

Modelling influence in group decision making

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Abstract Group decision making has been widely studied since group decision making processes are very common in many fields. Formal representation of the experts' opinions, aggregation of assessments or selection of the best alternatives has been some of main areas addressed by scientists and researchers. In this paper, we focus on another promising area, the study of group decision making processes from the concept of influence and social networks. In order to do so, we present a novel model that gathers the experts' initial opinions and provides a framework to represent the influence of a given expert over the other(s). With this proposal it is feasible to estimate both the evolution of the group decision making process and the final solution before carrying out the

group discussion process and consequently foreseeing possible actions.

Keywords Group decision making · Aggregation operators · Social network · Influence

1 Introduction

Decision making can be defined as a set of activities whose aim is to find satisfactory solutions for a given problem. This problem usually involves the analysis of a finite set of alternatives. From this analysis, decision makers, judges or expert(s) should decide which of the alternatives is the best one to solve an issue. Decision making can be approached from two points of view:

- Individual decision making: in which an expert must provide the best alternative from a set of possible solution, either evaluating the utility of each alternative or assessing a set of criteria that described them.
- Group decision making: in which a group of experts must reach a solution to a given problem.

Although decision making has been widely studied, only a few areas have drawn the attention of researchers. For instance, the formal representation of the experts' opinions (Kacprzyk and Roubens 1988; Kitainik 1993; Luce and Suppes 1965; Pérez-Asurmendi and Chiclana 2014; Tanino 1984, 1990; Ureña et al. 2015), the aggregation of the information (Pérez et al. 2014; Seo and Sakawa 1985) and the selection of the best alternative(s) (Cabrerizo et al. 2015; Morente-Molinera et al. 2016) have been developed in depth by many authors.

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Another promising field is the consensus in decision making problems (Alonso et al. 2010; Cabrerizo et al. 2010; Calza et al. 2015; Ciasullo et al. 2015; Gong et al. 2015; Kacprzyk et al. 1997; Mata et al. 2009, 2014). In this kind of problems, the aim is to reach a given course of action that the majority of experts approve and that the minority agree to consent. Thus, the minority feel that their opinions also contribute to reach the best possible decision for the group.

In this paper, we focus on another interesting area related to group decision making. We present a model based on ‘influence’ concept that let us estimate both the evolution of the group decision making process and the final solution from the experts’ initial opinions.

The utility of this field is propitious since on the one hand, we can predict beforehand the final decision of a given problem and, on the other hand, this model will let us know how the opinion of one expert affects their partners’ opinions throughout the process.

Few papers have dealt with the modelling of influences between people through discussion and exchanges of opinions (Friedkin and Johnsen 1990; Friedkin et al. 1999) and our aim is to apply these studies to group decision making processes.

A Group Decision Making (GDM) process usually entails an activity in which experts interact with each other to reach a final solution. These interactions consist of an exchange of opinions and information. Because of these interactions, experts with a wider background, experience and knowledge will be recognised and empowered with a higher status. This status reflects the influence of a given expert over the other(s). Hence, with a suitable representation for the influence among the experts and their initial opinions, we may be able to predict both the evolution of the experts’ opinions and their final decisions.

Social network analysis (Scott and Carrington 2011; Wasserman and Faust 1994) studies relationships among social entities or actors as well as patterns and implications of these relationships. These relationships may be of many sorts: economic, political, interactional, affective, etc. Social network analysis is based on the idea that actors and their actions are interdependent and the network structural environment provides opportunities for or constraints on individual actions.

Using social network analysis in GDM problems can produce important benefits since it allows us to model and study the relationship between experts and, therefore, infer or estimate the influence among themselves. The aim of this paper is to model this situation.

The structure of this paper is the following: First of all, in Sect. 2 we review some preliminaries; then the model is presented in Sect. 3. Finally, Sect. 4 shows an example and the conclusions are pointed out in Sect. 5.

2 Preliminaries

In this section, we sum up some aspects needed to understand our proposal such as group decision making, consistency and social influence networks theory.

2.1 Group decision making

In Group Decision Making problems (GDM), a group of experts must decide which alternative(s) is the best one to deal with an issue. For this reason, each one of them provides an assessment of every possible alternative. To represent these judgements, some structures are proposed. The most common ones are the following:

- A preference ordering of alternatives (Seo and Sakawa 1985): in this case, experts provide their preferences on a set of alternatives as an ordered vector of alternatives, from the best one to the worst one.
- Utility vectors (Luce and Suppes 1965; Tanino 1990): with this representation experts provide their preferences as a set of utility values for each alternative. The greater the value, the more preferred the alternative.
- Preference relations (Kacprzyk 1986; Kacprzyk and Roubens 1988; Kitainik 1993; Tanino 1984): based on the idea of pairwise comparisons, experts’ preferences are described by means of preference relations in which each value represents the preference of one alternative over the other.

Once experts have provided their opinion, we must accomplish the selection of the best alternatives. This process usually involves two phases (Alonso et al. 2009; Pérez et al. 2010; Roubens 1997; Wu et al. 2015):

- An aggregation phase: in this step, experts’ opinions are aggregated to obtain a global opinion.
- A selection phase: in this phase the best alternative(s) is chosen and presented as the decision making solution.

One of the most used preference structures in GDM is the preference relation. In preference relations, we find two kinds of interpretation regarding preference assessments: it can be additive reciprocal and multiplicative reciprocal. In this contribution, we have applied the additive preference relation and its definition is the following one:

Definition 1 (Bezdek et al. 1978; Chiclana et al. 2009; Nurmi 1981) An additive reciprocal fuzzy preference relation R on a finite set of alternatives X is a fuzzy relation in $X \times X$ with membership function $\mu_R : X \times X \rightarrow [0, 1]$,

$\mu_R(x_i, x_j) = r_{ij}$, verifying

$$r_{ij} + r_{ji} = 1 \quad \forall i, j \in \{1, \dots, n\}.$$

When cardinality of X is small, the reciprocal fuzzy preference relation may be conveniently denoted by the matrix $R = (r_{ij})$ and the following interpretation is also usually assumed:

- $r_{ij} = 1$ indicates the maximum degree of preference for x_i over x_j .
- $r_{ij} \in]0.5, 1[$ indicates a definite preference for x_i over x_j .
- $r_{ij} = 0.5$ indicates indifference between x_i and x_j .
- $r_{ij} \in]0, 0.5[$ indicates a definite preference for x_j over x_i .

Another important aspect related to GDM is opinion aggregation to compute a global opinion over a set of alternatives. Many operators have been proposed to carry out this computation highlighting the family of OWA operators.

In this section, the OWA (Yager 1988), IOWA (Yager and Filev 1998) and I-IOWA operators are introduced (Chiclana et al. 2007).

Definition 2 An OWA operator of dimension n is a mapping $\phi: \mathbb{R}^n \rightarrow \mathbb{R}$, which has an associated set of weights $W = (w_1, \dots, w_n)^T$, such that $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$,

$$\phi(a) = \phi(p_1, \dots, p_n) = \sum_{i=1}^n w_i p_{\sigma(i)} \quad (1)$$

and $\sigma: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ is a permutation function such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \dots, n-1$.

