but does not allow ignore rating decrements at current or previous periods.

Period t=5: All the suppliers not only obtain equal dynamic rating but also a_2 and a_5 present equal improvement from $t = 4$ to $t = 5$: $D_5(a_2) = D_5(a_5)$. Despite $D_5(a_1) > D_5(a_2)$, their indexes are $I_5(a_1) < I_5(a_2)$ because $I_4(a_1)$ was negative. Furthermore, as $D_5(a_2) = D_5(a_5)$ and $I_4(a_5) < 0$, $I_4(a_2) > 0$ then $I_5(a_2)$ is better than $I_5(a_5)$. Therefore the dynamic index provides for a_1 and a_5 an *average attitude* while for a_2 presents an *optimistic attitude*.

Table 3 Results at each period

Period	A_t	m_1	m_2	m_3	m_4	R_t	E_t	I_t
$t=1$	a_1	0.700	0.900	0.100	0.300	0.4000	0.40000	-
	a_2	0.100	0.200	0.100	0.050	0.1000	0.10000	-
	a_3	0.200	0.300	0.500	0.050	0.2500	0.25000	-
	a_4	0.100	0.200	0.500	0.500	0.4000	0.40000	-
	a_5	0.900	0.950	0.900	0.900	0.8200	0.82000	-
	a_6	0.450	0.150	0.850	0.150	0.5000	0.50000	-
$t=2$	a_1	0.300	0.600	0.750	0.600	0.6450	0.78700	-
	a_2	0.150	0.200	0.900	0.800	0.6450	0.68050	0.350000
	a_3	0.900	0.550	0.400	0.700	0.6000	0.68050	0.100000
	a_4	0.450	0.150	0.850	0.150	0.5000	0.70000	-
	a_5	0.900	0.800	0.410	0.400	0.6450	0.93610	-
	a_6	0.800	0.700	0.200	0.800	0.6000	0.80000	-
	a_7	0.800	0.700	0.900	0.900	0.8500	0.85000	-
$t=3$	a_1	0.700	0.650	0.800	0.950	0.8200	0.96166	-
	a_2	0.800	0.700	0.800	0.900	0.8200	0.94249	0.624625
	a_3	0.500	0.800	0.900	0.800	0.8000	0.94000	0.480000
	a_4	0.900	0.900	0.900	0.650	0.8000	0.94000	0.370000
	a_5	0.100	0.200	0.100	0.050	0.1000	0.94249	-0.624625
	a_6	1.000	0.500	1.000	0.500	0.7000	0.94000	0.190000
	a_7	0.800	0.700	0.900	0.900	0.8500	0.97750	-
$t=4$	a_1	0.200	0.100	0.050	0.100	0.1000	0.96500	-0.550472
	a_2	0.100	0.200	0.500	0.500	0.4000	0.96500	0.352802
	a_5	0.800	0.550	0.300	0.300	0.4000	0.96500	-0.463750
$t=5$	a_1	0.950	0.950	1.000	0.900	0.9450	0.99800	0.655194
	a_2	0.950	0.850	1.000	0.950	0.9450	0.99800	0.705525
	a_5	0.850	0.950	0.900	1.000	0.9450	0.99800	0.151515

4.2 Ranking Alternatives and Results Analysis

In Table 4 is depicted the summary of rankings obtained with the original DMCDM framework as well as with the improved one using the new index.

Table 4 Suppliers rankings

Period	Original framework	Discriminative dynamic index
$t=1$	$a_5 \succ a_6 \succ a_1 = a_4 \succ a_3 \succ a_2$	-
$t=2$	$a_5 \succ a_7 \succ a_6 \succ a_1 \succ a_3 = a_4 \succ a_2$	$a_5 \succ a_7 \succ a_6 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
$t=3$	$a_7 \succ a_1 \succ a_2 = a_5 \succ a_3 = a_4 = a_6$	$a_7 \succ a_1 \succ a_2 \succ a_5 \succ a_3 \succ a_4 \succ a_6$
$t=4$	$a_1 = a_2 = a_5$	$a_2 \succ a_5 \succ a_1$
$t=5$	$a_1 = a_2 = a_5$	$a_2 \succ a_1 \succ a_5$

In Sect. 2.2 were summarized situations in which the original framework can not discriminate alternatives consequently the *main objective of the DMCDM was not accomplished* since the *most desirable alternatives can not be selected considering their current and past performance*.

However it is remarkable that in all circumstances the discriminative dynamic index ranks the alternatives taking into account the desired *pessimistic, averaging* or *optimistic* attitude. Consequently our proposal support the DMCDM by improving the original framework in such way that the crucial purpose of DMCDM is achieved.

5 Conclusion

In this contribution, we focused on the DMCDM problems. To support consistent decisions in cases in which the framework in [4] is not effective, we introduced a novel discriminative dynamic index in a general resolution procedure for DMCDM. It uses an aggregation process based on associative bipolar operators. This features allows to exploit their associativity property to represent the rating behavior of alternatives over different periods as well as to model effects of rating changes above and below neutral element on the final aggregated value.

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