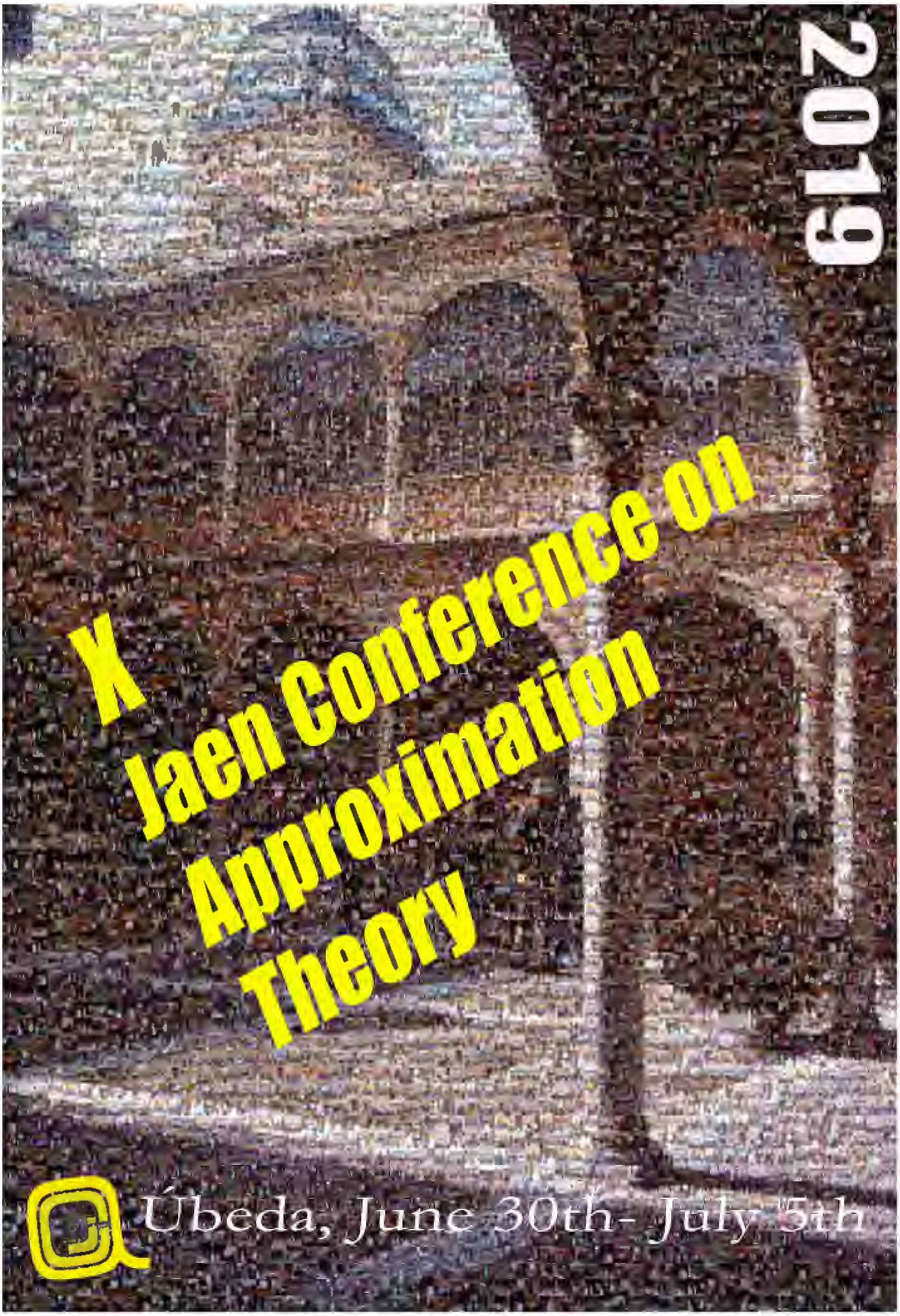

UNED

DIPUTACIÓN DE JAÉN

Universidad de Jaén

El cartel del X Jaen Conference on Approximation Theory es una imagen del patio central del Hospital de Santiago de Úbeda, sede los primeros encuentros de aproximación de la Universidad de Jaén. El dibujo es una buena aproximación de la imagen real y se ha obtenido a través de un mosaico construido con cientos de fotografías de todos los encuentros realizados desde el año 2000. Con ello se ha intentado expresar la idea de que dada una imagen, con un gran significado para gran parte de los asistentes al encuentro, es posible construir otra, lo más aproximada posible a la anterior, utilizando para ello todos los recuerdos generados durante los últimos veinte años. De alguna manera, en el cartel de este año están presentes todos aquellos profesores y acompañantes que han participado en los eventos de los años anteriores.

The poster of the X Jaen Conference on Approximation Theory represents the central courtyard of the Hospital de Santiago in Úbeda, where the University of Jaén held the first meetings on the Approximation Theory. This design, which constitutes a good approximation to the real image, has been obtained through a mosaic built with hundreds of photographs of all meetings organised since 2000. Thus, from an image full of meaning for the participants in these meetings, it has been possible to create a very similar one which gathers memories of the last twenty years. This way, all professors and accompanying people who participated in the past editions of the conference are present in this poster.

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  Justus Liebig University Giessen,
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- Andriy Prymak,
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- A. Sri Ranga,
  UNESP - Universidade Estadual Paulista,
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Introduction