In order to obtain the associated weighting vector, we use the method proposed by Yager (1983, 1988) to give semantic or meaning to weights by means of quantifier guided aggregations. To implement the concept of fuzzy majority in the aggregation phase, we use a fuzzy linguistic quantifier (Zadeh 1983), which indicates the proportion of satisfied criteria “necessary for good solution” (Yager 1996) (see the Appendix 1 for further details). In case of a regular increasing monotone (RIM) quantifier Q , the weights are computed using the following expression (Yager 1988):

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \dots, n.$$

Mitchell and Estrakh (1997) described a modified OWA operator in which the input arguments are not re-arranged according to their values but rather using a function of the arguments. Inspired by this work, Yager and Filev introduced

in (Yager and Filev 1998) a more general type of OWA operator, which they named the Induced OWA (IOWA) operator:

Definition 3 An IOWA operator of dimension n is a mapping $\Phi_W: (\mathbb{R} \times \mathbb{R})^n \rightarrow \mathbb{R}$, which has an associated set of weights $W = (w_1, \dots, w_n)$, so that $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$,

$$\Phi_W(\langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle) = \sum_{i=1}^n w_i p_{\sigma(i)},$$

and $\sigma: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ is a permutation function such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \dots, n-1$.

In the above definition the reordering of the set of values to aggregate, $\{p_1, \dots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \dots, u_n\}$ associated with them, which is based upon their magnitude. Yager and Filev called the vector of values (u_1, \dots, u_n) , the order inducing vector and $\{p_1, \dots, p_n\}$, the values of the argument variable (Yager and Filev 1998, 1999; Yager 2003). Thus, the main difference between the OWA operator and the IOWA operator is the reordering step of the argument variable. In the case of OWA operator this reordering is based upon the magnitude of the values to be aggregated, while in the case of IOWA operator an order-inducing vector is used as the criterion to induce that reordering.

In many cases, each expert $e_k \in E$ is assigned an importance degree u_k to him/her, which without loss of generality can be assumed to belong to the unit interval, i.e. $u_i \in [0, 1] \quad \forall i$, and that there is some i such that $u_i = 1$. This can always be assured by taking the normalised importance degrees that result when dividing them by the maximum importance degree. Thus, importance degree can be interpreted as a fuzzy set membership function, $\mu_I: E \rightarrow [0, 1]$, in such a way that $\mu_I(e_k) = u_k \in [0, 1]$ denotes the importance degree of the opinion provided by the expert e_k . In the area of quantifier-guided aggregations, Yager (1996) presents a procedure to evaluate the overall satisfaction of Q important criteria (experts) by an alternative x . In this procedure, once the satisfaction values to be aggregated have been ordered, the weighting vector associated with an OWA operator using a linguistic quantifier Q is calculated following the expression

$$w_k = Q\left(\frac{S(k)}{S(n)}\right) - Q\left(\frac{S(k-1)}{S(n)}\right) \quad (2)$$

being $S(k) = \sum_{l=1}^k u_{\sigma(l)}$, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight with those experts with zero importance degree. This procedure was extended by Yager to the case of induced aggregation (Yager 2003) and later by Chiclana et al. (2007)

to introduce the Importance IOWA (I-IOWA) operator. In this case, each component in the aggregation consists of a tuple (u_k, p_k) where p_k is the argument value to aggregate and u_k is the importance weight value associated with p_k and also the order-inducing value. This is summarised in the following definition:

Definition 4 Let $E = \{e_1, \dots, e_m\}$ be a set of experts and $U = (u_1, \dots, u_m) \in [0, 1]^m$ the vector of their associated importance degrees. An I-IOWA operator of dimension n , Φ_W^I , is an IOWA operator whose order-inducing vector is the vector of importance degrees and following associated set of weights $W = (w_1, \dots, w_n)$:

$$\Phi_W^I(\langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle) = \sum_{i=1}^n w_i p_{\sigma(i)},$$

with

$$w_k = Q\left(\frac{S(k)}{S(n)}\right) - Q\left(\frac{S(k-1)}{S(n)}\right), \quad (3)$$

where $S(k) = \sum_{l=1}^k u_{\sigma(l)}$, and σ is the permutation such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \dots, n-1$.

With the I-IOWA operator, we accomplish the selection phase. To do so, some authors (Chiclana et al. 2007) have proposed the use of a quantifier-guided choice degree of alternatives, a dominance degree. With this degree it is computed the best acceptable alternative for the majority (Q).

Definition 5 The quantifier-guided dominance degree, $QGDD_i$, quantifies the dominance that one alternative has over all the others and is defined as follows:

$$QGDD_i = \phi_Q(p_{i1}^c, p_{i2}^c, \dots, p_{in}^c).$$

So, for each alternative, x_i , we obtain a value that quantifies the dominance that this alternative has over all the others in a fuzzy majority sense.

With this degree we can sort out the alternatives and choose the best one(s),

$$X^{sol} = \left\{ x_i \mid x_i \in X, QGDD_i = \sup_j QGDD_j \right\}.$$

An example of how to use these operators in GDM problems is shown in Appendix 2.

2.2 Consistency on preference relations

Although preference relations are one of the most used preference structures, they also have some disadvantages. First

of all, the way of providing preferences limits experts in their global perception of the alternatives and as a consequence, the provided preferences could be inconsistent.

In Chiclana et al. (2008) the U -Consistency Method, useful to tackle the problem of incomplete information in reciprocal fuzzy preference relations, is presented. This method is based on the modelling of consistency of preferences via a self-dual almost continuous uninorm operator (Chiclana et al. 2007) and computes the missing pairwise preference values from the known ones with the aim of maintaining or maximising the expert's global consistency. This process is applied as well to complete preference relations as it is our case. With this process, we can assure that the values provided by the expert are compatible with the rest of information and, moreover, we can replace those ones that are not compatible with the estimated values.

In order to deal with the inconsistency, we must understand that this representation assumes three fundamental and hierarchical levels of rationality:

- The first level of rationality requires indifference between any alternative x_i and itself.
- The second one requires that if an expert prefers x_i to x_j , that expert should not simultaneously prefer x_j to x_i . This asymmetry condition is viewed as an 'obvious' condition/criterion of consistency for preferences. This rationality condition is modelled by the property of reciprocity in the pairwise comparison between any two alternatives, which is seen by Saaty (1980) as basic in making paired comparisons.
- Finally, the third one is associated with the transitivity in the pairwise comparison among any three alternatives.

A preference relation verifying the third level of rationality is usually called a consistency preference relation and any property that guarantees the transitivity of the preferences is called a consistency property. However, many properties or conditions have been suggested in the literature of fuzzy preference to model the third level of rationality. Among these properties we can cite (restricted) max–min transitivity, restricted max–max transitivity, additive transitivity and multiplicative transitivity (Chiclana et al. 2007; Dubois and Prade 1980; Tanino 1988), and we have used Tanino's multiplicative transitivity (Tanino 1988).

Next, we review some definitions to understand how to improve the consistency of a relationship.

