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DETERMINISTIC AND RANDOM BERNSTEIN POLYNOMIALS

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Abstract

The Bernstein polynomials $B_n f$ of a function f are the paradigmatic examples of positive linear operators. Since the 90's, it is known that the rate of convergence from $B_n f$ to f in the usual sup-norm is characterized as

$$K_1 \omega_2^{\varphi}\left(f, \frac{1}{\sqrt{n}}\right) \leq \|B_n f - f\| \leq K_2 \omega_2^{\varphi}\left(f, \frac{1}{\sqrt{n}}\right),$$

where $\omega_2^{\varphi}(f, \cdot)$ stands for the Ditzian-Totik second modulus of smoothness with weight function $\varphi(x) = \sqrt{x(1-x)}$. Several estimates for the upper constant K_2, depending on the degree of smoothness of f, have been obtained by many authors. However, as far as we know, no explicit estimate for the lower constant K_1 has been given, yet.

The first aim of this talk is to obtain upper and lower estimates for K_1. To do this, we use different tools, such as iterates of the operator B_n, together with their corresponding probabilistic representations, various expressions of the derivatives of $B_n f$, particularly that in terms of the Krawtchouk polynomials (the orthogonal polynomials with respect to the binomial law), and accurate estimates of inverse moments of suitable random variables.

In the second place, we consider random Bernstein polynomials, which are obtained when we replace the equidistant deterministic nodes $k/n, k = 0, 1, ..., n$ by random nodes. We show that such random polynomials uniformly converge in probability to f, giving at the same time rates of convergence. These results are illustrated in the case that the random nodes are the uniform order statistics.

Keywords: Bernstein polynomials, Ditzian-Totik modulus, Krawtchouk polynomials, random Bernstein polynomials, strong converse inequality, uniform convergence in probability.

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