This is the tenth edition of the Jaen Conference on Approximation Theory, an activity within the Jaen Approximation Project. This project organized ten editions of the so-called Ubeda Meeting for ten consecutive years, from 2000 to 2009, and nowadays issues the Jaen Journal on Approximation. This periodical, launched in 2009, was recently invited to be indexed in Emerging Sources Citation Index, Thomson Reuters.

The Conference is devoted to some significant aspects on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and the Applications of these fields in other areas. The main objective is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference program has been designed to keep joined the group during four days taking special care of both scientific and social activities.

The Conference features three invited speakers who will give plenary lectures: Elena E. Berdysheva, Andriy Prymak and A. Sri Ranga. Short talks and a poster session are scheduled as well.

Finally, but also important, the Conference provides to participants the possibility to visit World Heritage Sites and taste a wide culinary variety. We will do all the best for accompanying people to enjoy the Conference. We are grateful to all those who have made this project a reality; the University of Jaén (Departamento de Matemáticas), Diputación Provincial de Jaén, Ayuntamiento de Úbeda, Ayuntamiento de Quesada, Ayuntamiento de Alcalá la Real, UNIA (Sede de Baeza) and Centro Asociado de la UNED de la provincia de Jaén.

Here we emphasize our commitment to keep on working to improve our university and our province.

The Organizing Committee

Scientific Program

	June, 30th-Sunday
21:15-	**Dinner** (Parador de Úbeda)

	July, 1st-Monday	
	SESSION 1 (Chairperson D. Leviatan)	UNED (Úbeda)
9:30- 10:30	*Conference 1* Elena E. Berdysheva (p. 3)	
10:30-11:00	J. Szabados (p. 27)	
11:00-11:30	**Coffee Break**	
	SESSION 2 (Chairperson J. Szabados)	
11:30-12:00	G. Mastroianni (p. 20)	
12:00-12:30	V. Leonessa (p. 16)	
12:30-13:00	Mirella Cappelletti Montano (p. 11)	
13:00-13:45	**Opening Ceremony**	
14:30-	**Lunch** (Parador de Úbeda)	
18:00-	**Visit to Úbeda** (Meeting point: Parador de Úbeda)	
20:00-	**Visit to the olive oil factory *La Carrera***	
21:00-	**Dinner** (La Carrera)	