Definition 6 (Chiclana et al. 2007) Let U be a representable uninorm operator with strong negation $N(x) = 1 - x$ and let R be an additive reciprocal fuzzy preference relation on a finite set of alternatives X , $\mu_R : X \times X \rightarrow [0, 1]$, where $\mu_R(x_i, x_j) = r_{ij}$ represents the preference degree of the alternative x_i over x_j ; then the fuzzy preference relation R

on a finite set of alternatives is consistent with respect to U (U -Consistent) if

$$\forall i, j, k : (r_{ij}, r_{kj}) \notin \{(0, 1), (1, 0)\} \Rightarrow r_{ij} = U(r_{ik}, r_{kj}).$$

Tanino's multiplicative transitivity property (Tanino 1988) under reciprocity is the restriction to the region $[0, 1] \times [0, 1] \setminus \{(0, 1), (1, 0)\}$ of the following well-known and like representable uninorm (Klement et al. 1996)

$$U(x, y) = \begin{cases} 0, & (x, y) \in \{(0, 1), (1, 0)\} \\ \frac{xy}{xy + (1-x)(1-y)}, & \text{otherwise.} \end{cases}$$

In order to improve the quality of the information provided by the experts we use this transitivity. Given a fuzzy preference relation R and U a representable uninorm (with $N(X) = 1 - x$), the preference value r_{ik} ($i \neq k$) is partially U -estimated using an intermediate alternative x_j such that $(r_{ij}, r_{jk}) \notin \{(0, 1), (1, 0)\}$ as follows:

$$ur_{ik}^j = U(r_{ij}, r_{jk}).$$

The average of the partially U -estimated values obtained using all possible intermediate alternatives can be seen as the global consistency-based estimated value

$$ur_{ik} = \frac{\sum_{j \in R_{ik}^{01}} ur_{ik}^j}{\#R_{ik}^{01}},$$

where $R_{ik}^{01} = \{j \neq i, j \mid (r_{ij}, r_{jk}) \notin \{(0, 1), (1, 0)\}\}$.

With this definition, we estimate the missing values of a relationship or as in our case to improve the consistency of given relationship.

In our model, it is also very important to measure the U -Consistency of a preference relation. A preference relation is completely U -Consistency if ur_{ik} agrees with r_{ik} ($ur_{ik} = r_{ik}$). However, experts are not always fully consistent and because of this, it is possible to calculate the difference between the actual value r_{ik} and the estimated one ur_{ik} :

$$\varepsilon ur_{ik} = |ur_{ik} - r_{ik}|.$$

From this formula, the U -Consistency level associated with a preference value r_{ik} is defined, and then, the U -consistency level of the whole reciprocal fuzzy preference relation R .

Definition 7 (Chiclana et al. 2008) Given U a representable uninorm with $N(x) = 1 - x$, the U -consistency level associated with a preference value r_{ik} is defined as

$$UCL_{ik} = 1 - \varepsilon ur_{ik},$$

Definition 8 (Chiclana et al. 2008) Given U a representable uninorm with $N(x) = 1 - x$, the U -consistency level of a

fuzzy preference relation R measured in $[0, 1]$ is given as follows:

$$UCL_R = \frac{\sum_{i,j=1, i \neq j}^n UCL_{ik}}{n(n-1)}.$$

2.3 Social influence networks theory

Social Influence Network Theory began with French's formal theory of social power (French 1956) and it has been developed by several authors (Friedkin and Johnsen 1990; Friedkin et al. 1999; Harary 1959). The aim of this theory was to model the process of influence among actors belonging to the same social network. The following expression is a recursive definition of the influence process:

$$y^{(t)} = AWy^{(t-1)} + (I - A)y^{(1)}$$

for $t = 2, 3, \dots$, where $y^{(1)}$ is an $N \times 1$ vector of actors' initial opinions on an issue, $y^{(t)}$ is an $N \times 1$ vector of actors' opinions of time t , $W = [w_{ij}]$ is an $N \times N$ matrix of interpersonal influences ($0 \leq w_{ij} \leq 1$, $\sum_j w_{ij} = 1$), and $A = \text{diag}(a_{11}, a_{22}, \dots, a_{NN})$ is an $N \times N$ diagonal matrix of actors' susceptibilities to interpersonal influence on the issue ($0 \leq a_{ii} \leq 1$).

This model is defined from the following assumptions (Friedkin et al. 1999):

1. Cognitive weighted averaging: Actors are assumed to form their revised opinions through a weighted averaging of influences on them. This influence is not only exogenous because of the other actors' opinions, but also endogenous from their own initial opinion. The relative weight of the endogenous and exogenous influences for each actor is determined by $A = [a_{ii}]$, the coefficients of susceptibility to social influence.
2. Fixed social structure: The social structure of the group of actors, the network, is represented by the matrix W , is assumed to be fixed during the entire process of opinion formation.
3. Determinism: Given the direct influence matrix W and the group members' initial opinion $y^{(1)}$, the subsequent opinion changes in the group are completely determined.
4. Continuance: The process of opinion formation in the group continues until all changes of opinions that may occur have played themselves out.
5. Decomposability: The opinion formation process can be divided into time periods, defined by the times $t = 1, 2, 3, \dots$, that may not be of the same length in real time.
6. Simultaneity: In each time period, simultaneous linear equations yield an accurate prediction of all the influence events that occur during that period.

If W is known then A is computed as

$$a_{ii} = 1 - w_{ii}.$$

Assuming the process reaches an equilibrium, then $\lim_{t \rightarrow \infty} y^{(t)} = y^{(\infty)} = k < \infty$ and the original equation becomes

$$y^{(\infty)} = Vy^{(1)},$$

where V describes the total interpersonal effects that transform initial opinions into final opinions. Then V is calculated by using the following formula:

$$V = (I - AW)^{-1} (I - A).$$

3 Modelling influence in group decision making

In order to model an influence network in a GDM process, we propose to carry out the following phases:

1. Providing information: Experts' opinions are provided by means of reciprocal fuzzy preference relations. However, as we said earlier, one of the disadvantages of this structure is that experts do not always provide consistent assessments. Thus, we need to improve the consistency of this information by means of a U -Consistency Based Method (Chiclana et al. 2008). This method was originally presented to estimate missing pairwise preference values; however, this method is also used to improve the consistency of a given relationship. Finally, we transform the preference relation into utility vectors.
2. Modelling influence: In this phase we apply an influence model to estimate the evolution of the experts' opinions with regards to the other experts' opinions. Experts are supposed to discuss their opinion among them, and depending on the influence between them, their opinions are modified. In order to model this, we use the model reviewed in Sect. 2.3. In this model, the influence between experts is modelled by means of a social influence network. From this model, and with the experts' opinions, it is possible to infer the experts' final opinions.
3. Obtaining the solution: Once that final opinions have been calculated, the solution is computed. To do so, experts' utility vectors are aggregated by means of an I-IOWA operator. The I-IOWA is a weighted aggregation operator in which the information is aggregated based on the importance of the information sources. In our proposal, this importance is obtained by computing the consistency of the information that was initially provided by the experts. The more consistent it is, the more important the information is.

