Social Network Decision Making with Linguistic Trustworthiness–Based Induced OWA Operators

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Classic aggregation operators in group decision making such as the ordered weighted averaging (OWA), induced ordered weighted averaging (IOWA), C-IOWA, P-IOWA, and I-IOWA have shown to be successful tools to provide flexibility in the aggregation of preferences. However, these operators do not take advantage of information related to the interaction between experts. Experts involved in a group decision-making problem may have developed opinions about the reliability of other experts' judgments, either because they have previous history of interaction with each other or because they have knowledge that informs them on the reliability of other colleagues in the group in solving decision-making problems in the past. In this paper, and within the framework of social network decision making, we present three new social network analysis based IOWA operators that take advantage of the linguistic trustworthiness information gathered from the experts' social network to aggregate the social group preferences. Their use is analysed with simple but illustrative examples.

1. INTRODUCTION

In this paper, we deal with group decision-making (GDM) problems, which are usually solved using the following two steps procedure:[1,2]

(i) an aggregation step to collectively fuse the experts' opinions and
(ii) an exploitation step to obtain a final ranking of the available alternatives from which a group solution is derived.

Most GDM mathematical models ignore in their architecture the implementation of information related to their past/present interaction/relationship. In other words, GDM models tend to assume that experts are completely independent,

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unknown to each other, and therefore neglect the existence of any links whatsoever between them. However, this is far from reality as people engaged in solving GDM problems are probably known to each other or have some kind of prior knowledge that informs them on the reliability of other experts in the group in solving decision-making problems in the past.[3] Consequently, it would be more realistic and practical to apply in these cases decision models capable of implementing information reflecting such type of relationship. A promising and presently quite relevant mathematical methodology able to capture decision makers' relationships is based on the use of graph models and social network analysis (SNA). This is the focus of the present paper.

A GDM problem can generally classed as homogeneous or heterogeneous. In the first case, all decision makers have associated equal importance weights/degrees whereas unequal importance weights/degrees apply in the second case. In any case, the degree of importance/relevance of experts within a group is usually assumed to be provided beforehand or easily derived from some kind of reliable source, which are subsequently used to collectively fuse the expert's preferences on the problem to solve. In classical preference modeling, the set of numerical values $\{1, 0.5, 0\}$, or its equivalent $\{1, 0, -1\}$,[4] is used to represent when an alternative (first) is preferred to another alternative (second), when both alternatives are considered equally preferred (indifference), and when the second alternative is preferred to the first one, respectively. This classical preference modeling constitutes the simplest numeric discrimination model of preferences, and it proves insufficient in many decision-making situations as Fishburn pointed out in Ref. 4. Thus, in many cases it might be necessary in the implementation of some kind of "intensity of preference" between alternatives.

The concept of fuzzy set, which extends the classical concept of set, when applied to a classical relation leads to the concept of a fuzzy relation, which in turn allows the implementation of intensity of preferences.[5] The numeric scale used to evaluate intensity of preferences within the fuzzy framework is the whole unit interval $[0, 1]$ instead of $\{1, 0.5, 0\}$. Notice that this is argued, though, to assume unlimited computational abilities and resources from the individuals.[6] An alternative approach to preference modeling was proposed by Zadeh in Ref. 7. He argued that subjectivity, imprecision, and vagueness in the articulation of opinions pervade real-world decision applications, and individuals usually find difficult to evaluate their preference using exact numbers. Indeed, he continued by claiming that individuals might feel more comfortable using words by means of linguistic labels or terms to articulate their preferences.[7] Furthermore, humans exhibit a remarkable capability to manipulate perceptions and other characteristics of physical and mental objects, without any exact numerical measurements and complex computations.[8–10] Therefore, in this paper, the individuals' preferences between pair of alternatives will be assumed to be given in the form of linguistic labels.[11] In particular, both experts' preference opinions and experts' assessments about their partners will be modeled using the 2-tuple linguistic framework[12,13] and, therefore, a first objective here is for SNA concepts that are defined for the case of crisp numerical information to be reinterpreted and defined appropriately within this linguistic computational model.

As aforementioned, the degree of importance/relevance of experts within a group is usually assumed to be provided beforehand and are subsequently used to collectively fuse the experts' preferences on the problem to solve. Relevant fusion operators applicable to the linguistic framework that has been presented in the relevant literature include the minimum operator,[14] the exponential function,[15] the t-norm operator,[16] the ordered weighted averaging (OWA) operator,[17] the induced OWA (IOWA),[18] the I-IOWA, P-IOWA, and C-IOWA,[19] and the type-1 OWA operator.[20–22]

The importance of the expert is assumed here as not been provided beforehand. Consequently, a second objective of this paper is to derive experts' importance degrees from the experts' social structure. When a group of experts are gathered, they discuss the alternatives and listen to other experts' opinions. This exchange of information usually entails that experts have enough knowledge to appraise the trustworthiness and expertise of their partners. These judgments can be represented by means of a social network structure from which the importance of each expert can be derived by using SNA.[23–25] As it will be presented later in the paper, three new IOWA operators are possible to define the node in-degree centrality IOWA operator ($C_{iD}^{\prime l} - IOWA$), the node proximity degree IOWA operator ($P_P - IOWA$), and the node rank prestige IOWA operator ($P_R - IOWA$). These operators will make possible to aggregate the information by implementing the experts' social interactions and judgments and, therefore, can be considered as more flexible and realistic since the more reliable the experts judgments are, the more support by partners they will receive.

This paper is set out as follows: In Section 2, we summarize the background needed to understand the new operators. In Section 3, we present the framework of group decision-making process that takes advantage of SNA, which it is illustrated with their application to an example of social network decision making. Finally, our conclusions and future works will be pointed out in Section 4.

2. BACKGROUND

In this section, we will review the necessary preliminaries to understand the operators presented in Section 3. First of all, we will provide a summary of the linguistic computational methods. Second we will look over the SNA, and finally we will review the use of the IOWA operators in group decision making as our proposal is based on these operators.