	ACCOMPANYING PEOPLE PROGRAM
10:30-	Meeting point: Parador de Úbeda

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	July, 2nd-Tuesday	
	SESSION 3 (Chairperson H. Mhaskar)	UNED (Úbeda)
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10:30-11:00	Feng Dai (p. 13)	
11:00-11:30	**Coffee Break**	
	SESSION 4 (Chairperson G. Mastroianni)	
11:30-12:00	H. Mhaskar (p. 23)	
12:00-12:30	Miguel Piñar (p. 24)	
12:30-13:00	Juan Jesús Merino (p. 21)	
14:00-	**Lunch** (San Lorenzo Church)	
18:00-	**Visit to Baeza**	
20:30-	**Cocktail Dinner** (UNIA, Baeza)	

	July, 3rd-Wednesday
8:45-	**Visit to Alcalá la Real and Castillo de la Mota**
	Lunch (Bodegas Marcelino Serrano)
21:00-	**Gala dinner**

	July, 4th-Thursday	
	SESSION 5 (Chairperson F. Dai)	UNED (Úbeda)
9:30-10:30	*Conference 4* A. Sri Ranga (p. 7)	
10:30-11:00	K. G. Ivanov (p. 14)	
11:00-11:40	**Coffee Break and Poster Session**	
	SESSION 6 (Chairperson E. E. Berdysheva)	
11:40-12:10	M. E. Marriaga (p. 18)	
12:10-12:40	F. Weisz (p. 29)	
13:00-	**Trip to Quesada**	
	Lunch (Quesada)	
	Visit to Zabaleta and Miguel Hernández museums	
	Dinner (Quesada)	

Poster Session

Session: 4th-Thursday, 11:00–11:40,

- Yamilet Quintana (p. 25)
- Joaquín Jódar Reyes (p. 31)

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Invited Lectures

Metric approximation of set-valued functions of bounded variation

Elena E. Berdysheva, Nira Dyn, Elza Farkhi and Alona Mokhov

Abstract

We study approximation of set-valued functions (SVFs) — functions mapping a real interval to compact sets in $\mathbb{R}^d$. In addition to the theoretical interest in this subject, it is relevant to various applications in fields where SVFs are used, such as economy, optimization, dynamical systems, geometric modeling.

The earlier works in this area are mainly concerned with approximation of set-valued functions with convex images, for which the tools of Minkowski linear combinations of sets and the Aumann integral [1] of set-valued functions are effective. Yet these techniques posses the property of convexification: the resulting approximation is always a function with convex images, even if the function to be approximated is not. For example, R.A. Vitale [4] studied an adaptation of the classical Bernstein polynomial operator based on Minkowski linear combinations; the limit SVF in this approach consists of convex hulls of the values of the original function. Clearly, such methods are useless for the approximation of SFVs with general, not necessarily convex images.

N. Dyn, E. Farkhi and A. Mokhov developed a new approach that is free of convexification — the so-called metric linear combinations and the metric integral, see [3]. They introduced and studied adaptations of classical approximation operators based on these tools for continuous SFVs.

The next step is to study SFVs that are not necessarily continuous. In our joint work we consider SVFs of bounded variation in the Hausdorff metric and adopt different types of classical approximation operators to this setting. Some of the results can be found in [2].

Keywords: set-valued functions, metric linear combinations, metric integral, Bernstein operator.

AMS Classification: 26E25, 28B20, 41A35, 41A36.

Bibliography

[1] **R. J. Aumann**, Integrals of set-valued functions, *J. Math. Anal. Appl.* **12** (1965) 1–12.

[2] **E. E. Berdysheva, N. Dyn, E. Farkhi, A. Mokhov**, Metric approximation of set-valued functions of bounded variation, *J. Comput. Appl. Math.* **349** (2019) 251–264.

[3] **N. Dyn, E. Farkhi, A. Mokhov**, The metric integral of set-valued functions, *Set-Valued Var. Anal.* **26** (2018) 867–885.

[4] **R. A. Vitale**, Approximations of convex set-valued functions, *J. Approx. Theory* **26** (1979) 301–316.