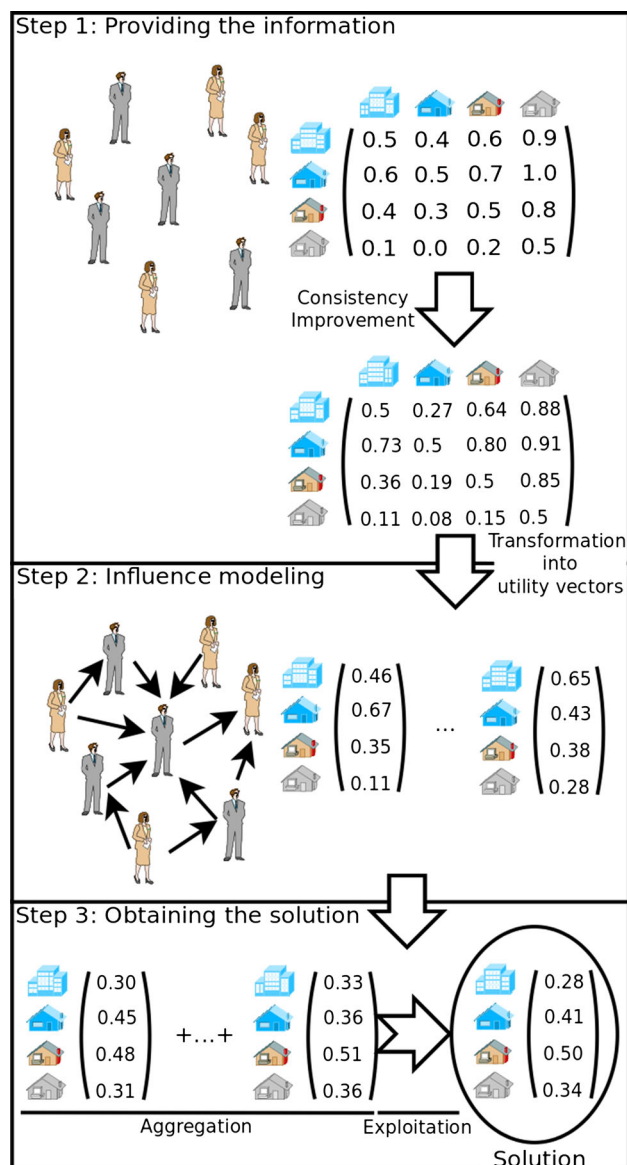


Fig. 1 Model's phases

A representation of model's phases can be seen in Fig. 1. In the following subsections we explain thoroughly these phases.

3.1 Providing the information

In this phase, a group of experts $E = \{e_1, \dots, e_m\}$ provide their opinions about a set of alternatives $X = \{x_1, \dots, x_n\}$ by means of reciprocal preference relations. The use of these representation provides some advantages since they allow experts to focus exclusively on two alternatives at a time.

Then, the information provided by the experts is improved by means of the consistency model reviewed in Sect. 2.2.

Example 1 : Let P_1 be the opinion of the expert e_1 :

$$P_1 = \begin{pmatrix} 0.5 & 0.4 & 0.6 & 0.9 & 0.7 & 0.8 \\ 0.6 & 0.5 & 0.7 & 1 & 0.8 & 0.9 \\ 0.4 & 0.3 & 0.5 & 0.8 & 0.6 & 0.7 \\ 0.1 & 0 & 0.2 & 0.5 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.4 & 0.7 & 0.5 & 0.6 \\ 0.2 & 0.1 & 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$$

then the preference relation obtained after applying the U -consistency method is

$$P'_1 = \begin{pmatrix} 0.5 & 0.27 & 0.64 & 0.89 & 0.74 & 0.82 \\ 0.73 & 0.5 & 0.8 & 0.92 & 0.85 & 0.89 \\ 0.36 & 0.2 & 0.5 & 0.85 & 0.62 & 0.74 \\ 0.11 & 0.083 & 0.15 & 0.5 & 0.2 & 0.27 \\ 0.26 & 0.15 & 0.38 & 0.8 & 0.5 & 0.64 \\ 0.18 & 0.11 & 0.26 & 0.73 & 0.36 & 0.5 \end{pmatrix}.$$

Some of the obtained results are the following:

$$\begin{aligned} ur_{12} &= \frac{ur_{12}^3 + ur_{12}^4 + ur_{12}^5 + ur_{12}^6}{4} = \frac{1.08}{4} = 0.27. \\ ur_{13} &= \frac{ur_{13}^2 + ur_{13}^4 + ur_{13}^5 + ur_{13}^6}{4} = \frac{2.56}{4} = 0.64. \\ &\dots \\ ur_{65} &= \frac{ur_{65}^1 + ur_{65}^2 + ur_{65}^3 + ur_{65}^4}{4} = \frac{1.44}{4} = 0.36. \end{aligned}$$

3.2 Modelling influence

Once experts' opinions have been transformed, we model the influence between experts. Experts are willing to discuss their decisions with each other. Because of these interactions, their opinions are influenced by other experts' opinions and they also influence the other ones. To apply the model summarised in Sect. 2.3 we accomplish the following steps:

1. Transform the preference relationships into utility vectors.
2. Apply the influence model for each experts' opinion.

3.2.1 Transform the preference relations into utility vectors

In order to carry out the next step, information must be transformed into utility vectors first. To do so, we propose to use a quantifier-guided choice degree of alternatives, a dominance degree, to compute the best acceptable alternative for the majority (Q). The quantifier-guided dominance degree $QGDD_i$ was reviewed in Sect. 2.1.

At the end of this step we have m utility vectors, each one with the evaluation of the expert i of the n alternatives:

$$U_i = \begin{pmatrix} u_{i1} \\ \dots \\ u_{in} \end{pmatrix}, i = 1, \dots, m.$$

3.2.2 Apply the influence model for each experts' opinion

In this step our aim is to model how the experts' opinions evolve by the interaction of each other. Experts may differ in some of the opinions and it is expected that they talk to each other to clarify, defend and modify their views.

As seen in Sect. 2.3 we need to know the experts' initial opinions and the matrix W . The influence model is applied on one alternative; however, in group decision making, experts provide their opinions about a set of alternatives $X = \{x_1, \dots, x_n\}$. Therefore, we have to decompose it in n problems, each one for each alternative.