2.1. 2-Tuple Linguistic Computational Model

Although the most usual representation of information in computer science is by means of numbers, many aspects of different activities in the real world are assessed in a qualitative form, with vague or imprecise knowledge, rather than in a quantitative one. In that case, a better approach may be to use linguistic variables instead of numerical ones.

Let $\mathcal{S} = \{s_0, \ldots, s_g\}$ be a set of linguistic labels ($g \geq 2$), with semantic underlying a ranking relation that can be precisely captured with a linear order, i.e.,

Table I. Seven linguistic labels and their semantic meanings.

Linguistic label	Semantic meaning
s_0	x_j is absolutely preferred to x_i
s_1	x_j is highly preferred to x_i
s_2	x_j is slightly preferred to x_i
s_3	x_i and x_j are equally preferred
s_4	x_i is slightly preferred to x_j
s_5	x_i is highly preferred to x_j
s_6	x_i is absolutely preferred to x_j

$s_0 < s_1 < \cdots < s_g$. Table I provides an example with seven linguistic labels and their corresponding semantic meanings for the comparison of the ordered pair of alternatives (x_i, x_j).

The number of labels is usually assumed odd with the central label $s_{g/2}$ standing for the state of indifference when comparing two alternatives, and the remaining labels located symmetrically around that central assessment to guarantee that asymmetric property is verified and preferences are represented by weak ordering to avoid "inconsistent" situations where an expert could prefer two alternatives at the same time.[26] Thus, if the linguistic assessment associated with the pair of alternatives (x_i, x_j) is $s_{ij} = s_h \in \mathcal{S}$, then the linguistic assessment corresponding to the pair of alternatives (x_j, x_i) would be $s_{ji} = s_{g-h}$. Therefore, the operator defined as $N(s_h) = s_k$ with $(k + h) = g$ is a negator operator because $N(N(s_h)) = N(s_k) = s_h$.

The main two representation formats of linguistic information are[11] the cardinal, which is based on the use of fuzzy set characterized with membership functions and that are mathematically processed using Zadeh's *extension principle*,[7] and the ordinal, which is based on the use of *2-tuples symbolic methodology*.[12] The second one will be described with more detail as it will be used herein.

The 2-tuple linguistic model takes as a basis the symbolic representation model based on indexes and in addition defines the concept of symbolic translation to represent the linguistic information by means of a pair of values called linguistic 2-tuple, (s_b, λ_b), where $s_b \in \mathcal{S}$ is one of the original linguistic terms and λ_b is a numeric value representing the symbolic translation. This representation structure allows, on the one hand, to obtain the same information as with the symbolic representation model based on indexes without losing information in the aggregation phase. On the other hand, the result of the aggregation is expressed on the same domain as the one of the initial linguistic labels and, therefore, the well-known retranslation problem of the above methods is avoided.

DEFINITION 1 (Linguistic 2-Tuple Representation). *Let $a \in [0, g]$ be the result of a symbolic aggregation of the indexes of a set of labels assessed in a linguistic term set $\mathcal{S} = \{s_0, \ldots, s_g\}$. Let $b = round(a) \in \{0, \ldots, g\}$. The value $\lambda_b = a - b \in [-0.5, 0.5)$ is called a symbolic translation, and the pair of values (s_b, λ_b) is called the* 2-tuple linguistic representation *of the symbolic aggregation a.*

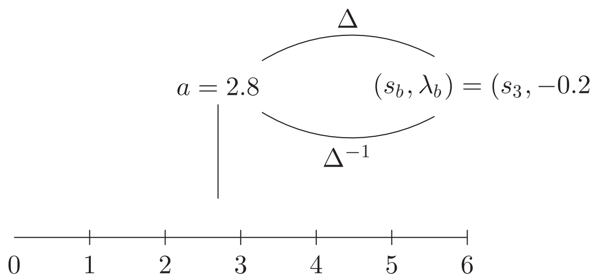


Figure 1. Ordinal linguistic representation: symbolic translation and 2-tuples.

The 2-tuple linguistic representation of symbolic aggregation can be mathematically formalized with the following mapping:

$$\begin{aligned} \Delta : [0, g] &\longrightarrow \mathcal{S} \times [-0.5, 0.5) \\ \Delta(a) &= (s_b, \lambda_b). \end{aligned} \tag{1}$$

Based on the linear order of the linguistic term set and the complete ordering of the set $[-0.5, 0.5)$, it is easy to prove that Δ is strictly increasing and continuous and, therefore, its inverse function exists:

$$\begin{aligned} \Delta^{-1} : \mathcal{S} \times [-0.5, 0.5) &\longrightarrow [0, g] \\ \Delta^{-1}(s_b, \lambda_b) = b + \lambda_b &= a. \end{aligned} \tag{2}$$

The following negation operator is defined:

$$N(\Delta(a)) = \Delta(g - a). \tag{3}$$

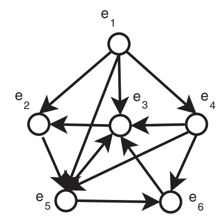
Figure 1 illustrates the application of the 2-tuple function Δ and its inverse Δ^{-1} for a linguistic term set of cardinality seven. The value of the symbolic translation is assumed to be 2.8, which means that $round(2.8) = 3$ and therefore it can be represented with the 2-tuple $(s_3, -0.2)$.

2.2. Social Network Analysis

SNA[23–25] studies the relationships between social entities such as members of a group, corporations, or nations and give us a background that allows us, among other things, to examine the structural and locational properties including centrality, prestige, and structural balance. SNA has been successfully applied to a wide range of areas including social sciences,[27] epidemiology,[28] economics,[29] and marketing.[30]

In SNA, the words "social networks" refers to the set of actors and the ties among them. In these networks, each individual has ties to other individuals, each of whom in turn is tied to others. The aim of SNA is to model these relationships to depict the structure of a group to, for instance, study the impact of this structure on

Table II. Different notations in SNA.