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• X Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 30th - July 5th, 2019

Polynomial approximation on C^2 domains*

Feng Dai and Andriy Prymak

Abstract

We introduce appropriate computable moduli of smoothness to characterize the rate of best approximation by multivariate polynomials on a connected and compact C^2-domain $\Omega \subset \mathbb{R}^d$. This new modulus of smoothness is defined via finite differences along the directions of coordinate axes, and along a number of tangential directions from the boundary. With this modulus, we prove both the direct Jackson inequality and the corresponding inverse for best polynomial approximation in $L_p(\Omega)$. The Jackson inequality is established for the full range of $0 < p \leq \infty$, while its proof relies on (i) Whitney type estimates with constants depending only on certain parameters; and (ii) highly localized polynomial partitions of unity on a C^2-domain. Both (i) and (ii) are of independent interest. In particular, our Whitney type estimate (i) is established for directional moduli of smoothness rather than the ordinary moduli of smoothness, and is applicable to functions on a very wide class of domains (not necessarily convex). It generalizes an earlier result of Dekel and Leviatan on Whitney type estimates on convex domains. The inverse inequality is established for $1 \leq p \leq \infty$, and its proof relies on a new Bernstein type inequality associated with the tangential derivatives on the boundary of Ω. Such an inequality also allows us to establish the Marcinkiewicz-Zygmund type inequalities, positive cubature formula, as well as the inverse theorem for Ivanov's averaged moduli of smoothness on general compact C^2-domains.

Keywords: C^2-domains, polynomial approximation, modulus of smoothness, Jackson inequality, inverse theorem, tangential Bernstein inequality, Marcinkiewicz-Zygmund inequality, positive cubature formula.

AMS Classification: 54C40, 14E20, 46E25, 20C20.

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• X Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 30th - July 5th, 2019

A class of orthogonal polynomials on the unit circle and related special functions

A. Sri Ranga

Abstract

The class of orthogonal polynomials $\Phi_n(b;.)$ that we consider are those with respect to the probability measures

$$d\mu^{(b)}(e^{i\theta}) = \frac{4^{\Re(b)}|\Gamma(b+1)|^2}{\Gamma(b+\bar{b}+1)} \frac{1}{2\pi} \left[e^{\pi-\theta}\right]^{\Im(b)} \left[\sin^2(\theta/2)\right]^{\Re(b)} d\theta,$$

on the unit circle, where $b = \lambda + i\eta$ and $\lambda > -1/2$. We will give information about a sequence $\{R_n(b.)\}_{n\geq 0}$ of para-orthogonal polynomials and the associated sequence of polynomials $\{P_n(b;.)\}_{n\geq 0}$ which satisfy a R_{II} type recurrence relation. All three sets of polynomials $\Phi_n(b;.)$, $R_n(b.)$ and $P_n(b;.)$ can be expressed as hypergeometric polynomials. Generating functions for all these sets of polynomials are also considered, which in the case of $P_n(b;.)$, lead to a subfamily of Whittaker functions. This subfamily of Whittaker functions also include Coulomb wave functions and Bessel functions. Finally, properties of associated Sobolev orthogonal polynomials, based on a concept which we have called *coherent pair of measures of the second kind on the unit circle*, are also looked at. Some sequences of polynomials that arise in this study are found to be related to continuous dual Hahn polynomials.

Bibliography

[1] **M.E.H. Ismail, A. Sri Ranga**, R_{II} type recurrence, generalized eigenvalue problem and orthogonal polynomials on the unit circle, *Linear Algebra Appl.* **562** (2019), 63-90.

[2] **F. Marcellán, A. Sri Ranga**, Sobolev orthogonal polynomials on the unit circle and coherent pairs of measures of the second kind, *Results Math.* **71** (2017), 1127-1149.

[3] **A. Martínez-Finkelshtein, L.L. Silva Ribeiro**, A. Sri Ranga and M. Tyaglov, Complementary Romanovski-Routh polynomials: From orthogonal polynomials on the unit circle to Coulomb wave functions, *Proc. Amer. Math. Soc.* **147** (2019), 2625-2640.