First of all, we need to compute the matrix V as we explained in Sect. 2.3:

$$V = (I - AW)^{-1} (I - A),$$

and then, we apply the influence model for each alternative:

$$y_{x_i}^{(\infty)} = V y_{x_i}^{(1)}, i = 1, \dots, n.$$

We must realise the representation of the information used by the influence model is not the same as the representation used by the experts to provide their opinions. However, this information is easily transformed since $y_{x_1}^{(1)}$ is the initial opinion of all the experts regarding the alternative x_1 , $y_{x_2}^{(1)}$ is the initial opinion of all the experts regarding the alternative x_2 and so on. Therefore, after transforming this information, we obtain

$$y_{x_i}^{(1)} = \begin{pmatrix} u_{1i} \\ \dots \\ u_{mi} \end{pmatrix}, i = 1, \dots, n.$$

Once we have applied the influence model, we obtain the experts' final opinions:

$$y_{x_i}^{(\infty)} = \begin{pmatrix} y_{1i}^{(\infty)} \\ \dots \\ y_{mi}^{(\infty)} \end{pmatrix}, i = 1, \dots, n,$$

and we easily use this information to obtain the final vectors $U_1^{(\infty)}, \dots, U_m^{(\infty)}$:

$$U_i^{(\infty)} = \begin{pmatrix} y_{i1}^{(\infty)} \\ \dots \\ y_{in}^{(\infty)} \end{pmatrix}, i = 1, \dots, m,$$

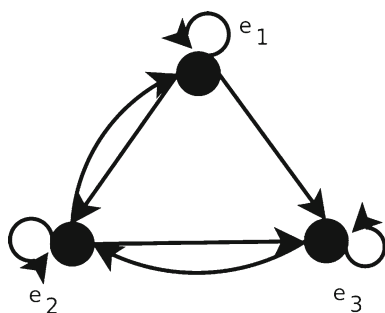


Fig. 2 Graph representation of matrix W

All these operations are described in depth in the following example.

Example 2 : Let $E = \{e_1, e_2, e_3\}$ be the group of experts and let $X = \{x_1, x_2, x_3\}$ be the group of alternatives. Experts express their assessment by means of preference relations which have been transformed into utility vectors:

$$U_1 = \begin{pmatrix} 0.3 \\ 0.4 \\ 0.1 \end{pmatrix}, \dots, U_2 = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.4 \end{pmatrix}, \dots, U_3 = \begin{pmatrix} 0.6 \\ 0.2 \\ 0.3 \end{pmatrix}.$$

In order to apply the influence model, the model is applied to each alternative. In this example, we work with alternative x_1 . First of all, $y_{x_1}^{(1)}$ is obtained from the information provided by the experts:

$$y_{x_1}^{(1)} = \begin{pmatrix} 0.3 \\ 0.1 \\ 0.6 \end{pmatrix}$$

where 1 is the initial period and 0.3, 0.1 and 0.6 are the e_1 's initial assessment, e_2 's initial assessment and e_3 's initial assessment about alternative x_1 , respectively.

Next we define the matrix that represents the interpersonal influence of the group of experts (see Fig. 2):

$$W = \begin{pmatrix} 0.2 & 0.2 & 0.6 \\ 0.3 & 0.3 & 0.4 \\ 0 & 0.2 & 0.8 \end{pmatrix}.$$

The main diagonal of the matrix W , that is, w_{ii} , represents the experts' susceptibility to interpersonal influence. For instance, if an expert has $w_{ii} = 1$ that means that this person is not subjected to any interpersonal influence. For example, $w_{12} = 0.2$ indicates the direct relative influence of the expert 2 on expert 1 is 0.2.

Now we apply the model. First of all, we calculate V and we obtain:

$$V = \begin{pmatrix} 0.25 & 0.09 & 0.66 \\ 0.07 & 0.41 & 0.52 \\ 0.00 & 0.02 & 0.98 \end{pmatrix}.$$

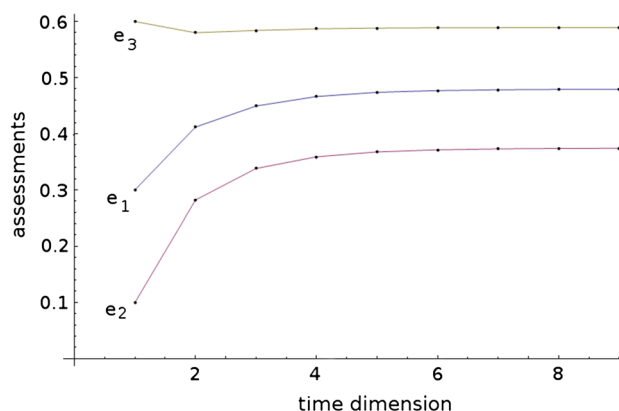


Fig. 3 Evolution of the alternative x_1

Matrix V represents the network influence on every experts. It is very important to realise that V and W are two different matrices and represent different aspect of interpersonal influence among experts. W represents the distribution of the direct relative interpersonal influence among experts. However, V takes into account all the flows of interpersonal influence, both direct and indirect. For instance, $v_{12} = 0.09$ what indicates that the 9 per cent of expert 1's final opinion is determined by expert 2.

Now, if we apply the influence model

$$y_{x_1}^{(\infty)} = V y_{x_1}^{(1)}$$

and we obtain the following solution for the alternative x_1 :

$$y_{x_1}^{(\infty)} = \begin{pmatrix} 0.48 \\ 0.37 \\ 0.59 \end{pmatrix}.$$

Looking at Fig. 3 we can study the evolution of alternative x_1 . So, expert e_1 starts with an assessment of 0.3 about the alternative x_1 , expert e_2 starts with 0.1, and e_3 with 0.6. The final experts' opinions are 0.48, 0.37 and 0.59, respectively. As can be seen, the most influential expert is e_3 whose opinion scarcely changes, and on the other hand, experts e_1 and e_2 are greatly affected by the opinion of e_3 that is the reason why their opinions change largely from 0.3 and 0.1 to 0.48 and 0.37, respectively.

3.3 Obtaining the solution

The aim of this final phase is to obtain the solution of the GDM problem, i.e., a global utility vector that provides us with an ordering vector of alternatives from the best one to the worst one.

To obtain the solution, the utility vectors obtained in the previous phase are aggregated by means of an I-IOWA operator. This operator requires a vector of importance degrees. In this model, it seems logical to use the experts' consistency