Sociometric	Graph theoretic	Algebraic
$A = \begin{pmatrix} 0&1&1&1&1&0 \\ 0&0&0&0&1&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&1&1 \\ 0&0&1&0&0&1 \\ 0&0&1&0&0&0 \end{pmatrix}$		e_1Re_2 e_4Re_3 e_1Re_3 e_4Re_5 e_1Re_4 e_4Re_6 e_1Re_5 e_5Re_3 e_2Re_5 e_5Re_6 e_3Re_2 e_6Re_3

the behavior of the set of actors and/or the influence of this structure on individuals within this set.[23–25] There are three notational schemes to represent the set of actors and the relationships themselves (see Table II):

- *Sociometric*: In which relational data are often presented in two-ways matrices called sociomatrix or adjacency matrix.
- *Graph theoretic*: In which the network is viewed as a graph, consisting of nodes joined by lines.
- *Algebraic*: This notation presents the advantage that allow us to distinguish several distinct relationships and represent combinations of relationships.

By far, the primary notational scheme used in SNA is the sociometric. In this scheme, data are represented by the adjacency matrix *o* sociomatrix. The entries in this matrix indicate whether two nodes are related or not. For instance, a nondirectional relationship on a single set of actors or nodes $E = \{e_1, \ldots, e_n\}$ is a relationship $A \subseteq E \times E$ with a characteristic function, $\mu_A : E \times E \longrightarrow \{0, 1\}$, which is defined as follows:

$$\mu_A\left(e_i, e_j\right) = \begin{cases} 1 & \text{if } e_i \text{ is related to } e_j. \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Another useful view is the graph scheme, consisting of nodes joined by lines. Dichotomous graphs only take into account whether the node e_i is related to e_j or does not but not the strength of the relationship or how frequently e_i interacts with e_j. For directional graphs, the line that goes from actor e_i to actor e_j is considered different from the line that goes from e_j to e_i. In this case, instead of lines, arcs or directed lines are usually used.

Among other things, SNA analysis let us to study the importance of the nodes of a social network via the well-known centrality index.[25,31–34] The most important centrality indexes are the node centrality for an undirected dichotomous graph and the in-degree and out-degree centrality for a directed dichotomous graph.

Definition 2 (Centrality Index). *Let $G = (E, L)$ be an undirected dichotomous graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, and $L = \left\{l_1, \ldots, l_q\right\}$ be the set of lines between pairs of nodes. The number of lines that are incident with a node, $d\,(e_i)$, is* *known as the node centrality index, $C_D(e_i)$, i.e.,*

$$C_D(e_i) = d(e_i).$$

This measure depends on the cardinality of the set E. Thus, the following standardized node centrality index is normally used:

$$C'_D(e_i) = \frac{d(e_i)}{n-1}. \tag{5}$$

DEFINITION 3 (In-Degree and Out-Degree Centrality Indexes). *Let $G = (E, L)$ be a directed dichotomous graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, and $L = \{l_1, \ldots, l_q\}$ be the set of directed lines, or arcs, between pairs of nodes.*

- *The number of arcs originating at a node, $d^+(e_i)$, is known as the node out-degree centrality index, $C_{oD}(e_i)$, i.e.,*

$$C_{oD}(e_i) = d^+(e_i).$$

- *The number of arcs terminating at a node, $d^+(e_i)$, is known as the node in-degree centrality index, $C_{iD}(e_i)$, i.e.,*

$$C_{iD}(e_i) = d^-(e_i).$$

As with the centrality index, both out-degree centrality and in-degree centrality indexes depend on the cardinality of the set E, and therefore the corresponding standardized measures are

$$C'_{oD}(e_i) = \frac{d^+(e_i)}{n-1}, \tag{6}$$

$$C'_{iD}(e_i) = \frac{d^-(e_i)}{n-1}. \tag{7}$$

Notice that the adjacency matrix as defined above is a binary or crisp relationship. However, in many situations, it may not be suitable to represent the relationship in a crisp way because this relationship is not clear-cut defined or because it has associated a weighting value representing the strength of the relationship modeled. To cope with these situations, the previous definition of an adjacency matrix has been extended with the concepts of weighted adjacency matrix[23–25] and the concept of fuzzy adjacency relationships,[35–37] respectively.

DEFINITION 4. *A fuzzy adjacency relationship R on E is a relationship in $E \times E$ with a membership function $\mu_R : E \times E \longrightarrow [0, 1]$, $\mu_R(e_i, e_j) = r_{ij}$, interpreted as follows:*

- *$r_{ij} = 1$ indicates that e_i is definitely related to e_j.*
- *$r_{ij} \in\ ]0, 1[$ indicates that e_i is to a certain extent related to e_j.*
- *$r_{ij} = 0$ indicates that e_i is not related to e_j.*

2.3. IOWA Operators

It was mentioned in the Introduction that the first step of a GDM resolution process is that of aggregating the information from which to derive a group solution to the problem. In the case of fuzzy preferences, Yager's OWA operator[17] has been proved to be extremely useful because it allows to implement the concept of *fuzzy majority*.[38]

DEFINITION 5 (OWA Operator). *An OWA operator of dimension n is a function* $\phi : \mathbb{R}^n \longrightarrow \mathbb{R}$ *that has associated a set of weights or weighting vector* $W = (w_1, \ldots, w_n)$ *to it, so that* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$*, and is defined to aggregate a list of values* $\{p_1, \ldots, p_n\}$ *according to the following expression:*

$$\phi(p_1, \ldots, p_n) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)},$$

where $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ *is a permutation such that* $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$*, i.e.,* $p_{\sigma(i)}$ *is the ith highest value in the set* $\{p_1, \ldots, p_n\}$.