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Short Talks/Posters

Bernstein-Durrmeyer type operators on hypercubes

Mirella Cappelletti Montano

Abstract

This talk is based on joint works with F. Altomare (cf. [1]) and V. Leonessa (cf. [1, 5]) and will focus on Bernstein-Durrmeyer type operators acting on spaces of continuous and integrable functions defined on the d-dimensional hypercube Q_d of $\mathbb{R}^d$, $d \geq 1$.

We investigate (cf. [1]) a class of operators that generalize the Bernstein-Durrmeyer operators with Jacobi weights on $[0,1]$ (cf. [4, 6]). This class appears to be strictly connected with the study of certain degenerate second-order elliptic differential operators, often referred to as Fleming-Viot operators. By making mainly use of techniques arising from approximation theory, in fact, it is possible to show that the Fleming-Viot operators (pre)-generate positive semigroups both in the space of all continuous functions and in weighted L^p-spaces. In addition, those semigroups are approximated by iterates of the above mentioned Bernstein-Durrmeyer type operators. As a consequence, some regularity properties and the asymptotic behaviour of the semigroups can be inferred.

Inspired by [2, 3], we also present (cf. [5]) a further generalization of the operators in [1], by constructing Bernstein-Durrmeyer type operators defined by means of an arbitrary Borel measure μ on Q_d. Some approximation properties of this class of operators, both in the space of all continuous functions and in L^p-spaces with respect to μ, are discussed, together with an asymptotic formula.

Keywords: Bernstein-Durrmeyer operators, Jacobi weights, asymptotic formula, Fleming-Viot operator, Markov semigroup, approximation of semigroups.
AMS Classification: 47D06, 47F05, 41A36, 41A63.

Bibliography

[1] **F. Altomare, M. Cappelletti Montano and V. Leonessa**, On the positive semigroups generated by Fleming-Viot type differential operators, Comm. Pure Appl. Anal. **18**(1) (2019), 323–340.

[2] **E.E. Berdysheva**, Uniform convergence of Bernstein-Durrmeyer operators with respect to arbitrary measure, J. Math. Anal. Appl. **394** (2012), 324–336.

[3] **E.E. Berdysheva, K. Jetter**, Multivariate Bernstein Durrmeyer operators with arbitrary weight functions, J. Approx. Theory **162** (2010), 576–598.

[4] **H. Berens and Y. Xu**, On Bernstein-Durrmeyer polynomials with Jacobi weights, *in: C. K. Chui (Ed.), Approximation Theory and Functional Analysis, Academic Press, Boston*, 1991, 25–46.

[5] **M. Cappelletti Montano and V. Leonessa**, A generalization of Bernstein-Durrmeier operators on hypercubes by means of an arbitrary measure, Stud. Univers. Babeş-Bolyai Math. **64**(2) (2019), 239–252.

[6] **R. Paltanea**, Sur un opérateur polynomial défini sur l'ensemble des fonctions intégrables, Univ. Babeş-Bolyai, Cluj-Napoca, 83-2 (1983), 101–106.

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Integral norm discretization and related problems

Feng Dai

Abstract

In this talk, I will present a recent joint work with Prymak, Temlyakov and Tikhonov on the problem of replacing an integral norm with respect to a given probability measure by the corresponding integral norm with respect to a discrete measure for elements of finite dimensional spaces. I will also discuss discretization of the uniform norm of functions from a given finite dimensional subspace of continuous functions, with special focus on the case of the multivariate trigonometric polynomials with frequencies from a finite set with fixed cardinality.

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• X Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 30th - July 5th, 2019

Nonlinear n-term approximation on the sphere from shifts of the Newtonian kernel*

Kamen G. Ivanov and Pencho Petrushev

Abstract

The fundamental solution of the Laplace equation (Newtonian kernel) $\frac{1}{|x|^{d-2}}$ in dimension $d > 2$ or $\ln \frac{1}{|x|}$ if $d = 2$ is a basic building block in Potential theory. The main goal of this lecture is to study the rates of nonlinear n-term approximation of $\mathcal{H}^p, 0 < p < \infty$, and BMO functions on the unit sphere $\mathbb{S}^{d-1}$ from shifts of the Newtonian kernel with poles outside the unit ball $\overline{B^d} \subset \mathbb{R}^d$. We prove the following Jackson-type estimates.