Table 1 Experts' initial opinions

$P_1 = \begin{pmatrix} 0.50 & 0.40 & 0.60 & 0.90 & 0.70 & 0.80 \\ 0.60 & 0.50 & 0.70 & 1.00 & 0.80 & 0.90 \\ 0.40 & 0.30 & 0.50 & 0.80 & 0.60 & 0.70 \\ 0.10 & 0.00 & 0.20 & 0.50 & 0.30 & 0.40 \\ 0.30 & 0.20 & 0.40 & 0.70 & 0.50 & 0.60 \\ 0.20 & 0.10 & 0.30 & 0.60 & 0.40 & 0.50 \end{pmatrix}$	$P_2 = \begin{pmatrix} 0.50 & 0.70 & 0.80 & 0.60 & 1.00 & 0.90 \\ 0.30 & 0.50 & 0.60 & 0.40 & 0.80 & 0.70 \\ 0.20 & 0.40 & 0.50 & 0.30 & 0.70 & 0.60 \\ 0.40 & 0.60 & 0.70 & 0.50 & 0.90 & 0.80 \\ 0.00 & 0.20 & 0.30 & 0.10 & 0.50 & 0.40 \\ 0.10 & 0.30 & 0.40 & 0.20 & 0.60 & 0.50 \end{pmatrix}$	$P_3 = \begin{pmatrix} 0.50 & 0.69 & 0.12 & 0.20 & 0.36 & 0.90 \\ 0.31 & 0.50 & 0.06 & 0.10 & 0.20 & 0.80 \\ 0.88 & 0.94 & 0.50 & 0.64 & 0.80 & 0.98 \\ 0.80 & 0.90 & 0.36 & 0.50 & 0.69 & 0.97 \\ 0.64 & 0.80 & 0.20 & 0.31 & 0.50 & 0.94 \\ 0.10 & 0.20 & 0.02 & 0.03 & 0.06 & 0.50 \end{pmatrix}$
$P_4 = \begin{pmatrix} 0.50 & 0.10 & 0.36 & 0.69 & 0.16 & 0.26 \\ 0.90 & 0.50 & 0.84 & 0.95 & 0.62 & 0.76 \\ 0.64 & 0.16 & 0.50 & 0.80 & 0.25 & 0.39 \\ 0.31 & 0.05 & 0.20 & 0.50 & 0.08 & 0.14 \\ 0.84 & 0.38 & 0.75 & 0.92 & 0.50 & 0.66 \\ 0.74 & 0.24 & 0.61 & 0.86 & 0.34 & 0.50 \end{pmatrix}$	$P_5 = \begin{pmatrix} 0.50 & 0.34 & 0.25 & 0.82 & 0.75 & 0.87 \\ 0.66 & 0.50 & 0.25 & 0.18 & 0.82 & 0.91 \\ 0.75 & 0.75 & 0.50 & 0.94 & 0.91 & 1.00 \\ 0.18 & 0.82 & 0.06 & 0.50 & 0.34 & 0.75 \\ 0.25 & 0.18 & 0.09 & 0.66 & 0.50 & 0.82 \\ 0.13 & 0.09 & 0.00 & 0.25 & 0.18 & 0.50 \end{pmatrix}$	$P_6 = \begin{pmatrix} 0.50 & 0.13 & 0.18 & 0.34 & 0.75 & 0.09 \\ 0.87 & 0.50 & 0.66 & 0.82 & 0.91 & 0.25 \\ 0.82 & 0.34 & 0.50 & 0.75 & 0.87 & 0.82 \\ 0.66 & 0.18 & 0.25 & 0.50 & 0.75 & 0.91 \\ 0.25 & 0.09 & 0.13 & 0.25 & 0.50 & 0.97 \\ 0.91 & 0.75 & 0.18 & 0.09 & 0.03 & 0.50 \end{pmatrix}$
$P_7 = \begin{pmatrix} 0.50 & 0.55 & 0.45 & 0.25 & 0.70 & 0.30 \\ 0.45 & 0.50 & 0.70 & 0.85 & 0.40 & 0.80 \\ 0.55 & 0.30 & 0.50 & 0.65 & 0.70 & 0.60 \\ 0.75 & 0.15 & 0.35 & 0.50 & 0.95 & 0.60 \\ 0.30 & 0.60 & 0.30 & 0.05 & 0.50 & 0.85 \\ 0.70 & 0.20 & 0.40 & 0.40 & 0.15 & 0.50 \end{pmatrix}$	$P_8 = \begin{pmatrix} 0.50 & 0.70 & 0.75 & 0.95 & 0.60 & 0.85 \\ 0.30 & 0.50 & 0.55 & 0.80 & 0.40 & 0.65 \\ 0.25 & 0.45 & 0.50 & 0.70 & 0.60 & 0.45 \\ 0.05 & 0.20 & 0.30 & 0.50 & 0.85 & 0.40 \\ 0.40 & 0.60 & 0.40 & 0.15 & 0.50 & 0.75 \\ 0.15 & 0.35 & 0.55 & 0.60 & 0.25 & 0.50 \end{pmatrix}$	

degree as the order-inducing values. We assume that experts who show more consistency in their assessment are supposed to be more reliable, and, therefore, they should have a greater weight in the aggregation.

In order to compute the experts' consistency degree, vector W_c , we use the U -consistency level for each experts' preference relation by using the formula reviewed in Sect. 2.2:

$$W_c = (UCL_{P_1}, \dots, UCL_{P_m},)$$

where UCL_{P_i} is the U -consistency level of the preference relation P_i that has been provided by the expert e_i .

Once we have calculated this vector, we obtain the final solution, U_s , as follows:

$$U_s = \Phi_W^I(U_1^{(\infty)}, \dots, U_m^{(\infty)}) = \begin{pmatrix} u_s^1 \\ \vdots \\ u_s^n \end{pmatrix},$$

where the best alternative has the greater value, and the worst one, the smaller one.

4 Example of application of the influence model in group decision making

Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ be a set of eight experts and six alternatives $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. To show the model, we carry the following phases.

Phase 1: providing the information

There are eight experts, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$, with their respective initial preference relations (see Table 1).

Now we check the consistency and improve the quality of the information provided (see Table 2)

Phase 2: modelling influence

Preference relations are transformed into utility vectors. To do so, the dominance degree is computed. We use the weighting vector $W_{OWA} = (0.41, 0.17, 0.13, 0.11, 0.096, 0.087)$ calculated with the linguistic quantifier "most of" defined by $Q(r) = r^{1/2}$.

Finally, the utility vectors obtained from the preference relations are the following:

$$\begin{aligned} U_1 &= \begin{pmatrix} 0.74 \\ 0.84 \\ 0.66 \\ 0.31 \\ 0.58 \\ 0.49 \end{pmatrix} & U_2 &= \begin{pmatrix} 0.84 \\ 0.66 \\ 0.58 \\ 0.74 \\ 0.31 \\ 0.49 \end{pmatrix} & U_3 &= \begin{pmatrix} 0.61 \\ 0.48 \\ 0.86 \\ 0.81 \\ 0.7 \\ 0.27 \end{pmatrix} \\ U_4 &= \begin{pmatrix} 0.46 \\ 0.84 \\ 0.58 \\ 0.31 \\ 0.77 \\ 0.66 \end{pmatrix} & U_5 &= \begin{pmatrix} 0.70 \\ 0.68 \\ 0.88 \\ 0.56 \\ 0.55 \\ 0.29 \end{pmatrix} & U_6 &= \begin{pmatrix} 0.45 \\ 0.73 \\ 0.69 \\ 0.57 \\ 0.45 \\ 0.54 \end{pmatrix} \\ U_7 &= \begin{pmatrix} 0.54 \\ 0.65 \\ 0.58 \\ 0.57 \\ 0.49 \\ 0.49 \end{pmatrix} & U_8 &= \begin{pmatrix} 0.76 \\ 0.58 \\ 0.56 \\ 0.42 \\ 0.60 \\ 0.47 \end{pmatrix}. \end{aligned}$$

Each expert is supposed to talk with each other. Among them, there exists an influence network that represents the degree by which one expert's opinions is influenced by that of other ones. This influence network is represented by the matrix W and graphically described in Fig. 4):