An issue in the definition of the *OWA* operator is how to obtain the associated weighting vector. In Ref. 17, Yager proposed two ways to obtain it. The first approach is to use some kind of learning mechanism using some sample data, and the second approach is to try to give some semantics or meaning to the weights. The latter allowed applications in the area of quantifier-guided aggregations.[38]

Given a function $Q : [0, 1] \rightarrow [0, 1]$ such that $Q(0) = 0$, $Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$, an OWA aggregation guided by this function can be obtained as

$$\phi_Q(p_1, \ldots, p_n) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)},$$

where $\sigma : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ is a permutation such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $p_{\sigma(i)}$ is the ith largest value in the set $\{p_1, \ldots, p_n\}$ and

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n. \tag{8}$$

Yager[38] considered the parameterized family of regular increasing monotone quantifiers $Q(r) = r^a$ $(a \geq 0)$ for such representation. This family of functions guarantees that

(i) all the experts contribute to the final aggregated value (strict monotonicity property) and
(ii) associates, when $a \in [0, 1]$, higher weight values to the aggregated values with associated higher importance values (concavity property).[19]

In particular, the value $a = 1/2$ is used to represent the fuzzy linguistic quantifier *"most of."*

Mitchell and Estrakh[39] described a modified OWA operator in which the input arguments are not rearranged according to their values but rather using a function of the arguments. Inspired by this work, Yager and Filev[18] introduced a more general type of OWA operator, which they named the IOWA operator:

DEFINITION 6 (IOWA Operator). *An IOWA operator of dimension n is a function* $\Phi_W : (\mathbb{R} \times \mathbb{R})^n \longrightarrow \mathbb{R}$*, to which a set of weights or weighting vector is associated,* $W = (w_1, \ldots, w_n)$*, such that* $w_i \in [0, 1]$ *and* $\Sigma_i w_i = 1$*, and it is defined to aggregate the set of second arguments of a list of n 2-tuples* $\{\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\}$ *according to the following expression:*

$$\Phi_W\left(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\right) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)},$$

where $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ *is a permutation such that* $u_{\sigma(i)} \geq u_{\sigma(i+1)},\ \forall i = 1, \ldots, n-1$*, i.e.,* $\langle u_{\sigma(i)}, p_{\sigma(i)}\rangle$ *is the 2-tuple with* $u_{\sigma(i)}$ *the ith highest value in the set* $\{u_1, \ldots, u_n\}$*.*

In the above definition, the reordering of the set of values to aggregate, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated with them, which is based on their magnitude. Owing to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order-inducing variable and $\{p_1, \ldots, p_n\}$ as the values of the argument variable.[18] Obviously, an immediate consequence of Definition 6 is that if the order-inducing variable is the argument variable then the IOWA operator is reduced to the OWA operator.

Three types of IOWA operators were further presented in Ref. 19. The importance IOWA operator (I-IOWA), which applies the ordering of the argument values based on the importance of the information sources, the consistency IOWA operator (C-IOWA), which applies the ordering of the argument values based on the consistency of the information sources, and the preference IOWA operator (P-IOWA), which applies the ordering of the argument based on the relative preference values associated with each one of them.

2.4. Quantifier Nondominance Degree

Once the information has been aggregated, the exploitation step must be accomplished to obtain a global ranking of them. To do so, for instance, the following two choice degrees based on the concept of fuzzy majority are used:[1] the quantifier-guided dominance degree (QGDD) and the quantifier-guided nondominance degree (QGDNDD). These degrees are based on the use of the OWA operator, and the weights used are calculated by means of the quantifier that represents the fuzzy majority.

Definition 7 (Quantifier Guided Dominance Degree). *QGDD quantifies the dominance that the alternative x_i has over all the other alternatives in a fuzzy majority sense as*

$$QGDD(x_i) = \Phi_Q\left(p_{ij},\, j = 1, \ldots, m,\, j \neq i\right), \tag{9}$$

where Φ_Q is an OWA operator whose weights are defined using a relative quantifier Q, and whose components are the elements of the corresponding row of the matrix P.

The elements of the set

$$X^{QGDD} = \{x | x \in X,\, QGDD(x) = sup_{x \in X} QGDD(z)\}, \tag{10}$$

are called the maximum dominance elements of the fuzzy majority of X quantified by Q.

Nevertheless, we could also use the $QGNDD$. The $QDNDD$ is a generalization of Orlovski's nondominated alternative concept[40] and is defined as follows:

Definition 8 (Quantifier-Guided Nondominance Degree). *$QGNDD$ quantifies the degree to which the alternative x_i is not dominated by a fuzzy majority of the remaining alternative as*

$$QGNDD(x_i) = \Phi_Q\left(1 - p^s_{ji},\, j = 1, \ldots, m,\, j \neq i\right), \tag{11}$$

where $p^i_{ji} = max\left\{p_{ji} - p_{ij}, 0\right\}$ represents the degree to which x_i is strictly dominated by x_j.

The elements of the set

$$X^{QGNDD} = \{x | x \in X,\, QGNDD(x) = sup_{z \in X} QGNDD(z)\}, \tag{12}$$

are called maximal nondominated elements by the fuzzy majority of X quantified by Q.

3. SOCIAL NETWORK GROUP DECISION MAKING

In this section, we present the SNA GDM process, which resembles a classic one in that there still exist the aggregation and the exploitation steps still apply as illustrated in Figure 2. The main differences are though in how these steps are carried out and the information processed. In the SNA GDM process, it is necessary to gather information about the reliability of the judgments of the experts, for which each expert is required to provide their opinion about the trustworthiness of other experts' judgments. This information will be combined to form the experts' social network structure to which SNA will be carried out to compute experts' importance weights, which in turn will drive the fusion of the individual preferences to obtain

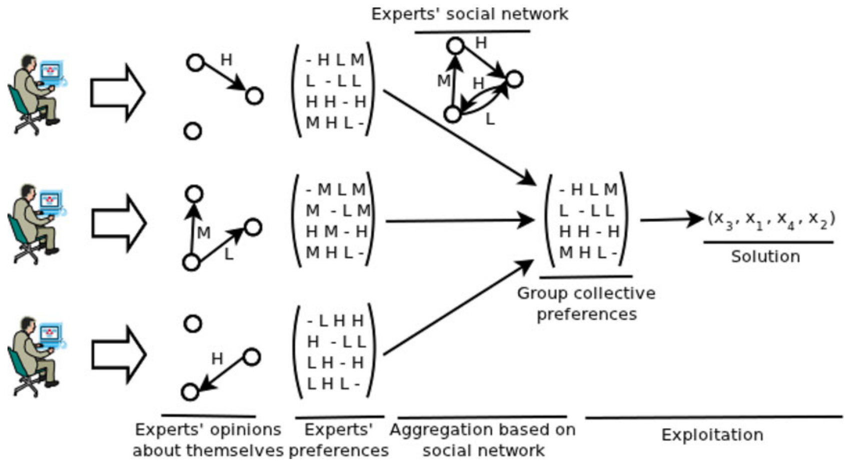


Figure 2. Graph representation of the SNA group decision process.

the collective preferences from which the final global ranking of alternatives. All these steps are elaborated in the following sections.