Theorem 1. *Let* $0 < p < \infty$, $s > 0$, $1/\tau = s/(d-1) + 1/p$. *If* $f \in \mathcal{B}_\tau^{s\tau}(\mathbb{S}^{d-1})$, *then* $f \in \mathcal{H}^p(\mathbb{S}^{d-1})$ *and for* $n \geq 1$

$$E_n(f)_{\mathcal{H}^p} \leq cn^{-s/(d-1)} \|f\|_{\mathcal{B}_\tau^{s\tau}},$$

where the constant $c > 0$ *depends only on* p, s, d.

Theorem 2. *Let* $s > 0$, $1/\tau = s/(d-1)$. *If* $f \in \mathcal{B}_\tau^{s\tau}(\mathbb{S}^{d-1})$, *then* $f \in \mathrm{BMO}(\mathbb{S}^{d-1})$ *and for* $n \geq 1$

$$E_n(f)_{\mathrm{BMO}} \leq cn^{-s/(d-1)} \|f\|_{\mathcal{B}_\tau^{s\tau}},$$

where the constant $c > 0$ *depends only on* s, d.

Here $\mathcal{H}^p(\mathbb{S}^{d-1}), 0 < p < \infty$, denotes the Hardy space on $\mathbb{S}^{d-1}$ ($\mathcal{H}^p = L^p$ for $1 < p < \infty$), $\mathrm{BMO}(\mathbb{S}^{d-1})$ denotes the Bounded mean oscillation space on $\mathbb{S}^{d-1}$ and $\mathcal{B}_p^{sq}(\mathbb{S}^{d-1})$ stands for the Besov space with parameters s, q, p on the sphere. $E_n(f)_{\mathfrak{B}}$ denotes the best nonlinear n-term approximation of f from shifts of the Newtonian kernel in the norm of the Banach space $\mathfrak{B}$.

The rates of approximation in Theorems 1 and 2 are optimal in terms of the Besov spaces. The main vehicle in establishing these results is the construction of highly localized frames

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for Besov and Triebel-Lizorkin spaces on the sphere whose elements are linear combinations of a fixed number of shifts of the Newtonian kernel.

The above results naturally extends to approximation of harmonic functions in B^d. According to Theorem 2 BMO is a natural replacement for L^∞ in best approximation relations.

Keywords: nonlinear approximation, harmonic functions, Newtonian kernel, Hardy spaces, Besov spaces, frame decomposition.

AMS Classification: 41A17, 41A25, 42C15, 42C40, 42B35, 42B30.

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A generalization of Kantorovich operators for convex compact sets and applications

Vita Leonessa

Abstract

Let K be a convex compact subset of some locally convex Hausdorff space X. Starting from a Markov operator $T : C(K) \to C(K)$, a real number $a \geq 0$, and a sequence $(\mu_n)_{n\geq1}$ of probability Borel measures on K, we shall construct a sequence of positive linear operators $(C_n)_{n\geq1}$ acting on certain function spaces on K.

The class of such operators contains some well-known operators as the Kantorovich ones on the unit interval, on the multidimensional hypercube or on the simplex or $\mathbb{R}^d$, together with several of their wide-ranging generalizations scattered in the literature.

We are interested on approximation as well as shape preserving properties of the operators C_n. Moreover, we also show that the operators C_n can be used for approximating the solutions to certain initial-boundary value differential problems.

Even if X can be also infinite dimensional, for the sake of simplicity in this talk we limit ourselves to the case $K \subset \mathbb{R}^d$, $(d \geq 1)$.

The talk is based on some joint works with Francesco Altomare, Mirella Cappelletti Montano, and Ioan Raşa (see [1, 2, 3]).