Table 2 Experts' opinions more consistent

$P'_1 = \begin{pmatrix} 0.50 & 0.27 & 0.64 & 0.89 & 0.74 & 0.82 \\ 0.73 & 0.50 & 0.80 & 0.92 & 0.85 & 0.89 \\ 0.36 & 0.20 & 0.50 & 0.85 & 0.62 & 0.74 \\ 0.11 & 0.08 & 0.15 & 0.50 & 0.20 & 0.27 \\ 0.26 & 0.15 & 0.38 & 0.80 & 0.50 & 0.64 \\ 0.18 & 0.11 & 0.26 & 0.73 & 0.36 & 0.50 \end{pmatrix}$	$P'_2 = \begin{pmatrix} 0.50 & 0.80 & 0.85 & 0.73 & 0.92 & 0.89 \\ 0.20 & 0.50 & 0.62 & 0.36 & 0.85 & 0.74 \\ 0.15 & 0.38 & 0.50 & 0.26 & 0.80 & 0.64 \\ 0.27 & 0.64 & 0.74 & 0.50 & 0.89 & 0.82 \\ 0.08 & 0.15 & 0.20 & 0.11 & 0.50 & 0.27 \\ 0.11 & 0.26 & 0.36 & 0.18 & 0.73 & 0.50 \end{pmatrix}$	$P'_3 = \begin{pmatrix} 0.50 & 0.69 & 0.13 & 0.20 & 0.36 & 0.89 \\ 0.31 & 0.50 & 0.06 & 0.10 & 0.20 & 0.78 \\ 0.87 & 0.94 & 0.50 & 0.63 & 0.79 & 0.98 \\ 0.80 & 0.90 & 0.37 & 0.50 & 0.69 & 0.97 \\ 0.64 & 0.80 & 0.21 & 0.31 & 0.50 & 0.94 \\ 0.11 & 0.22 & 0.02 & 0.03 & 0.06 & 0.50 \end{pmatrix}$
$P'_4 = \begin{pmatrix} 0.50 & 0.10 & 0.36 & 0.69 & 0.16 & 0.27 \\ 0.90 & 0.50 & 0.83 & 0.95 & 0.63 & 0.76 \\ 0.64 & 0.17 & 0.50 & 0.79 & 0.25 & 0.39 \\ 0.31 & 0.05 & 0.21 & 0.50 & 0.08 & 0.14 \\ 0.84 & 0.37 & 0.75 & 0.92 & 0.50 & 0.65 \\ 0.73 & 0.24 & 0.61 & 0.86 & 0.35 & 0.50 \end{pmatrix}$	$P'_5 = \begin{pmatrix} 0.50 & 0.56 & 0.15 & 0.62 & 0.69 & 0.93 \\ 0.44 & 0.50 & 0.18 & 0.85 & 0.60 & 0.82 \\ 0.85 & 0.82 & 0.50 & 0.82 & 0.93 & 0.97 \\ 0.38 & 0.15 & 0.18 & 0.50 & 0.54 & 0.82 \\ 0.31 & 0.40 & 0.07 & 0.46 & 0.50 & 0.81 \\ 0.07 & 0.18 & 0.03 & 0.18 & 0.19 & 0.50 \end{pmatrix}$	$P'_6 = \begin{pmatrix} 0.50 & 0.17 & 0.18 & 0.33 & 0.45 & 0.59 \\ 0.83 & 0.50 & 0.47 & 0.61 & 0.71 & 0.82 \\ 0.82 & 0.53 & 0.50 & 0.60 & 0.70 & 0.61 \\ 0.67 & 0.39 & 0.40 & 0.50 & 0.62 & 0.46 \\ 0.55 & 0.29 & 0.30 & 0.38 & 0.50 & 0.31 \\ 0.41 & 0.18 & 0.39 & 0.54 & 0.69 & 0.50 \end{pmatrix}$
$P'_7 = \begin{pmatrix} 0.50 & 0.30 & 0.40 & 0.45 & 0.51 & 0.66 \\ 0.70 & 0.50 & 0.53 & 0.45 & 0.73 & 0.68 \\ 0.60 & 0.47 & 0.50 & 0.40 & 0.54 & 0.66 \\ 0.55 & 0.55 & 0.60 & 0.50 & 0.44 & 0.60 \\ 0.49 & 0.27 & 0.46 & 0.56 & 0.50 & 0.37 \\ 0.34 & 0.32 & 0.34 & 0.40 & 0.63 & 0.50 \end{pmatrix}$	$P'_8 = \begin{pmatrix} 0.50 & 0.75 & 0.75 & 0.72 & 0.77 & 0.82 \\ 0.25 & 0.50 & 0.55 & 0.62 & 0.59 & 0.65 \\ 0.25 & 0.45 & 0.50 & 0.60 & 0.46 & 0.67 \\ 0.28 & 0.38 & 0.40 & 0.50 & 0.20 & 0.44 \\ 0.23 & 0.41 & 0.54 & 0.80 & 0.50 & 0.50 \\ 0.18 & 0.35 & 0.33 & 0.56 & 0.50 & 0.50 \end{pmatrix}$	

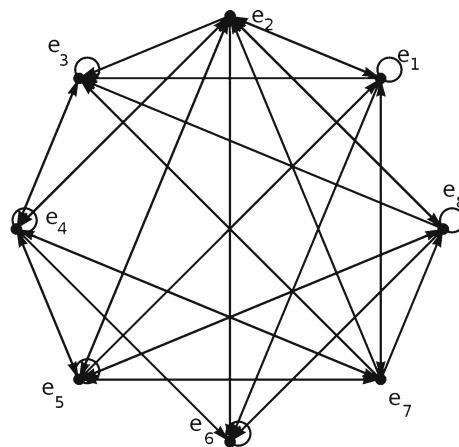
$$W = \begin{pmatrix} 0.2 & 0.1 & 0.25 & 0 & 0 & 0.15 & 0.3 & 0 \\ 0.1 & 0 & 0.18 & 0.05 & 0.34 & 0.24 & 0 & 0.09 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.23 & 0.34 & 0.14 & 0.09 & 0.1 & 0 \\ 0.2 & 0.08 & 0 & 0.021 & 0.05 & 0 & 0.049 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.2 & 0.09 & 0.05 & 0.32 & 0.04 & 0 & 0 & 0.3 \\ 0 & 0.07 & 0.27 & 0 & 0.5 & 0.04 & 0 & 0.12 \end{pmatrix}.$$

After applying the influence model, the utility vectors obtained are the following:

$$\begin{aligned} U'_1 &= \begin{pmatrix} 0.60 \\ 0.69 \\ 0.72 \\ 0.53 \\ 0.61 \\ 0.45 \end{pmatrix} & U'_2 &= \begin{pmatrix} 0.56 \\ 0.65 \\ 0.73 \\ 0.58 \\ 0.60 \\ 0.45 \end{pmatrix} & U'_3 &= \begin{pmatrix} 0.58 \\ 0.56 \\ 0.80 \\ 0.70 \\ 0.70 \\ 0.35 \end{pmatrix} \\ U'_4 &= \begin{pmatrix} 0.52 \\ 0.72 \\ 0.67 \\ 0.48 \\ 0.69 \\ 0.53 \end{pmatrix} & U'_5 &= \begin{pmatrix} 0.60 \\ 0.64 \\ 0.73 \\ 0.57 \\ 0.62 \\ 0.42 \end{pmatrix} & U'_6 &= \begin{pmatrix} 0.45 \\ 0.73 \\ 0.69 \\ 0.57 \\ 0.45 \\ 0.54 \end{pmatrix} \\ U'_7 &= \begin{pmatrix} 0.57 \\ 0.67 \\ 0.71 \\ 0.55 \\ 0.64 \\ 0.46 \end{pmatrix} & U'_8 &= \begin{pmatrix} 0.61 \\ 0.62 \\ 0.72 \\ 0.58 \\ 0.63 \\ 0.42 \end{pmatrix}. \end{aligned}$$

Phase 3: exploitation

In the final phase we aggregate the utility vectors obtained in the previous step by means of an I-IOWA operator. In order to

**Fig. 4** Matrix W of interpersonal influences

accomplish this phase, we need to compute vector W_c . This vector is computed from the experts' consistency degree and it is calculated as we explained in Sect. 2.2:

$$W_c = (0.94, 0.94, 0.99, 1, 0.86, 0.75, 0.76, 0.85).$$

This vector is used as a weighting vector to compute the solution.