3.1. Social Network Representation

The social network reliability between experts will be represented using the following graph $G = (E, L, W)$, where E is the group of expert, L the arcs between any two experts, and W the weights associated with each arcs representing the strength of reliability between experts. As said above, the linguistic 2-tuple computation model[12] is applied to both represent and compute the values of the relationships of trustworthiness between experts.

DEFINITION 9. *A linguistic 2-tuple adjacency relationship R_L on E is a relationship in $E \times E$ with a membership function $\mu_{R_L} : E \times E \longrightarrow \mathcal{S} \times [-0.5, 0.5)$.*

3.2. SNA Linguistic Trustworthiness–Based Induced OWA Operators

Next, we will adapt the previous different measures of centrality to the case of having 2-tuple linguistic inputs and their use to propose three new SNA linguistic based IOWA operators: $C_{iD}^{\prime l} - IOWA$, $P_P - IOWA$, and $P_R - IOWA$. To illustrate their use and to carry out a comparative study between them, the following example of the GDM problem with a social network linguistic 2-tuple trustworthiness adjacency relationship will be solved.

Example 1. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be a set of experts that express their opinions about the reliability of other experts' judgments using the linguistic term

$\mathcal{S} = \{s_0, s_1, s_2, s_3, s_4\}$ to assess experts' trustworthiness as follows:

$$R_L = \begin{pmatrix} - & (s_2^r, 0) & (s_1^r, 0) & - & - & - \\ - & - & (s_4^r, 0) & - & - & (s_3^r, 0) \\ (s_1^r, 0) & (s_2^r, 0) & - & - & - & - \\ - & - & - & - & (s_3^r, 0) & (s_1^r, 0) \\ (s_2^r, 0) & - & (s_2^r, 0) & - & - & - \\ - & (s_3^r, 0) & (s_1^r, 0) & - & - & - \end{pmatrix}$$

Notice that we have added the supscript r to the linguistic assessment of R_L to differentiate them from the linguistic preferences the experts provide on the set of alternatives as described below. The graph representation of this sociomatrix is given in Figure 3.

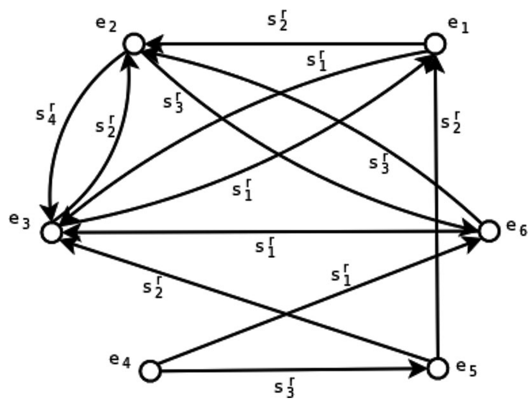


Figure 3. Graph representation of the sociomatrix.

Let us assume that the set off experts are to choose the best alternative from $X = \{x_1, x_2, x_3, x_4, x_5\}$ and that they provide the following preference relations using the same linguistic term set $\mathcal{S}$ as above:

$$P^1 = \begin{pmatrix} - & (s_3, 0) & (s_6, 0) & (s_5, 0) & (s_0, 0) \\ (s_3, 0) & - & (s_1, 0) & (s_2, 0) & (s_0, 0) \\ (s_0, 0) & (s_5, 0) & - & (s_0, 0) & (s_4, 0) \\ (s_1, 0) & (s_4, 0) & (s_6, 0) & - & (s_5, 0) \\ (s_6, 0) & (s_6, 0) & (s_2, 0) & (s_1, 0) & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & (s_2, 0) & (s_2, 0) & (s_2, 0) & (s_1, 0) \\ (s_4, 0) & - & (s_0, 0) & (s_0, 0) & (s_2, 0) \\ (s_4, 0) & (s_6, 0) & - & (s_1, 0) & (s_6, 0) \\ (s_4, 0) & (s_6, 0) & (s_5, 0) & - & (s_3, 0) \\ (s_5, 0) & (s_4, 0) & (s_0, 0) & (s_3, 0) & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & (s_3, 0) & (s_0, 0) & (s_1, 0) & (s_5, 0) \\ (s_3, 0) & - & (s_2, 0) & (s_3, 0) & (s_5, 0) \\ (s_6, 0) & (s_4, 0) & - & (s_5, 0) & (s_5, 0) \\ (s_5, 0) & (s_3, 0) & (s_1, 0) & - & (s_6, 0) \\ (s_1, 0) & (s_1, 0) & (s_1, 0) & (s_0, 0) & - \end{pmatrix}$$

$$P^4 = \begin{pmatrix} - & (s_6, 0) & (s_5, 0) & (s_4, 0) & (s_3, 0) \\ (s_0, 0) & - & (s_2, 0) & (s_6, 0) & (s_2, 0) \\ (s_1, 0) & (s_4, 0) & - & (s_2, 0) & (s_1, 0) \\ (s_2, 0) & (s_0, 0) & (s_4, 0) & - & (s_1, 0) \\ (s_3, 0) & (s_4, 0) & (s_5, 0) & (s_5, 0) & - \end{pmatrix}$$

$$P^5 = \begin{pmatrix} - & (s_1, 0) & (s_5, 0) & (s_5, 0) & (s_1, 0) \\ (s_5, 0) & - & (s_6, 0) & (s_5, 0) & (s_1, 0) \\ (s_1, 0) & (s_0, 0) & - & (s_0, 0) & (s_2, 0) \\ (s_1, 0) & (s_1, 0) & (s_6, 0) & - & (s_3, 0) \\ (s_5, 0) & (s_5, 0) & (s_4, 0) & (s_3, 0) & - \end{pmatrix}$$

$$P^6 = \begin{pmatrix} - & (s_2, 0) & (s_5, 0) & (s_1, 0) & (s_5, 0) \\ (s_4, 0) & - & (s_6, 0) & (s_2, 0) & (s_0, 0) \\ (s_1, 0) & (s_0, 0) & - & (s_4, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_2, 0) & - & (s_6, 0) \\ (s_1, 0) & (s_6, 0) & (s_1, 0) & (s_0, 0) & - \end{pmatrix}$$

We will assume the implementation of the fuzzy linguistic quantifier "most of" represented by $Q(r) = r^{\frac{1}{2}}$,[38] with the following weighting vector:

$$W = \{0.41, 0.17, 0.13, 0.11, 0.096, 0.087\}.$$

3.2.1. 2-Tuple Linguistic Trustworthiness In-Degree Centrality IOWA Operator

The in-degree centrality is also known as *prestige*.[25] Prestige of a node is based on counting only those nodes that are adjacent to it. The 2-tuple linguistic in-degree centrality index definition is given below.

DEFINITION 10 (2-Tuple Linguistic In-Degree Centrality Index). *Let $G = (E, L, W^L)$ be a directed linguistic graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, $L = \{l_1, \ldots, l_q\}$ be the set of directed lines, or arcs, between pairs of nodes, and $W^L = \{w_1^L, \ldots, w_q^L\}$ be the set of linguistic assessments attached to the lines (or arcs) with $w_i^L \in \mathcal{S}$. Let $R_L = (r_{ji})_{n\times n}$ be the sociomatrix associated with G, then the 2-tuple linguistic relative node in-degree centrality index is given as*

$$C'^{l}_{iD}(e_i) = \frac{1}{n-1} \sum_j \Delta^{-1}(r_{ji}).$$

As in the crisp case, experts who are directly supported by others will be more important than those ones who are scarcely supported. From this definition, it is possible to construct a new IOWA operator based on the 2-tuple linguistic in-degree centrality index.

DEFINITION 11 (2-Tuple Linguistic Trustworthiness In-Degree Centrality IOWA Operator). *Let $E = \{e_1, \ldots, e_n\}$ be a set of experts that provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_m\}$, by means of the linguistic preference relations, $\left\{P^1, \ldots, P^n\right\}$ and R_L be the sociomatrix representing the degree of trust between experts, then a $C'^{l}_{iD} - IOWA$ operator of dimension n is an IOWA operator whose set of order inducing values is the set of the relative node in-degree centrality indexes.*

Example 2. Resolution of Example 1 with $C'^{l}_{iD} - IOWA$ Operator.

1. **Aggregation step.** The experts' 2-tuple linguistic in-degree centrality indexes are

$$C'^{l}_{D} = \{0.60, 1.4, 1.6, 0, 0.6, 0.80\}$$

Therefore, the ordering of experts is the following:

$$e_3 \succ e_2 \succ e_6 \succ e_1 \sim e_5 \succ e_4$$

The collective 2-tuple linguistic preference relationships obtained using the $C'^{l}_{iD} - IOWA$ is

$$P = \begin{pmatrix} - & (s_3, -0.26) & (s_3, -0.45) & (s_2, 0.25) & (s_3, 0.23) \\ (s_3, 0.26) & - & (s_3, -0.48) & (s_3, -0.25) & (s_3, -0.34) \\ (s_3, 0.45) & (s_3, 0.48) & - & (s_3, -0.096) & (s_4, 0.40) \\ (s_4, -0.25) & (s_3, 0.25) & (s_3, 0.096) & - & (s_5, -0.37) \\ (s_3, -0.23) & (s_3, 0.34) & (s_2, -0.40) & (s_1, 0.37) & - \end{pmatrix}$$

2. **Exploitation step.** The 2-tuple linguistic quantifier-guided dominance choice degree associated with each one of the alternatives is

$$QGDD = \{(s_3, -0.18), (s_3, -0.02), (s_3, 0.43), (s_4, -0.43), (s_3, -0.43)\}.$$

Therefore, the collective ordering of alternatives obtained using the $C'^{l}_{iD} - IOWA$ is

$$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5.$$

3.2.2. *2-Tuple Linguistic Trustworthiness Proximity Degree IOWA Operator*

Prestige of a node can be extended by adding to the nodes that are adjacent to it those other nodes that are in the influence domain of the node of interest, i.e., those nodes that are both directly and indirectly linked to it. This is known as the node proximity degree.[25,41]

DEFINITION 12 (Node Proximity Degree). *Let $G = (E, L)$ be a directed dichotomous graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, and $L = \left\{l_1, \ldots, l_q\right\}$ be the set of* *directed lines, or arcs, between pairs of nodes. The node proximity degree is given as*

$$P_P(e_i) = \frac{I_i/(n-1)}{\sum_j d(e_j, e_i)/I_i}$$

where I_i is the number of nodes in the influence domain of the node e_i, $d\left(e_j, e_i\right)$ is the distance the node e_j is to e_i, and the summation is just over those nodes in the influence domain of the node e_i

In the case of interest in this paper where arcs have a linguistic label associated requires the implementation of such linguistic labels in the expression of $d\left(e_j, e_i\right)$. This is proposed to be done by transforming the 2-tuple linguistic sociomatrix into a crisp sociomatrix using the following $\alpha - cut$ approach.