The global opinion obtained after the aggregation is

$$U_s = \begin{pmatrix} 0.53 \\ 0.68 \\ 0.71 \\ 0.57 \\ 0.57 \\ 0.48 \end{pmatrix}$$

From this vector, the order of the alternatives, from the best one to worst one is

$$\{x_3, x_2, x_4, x_5, x_1, x_6\}.$$

5 Conclusion

In this proposal we have presented a novel approach to model the experts' interaction in a group decision making problem.

GDM classical models do not take into account the influence concept among experts to carry out the decision making. However, in real group decision making problems, the influence may play a key role to accomplish this task. For example, popular people in Social Networks are known to be able to establish trends or styles.

The new model takes into account the relationship among experts to create a social network. This social network allows us to represent the experts' social influence network. Moreover, because of the use of this methodology, we can infer and study the evolution of the experts' opinions as well as predicting the solution of the group decision making problem. In the future, we will study a better representation for the experts' social network to make easier its use. Besides, considering that Social Networks have a very relevant role in many areas of the modern society, this proposal could be very useful to address real problems in which it is required to predict the final result of a group decision making problem from the people's initial opinions without carrying out the GDM process. On the other hand, we also want to study as to how to model the influence in linguistic contexts (Massanet et al. 2014; Wu et al. 2015) and in heterogeneous decision making frameworks (Morente-Molinera et al. 2016; Pérez et al. 2014).

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Compliance with ethical standards

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Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Appendix 1: Fuzzy quantifiers and their use to model fuzzy majority

Fuzzy majority is a soft majority concept expressed by a fuzzy quantifier. This fuzzy quantifier is manipulated by means of a fuzzy logic-based calculus of linguistically quantified propositions. Hence, the use of fuzzy-majority-guided aggregation operators allows us to incorporate the concept of majority into the computation of the solution.

Quantifiers are used to represent the amount of items satisfying a given predicate. Classic logic defines two quantifiers, *there exists* and *for all*, however, this can be seen as an impor-

tant drawback because human discourse is much richer and more diverse. In order to provide a more flexible knowledge representation tool, Zadeh introduced the concept of fuzzy quantifiers (Zadeh 1983).

Zadeh suggested that the semantics of a fuzzy quantifier can be captured using fuzzy subsets for its representation. Moreover, he differentiated between two types of fuzzy quantifiers: absolute and relative ones. In this model, we have focused on relative quantifiers, such as *most*, *at least half*, etc., since they can symbolise any quantifier of natural language. These quantifiers can be represented by fuzzy subsets of the unit interval $[0, 1]$. For any $r \in [0, 1]$, $Q(r)$ indicates the degree in which the proportion r is compatible with the meaning of the quantifier it represents.

A relative quantifier $Q : [0, 1] \rightarrow [0, 1]$ satisfies

$$Q(0) = 0 \exists r \in [0, 1] \text{ such that } Q(r) = 1$$

Yager (1996) identified two categories of relative quantifiers: regular increasing monotone (RIM) quantifiers and regular decreasing monotone (RDM) quantifiers. The first category is characterised by quantifiers such as *all*, *most*, *many*, *at least* α ; and the second one by *at most one*, *few*, *at most* α , being the former one, the category used in this proposal.

A RIM quantifier satisfies

$$\forall a, b \text{ if } a > b \text{ then } Q(a) \geq Q(b).$$

Yager (1996) considers the parameterised family of RIM quantifiers

$$Q(r) = r^\alpha, \alpha \geq 0$$

When this family of RIM quantifiers is used with OWA and IOWA operators, it is important to realise that $\alpha < 1$ to associate high weighting values with high consistent ones. In particular, in this paper, we use the RIM function $Q(r) = r^{1/2}$.

Appendix 2: Example of group decision making

Let $E = \{e_1, e_2, e_3, e_4\}$ be the group of four experts and $X = \{x_1, x_2, x_3, x_4\}$ be the set of four alternatives. This group of experts, E , express their preferences about the set of alternatives, X , by means of fuzzy preference relations, $\{P_1, P_2, P_3, P_4\}$, $P_k = [p_{ij}^k]$, $p_{ij}^k \in [0, 1]$, which are additive reciprocal.

Consider the following preferences over the set of alternatives X :

$$P_1 = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 0.1 \\ 0.7 & 0.5 & 0.6 & 0.6 \\ 0.3 & 0.4 & 0.5 & 0.2 \\ 0.9 & 0.4 & 0.8 & 0.5 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0.5 & 0.4 & 0.6 & 0.2 \\ 0.6 & 0.5 & 0.7 & 0.4 \\ 0.4 & 0.3 & 0.5 & 0.1 \\ 0.8 & 0.6 & 0.9 & 0.5 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0.5 & 0.5 & 0.7 & 0 \\ 0.5 & 0.5 & 0.8 & 0.4 \\ 0.3 & 0.2 & 0.5 & 0.2 \\ 1 & 0.6 & 0.8 & 0.5 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0.5 & 0.4 & 0.7 & 0.8 \\ 0.6 & 0.5 & 0.4 & 0.3 \\ 0.3 & 0.6 & 0.5 & 0.1 \\ 0.2 & 0.7 & 0.9 & 0.5 \end{pmatrix},$$

and their respective expert's importance $I = \{0.75, 1, 0.5, 0.25\}$.

We use the fuzzy linguistic quantifier 'most of' defined by $Q(r) = r^{1/2}$ (Yager 1996), with its corresponding weighting vector $W = (0.5, 0.21, 0.16, 0.13)$. By using the I-IOWA operator, the following collective preference relation is computed:

$$P_c = \begin{pmatrix} 0.5 & 0.4 & 0.65 & 0.23 \\ 0.6 & 0.5 & 0.65 & 0.43 \\ 0.35 & 0.35 & 0.5 & 0.14 \\ 0.77 & 0.57 & 0.86 & 0.5 \end{pmatrix}.$$

Now, if we use the quantifier-guided dominance degree, $QGDD_i$ with the weighting vector $W_{QGDD} = (0.5, 0.21, 0.16, 0.13)$, the following utility vector is obtained:

$$U^c = \begin{pmatrix} 0.52 \\ 0.59 \\ 0.4 \\ 0.75 \end{pmatrix},$$

and, therefore, the final solution is

$$(x_4, x_2, x_1, x_3),$$

i.e. x_4 is the preferred alternative, x_2 is the second one, etc.

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