DEFINITION 13. *Let $G = \left(E, L, W^L\right)$ be a directed linguistic graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, $L = \left\{l_1, \ldots, l_q\right\}$ the set of directed lines, or arcs, between pairs of nodes, and $W^L = \left\{w_1^L, \ldots, w_q^L\right\}$ be the set of linguistic assessments attached to the lines (or arcs) with $w_i^L \in \mathcal{S}$. Given $\alpha_l \in \mathcal{S}$, the following function, $f_{\alpha_l - cut} : E \times E \longrightarrow \{0, 1\}$*

$$f_{\alpha_l - cut}\left(e_i, e_j\right) = \begin{cases} 1 & \text{if } \mu_{R_L}\left(e_i, e_j\right) \succeq \alpha_l \\ 0 & \text{if } \mu_{R_L}\left(e_i, e_j\right) \prec \alpha_l \\ - & \text{if } \mu_{R_L}\left(e_i, e_j\right) \text{ is not defined} \end{cases}$$

transform the directed linguistic graph G in a crisp graph.

The following introduces the IOWA operator based on the 2-tuple linguistic proximity degree.

DEFINITION 14 (2-Tuple Linguistic Trustworthiness Proximity Degree IOWA Operator). *Let $E = \{e_1, \ldots, e_n\}$ be a set of experts that provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_m\}$, by means of the linguistic preference relations, $\left\{P^1, \ldots, P^n\right\}$ and R_L be the sociomatrix representing the degree of trust between experts, then a $P_P - IOWA$ operator of dimension n is an IOWA operator whose set of order inducing values is the set of the of proximity degrees obtained from the crisp sociomatrix calculated from the application of the $\alpha_l - cut$ function over R_L.*

Example 3. Resolution of Example 1 with the $P_P - IOWA$ operator.

1. **Aggregation step.** First, using $\alpha_l = (s_0, 0)$ the following crisp sociomatrix is derived from the original matrix R_L via the above $\alpha_l - cut$ function:

$$R'_L = \begin{pmatrix} - & 1 & 1 & - & - & - \\ - & - & 1 & - & - & 1 \\ 1 & 1 & - & - & - & - \\ - & - & - & - & 1 & 1 \\ 1 & - & 1 & - & - & - \\ - & 1 & 1 & - & - & - \end{pmatrix}$$

The experts' proximity degree is

$$P_P = \{0.625, 0.71, 0.83, 0, 0.2, 0.55\}$$

Therefore, the ordering of experts is the following:

$$e_3 \succ e_2 \succ e_1 \succ e_6 \succ e_5 \succ e_4.$$

The collective 2-tuple linguistic preference relationships obtained using the $P_P - IOWA$ is

$$P = \begin{pmatrix} - & (s_3, -0.21) & (s_3, -0.42) & (s_2, 0.34) & (s_3, 0.12) \\ (s_3, 0.21) & - & (s_2, 0.36) & (s_3, -0.29) & (s_3, -0.35) \\ (s_3, 0.42) & (s_4, -0.36) & - & (s_3, -0.18) & (s_4, 0.40) \\ (s_4, -0.34) & (s_3, 0.29) & (s_3, 0.18) & - & (s_5, -0.36) \\ (s_3, -0.12) & (s_3, 0.35) & (s_2, -0.40) & (s_1, 0.36) & - \end{pmatrix}$$

2. **Exploitation step.** The 2-tuple linguistic quantifier–guided dominance choice degree associated with each one of the alternatives is

$$QGDD = \{(s_3, -0.17), (s_3, -0.07), (s_3, 0.43), (s_4, -0.45), (s_3, -0.38)\}$$

Therefore, the collective ordering of alternatives obtained using the $P_P - IOWA$ is

$$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5.$$

3.2.3. 2-Tuple Linguistic Trustworthiness Rank Prestige IOWA Operator

The aforementioned measures look at the in-degrees and the distance of the nodes, respectively. The status or rank prestige on the other hand combines the number of direct choices to a node with the status or rank of the nodes involved in that choice.[25,33,42]

DEFINITION 15 (Node Rank Prestige). *Let $G = \left(E, L, W^L\right)$ be a directed weighted graph, $E = \{e_1, \ldots, e_n\}$ be the set of nodes, $L = \left\{l_1, \ldots, l_q\right\}$ the set of directed lines, or arcs, between pairs of nodes, and $W^L = \left\{w_1^L, \ldots, w_q^L\right\}$ be the set of weights attached to the lines (or arcs). Let $R_L = \left(r_{ji}\right)_{n\times n}$ be the sociomatrix associated with G, then the node rank prestige index is given as*

$$P_R(e_i) = r_{1i} P_R(e_1) + r_{2i} P_R(e_2) + \cdots + r_{li} P_R(e_l) + \cdots + r_{ni} P_R(e_n)$$

In the case of having linguistic weights, the linguistic sociomatrix is transformed into a numeric one by means of function Δ^{-1}.

Therefore, the last operator we are going to present takes into account the status or the rank of the actors involved. Therefore, we have the following expression for the 2-tuple linguistic node rank prestige index:

$$P_R(e_i) = \Delta^{-1}(r_{1i}) P_R(e_1) + \cdots + \Delta^{-1}(r_{li}) P_R(e_l) + \cdots + \Delta^{-1}(r_{ni}) P_R(e_n)$$

The following introduces the corresponding 2-tuple linguistic IOWA operator.

DEFINITION 16 (2-Tuple Linguistic Trustworthiness Rank Prestige IOWA Operator). *Let $E = \{e_1, \ldots, e_n\}$ be a set of experts that provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_m\}$, by means of the linguistic preference relations, $\{P^1, \ldots, P^n\}$ and R_L be the sociomatrix representing the degree of trust between experts, then a $P_R - IOWA$ operator of dimension n is an IOWA operator whose set of order inducing values is the set of the of rank prestige degrees.*

If we compute all the rank prestige degrees, we will obtain n equations, all of which depend on all the indexes themselves:

$$\begin{aligned}
&P_R(e_1) = \Delta^{-1}(r_{11}) P_R(e_1) + \cdots + \Delta^{-1}(r_{n1}) P_R(e_n)\\
&P_R(e_2) = \Delta^{-1}(r_{12}) P_R(e_1) + \cdots + \Delta^{-1}(r_{n2}) P_R(e_n)\\
&\vdots\\
&P_R(e_n) = \Delta^{-1}(r_{1n}) P_R(e_1) + \cdots + \Delta^{-1}(r_{nn}) P_R(e_n)
\end{aligned}$$

We have a system of n linear equations with n unknowns. To resolve the system, we place the set of rank indexes in a vector $p = (P_R(v_1), P_R(v_2), \ldots, P_R(v_n))^T$. Then, we can rewrite the system of equations as

$$p = X'^T p$$

This equation is identical to a characteristic equation in which p is an eigenvector of X' corresponding to the eigenvalue value 1. Katz[42] noted that this system has no finite solution. Therefore, to find a solution, one must put some constraints on either X'^T or on the indexes themselves. Following Katz's recommendations to solve this system of equations, the sociomatrix X' is standardized to have column sums of unity. That way, the highest eigenvalue of the standardized sociomatrix X' will be the unity and the eigenvector associated with this eigenvalue will be the vector of rank prestige degrees, p.

Example 4. Resolution of Example 1 with the$P_R - IOWA$ Operator.

1. **Aggregation step.** The application of Katz' m approach leads to the following experts' rank prestige degrees:

$$P_R = \{0.59, 2.34, 1.78, 0, 0, 1\}$$

Therefore, the ordering of experts is the following:

$$e_2 \succ e_3 \succ e_6 \succ e_1 \succ e_4 \sim e_5$$

Table III. Results of the different IOWAs.

	Experts' ordering	Solution
$C_{iD}^{\prime l} - IOWA$	$e_3 \succ e_2 \succ e_6 \succ e_1 \sim e_5 \succ e_4$	$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5$
$P_P - IOWA$	$e_3 \succ e_2 \succ e_1 \succ e_6 \succ e_5 \succ e_4$	$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5$
$P_R - IOWA$	$e_2 \succ e_3 \succ e_1 \succ e_6 \succ e_5 \succ e_4$	$x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5$

The collective 2-tuple linguistic preference relationships obtained using the $P_R - IOWA$ is

$$P = \begin{pmatrix} - & (s_3, -0.47) & (s_3, 0.039) & (s_2, 0.49) & (s_2, 0.26) \\ (s_3, 0.47) & - & (s_2, -0.021) & (s_2, -0.0098) & (s_2, -0.067) \\ (s_3, -0.039) & (s_4, 0.021) & - & (s_2, -0.053) & (s_5, -0.34) \\ (s_4, -0.49) & (s_4, 0.0098) & (s_4, 0.053) & - & (s_4, -0.059) \\ (s_4, -0.26) & (s_4, 0.067) & (s_1, 0.34) & (s_2, 0.059) & - \end{pmatrix}$$

2. **Exploitation step.** The 2-tuple linguistic quantifier–guided dominance choice degree associated with each one of the alternatives is

$$QGDD = \{(s_3, -0.22), (s_3, -0.17), (s_3, 0.221), (s_4, -0.34), (s_3, 0.18)\}$$

Therefore, the collective ordering of alternatives obtained using the $P_P - IOWA$ is

$$x_4 \succ x_3 \succ x_5 \succ x_2 \succ x_1.$$

3.3. Comparison

In Table III, we can see the different solutions obtained by the three IOWAs presented in this paper.

In all there cases, e_4 is listed as last in terms of importance whereas e_3 and e_2 are listed in the first two positions, although the order is reversed for $P_R - IOWA$ when compared to $C_{iD}^{\prime l} - IOWA$ and $P_P - IOWA$. The cause of this discrepancy is due to the assignation by $P_R - IOWA$ of a status to each node, which is inferred from the status of the nodes that support it and the strength of such support. In this example, the difference that appears between the final ranking of e_2 and e_3 is mainly caused by nodes e_6 and e_1 as they show a greater support for e_2 than for e_3. The first two IOWAs, $C_{iD}^{\prime l} - IOWA$ and $P_P - IOWA$, also differ in relation to the ordering of experts e_6 and e_1. On the one hand, the support node e_6 receives from nodes e_4 and e_2 is greater than the supported received by e_1 from e_5 and e_3, and this explains why $C_{iD}^{\prime l} - IOWA$ lists of e_6 as more important than e_1. On the other hand, as $P_P - IOWA$ also takes into account influence domain of a node it is worth to notice here that the influence domain of node e_1 is higher than the influence domain of e_6, which explains why the ordering of importance is reversed in this case with respect to these two nodes. In all cases, though, the final rankings of the alternatives coincide although this cannot be generalized to all cases as the final ranking very much depend on the actual values of the linguistic preferences.

4. CONCLUSIONS

In this paper, we have studied the use of SNA in group decision-making problems. We have defined three new IOWA operators, $C'^{l}_{iD} - IOWA$, $P_P - IOWA$, and the $P_R - IOWA$, that are applicable to the case of dealing with experts linguistic information on the degree of reliability of other experts' judgments within the 2-tuple computational framework. The first IOWA operator, $C'^{l}_{iD} - IOWA$, is based on the node in-degree centrality index and presents the collective reliability gathered by the adjacent nodes to that node. The second IOWA operator, $P_P - IOWA$, takes into account not only the adjacent nodes but also the nodes that are linked to it indirectly. The third one, $P_R - IOWA$, computes the ranks or status of the nodes based on the nodes that have chosen them as a reliable source. The use of one operator or another will obviously depend on the nature of the problem and/or group of experts, as well as the type of importance degree to implement in the resolution of the GDM problem. In any case, the main advantage of these threes IOWAs is that the importance of the experts' judgments is obtained from the experts' themselves and, therefore, the assumption of these has been provided beforehand and is superfluous. These operators make possible to aggregate the information by implementing the experts' social interactions and judgments, and therefore can be considered as more flexible and realistic since the more reliable the experts judgments are, the more support by partners they will receive. However, one of the limitations of this proposal is that we only deal with positive trust relationships, so there is a need to extend this study to manage both positive and negative feedbacks about experts' trustworthiness.

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