Keywords: positive operators, shape preserving approximation, asymptotic formula, approximation of semigroups.

AMS Classification: 41A36, 47B65, 47D07.

Bibliography

[1] **F. Altomare, M. Cappelletti Montano, V.L., and I. Raşa**, A generalization of Kantorovich operators for convex compact subsets, Banach J. Math. Anal. **11**(3) (2017), 591–614.

[2] **F. Altomare, M. Cappelletti Montano, V.L., and I. Raşa**, Elliptic differential operators and positive semigroups associated with generalized Kantorovich operators, J. Math. Anal. Appl. **458**(1) (2018), 153–173.

[3] **M. Cappelletti Montano and V.L.**, Generalized Kantorovich operators on Bauer simplices and their limit semigroups, Numer. Funct. Anal. Optim. **38**(6) (2017), 723–737.

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• X Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, June 30th - July 5th, 2019

On coherent pairs of polynomial systems in two variables*

Francisco Marcellán, Misael E. Marriaga, Teresa E. Pérez
and Miguel A. Piñar

Abstract

Coherent pairs of measures were introduced in 1991 and constitute a very useful tool in the study of Sobolev orthogonal polynomials on the real line. In this work, coherence and partial coherence in two variables appear as the natural extension of the univariate case. Given two families of bivariate orthogonal polynomials expressed as *polynomial systems*, they are a *partial coherent pair* if there exists a polynomials of the second family can be given as a linear combination of the first partial derivatives of (at most) three consecutive polynomials of the first family. A *full coherent pair* is a pair of families of bivariate orthogonal polynomials related by means of partial coherent relations in each variable. Consequences of this kind of relations concerning both families of bivariate orthogonal polynomials are studied.

Keywords: bivariate orthogonal polynomials, classical and semiclassical orthogonal polynomials, coherent pairs.

Bibliography

[1] **C. F. Dunkl, Y. Xu**, Orthogonal polynomials of several variables, 2nd edition, Encyclopedia of Mathematics and its Applications, vol. **155**, Cambridge Univ. Press, Cambridge, 2014.

[2] **A. Iserles, P. E. Koch, S. P. Nørsett, J. M. Sanz-Serna**, On polynomials orthogonal with respect to certain Sobolev inner products, J. Approx. Theory **65** (1991), 151–175.

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[3] **H. L. Krall, I. M. Sheffer**, Orthogonal polynomials in two variables, Ann. Mat. Pura Appl. (4) **76** (1967), 325-376.

[4] **F. Marcellán, A. Branquinho, J. Petronilho**, Classical orthogonal polynomials: a functional approach, Acta Appl. Math. **34** (1994), 283-303.

[5] **F. Marcellán, M. E. Marriaga, T. E. Pérez, M. A. Piñar**, On bivariate classical orthogonal polynomials, Appl. Math. Comp. **325** (2018), 340–357.

[6] **F. Marcellán, M. E. Marriaga, T. E. Pérez, M. A. Piñar**, Matrix Pearson equations satisfied by Koornwinder weights in two variables, Acta Appl. Math. **153** (2018), 81–100.

[7] **M. E. Marriaga, T. E. Pérez, M. A. Piñar**, Three term relations for a class of bivariate orthogonal polynomials, Mediterr. J. Math. **14** (2017), no. 2, Art. 54, 26 pp.

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Hermite and Hermite–Fejér interpolation at Pollaczek zeros

G. Mastroianni and I. Notarangelo

Abstract

The Lagrange interpolation based on Pollaczek zeros has been considered in [1] (see also [2]). In this talk we study the uniform and L^p- convergence of the Hermite and Hermite–Fejér interpolation. Error estimates will be shown.

Keywords: Hermite interpolation, Hermite–Fejér interpolation, weighted polynomial approximation, orthogonal polynomials w.r.t. exponential weights, Pollaczek zeros.
AMS Classification: 41A05, 41A10.

Bibliography

[1] **G. Mastroianni and I. Notarangelo**, Lagrange interpolation with exponential weights on $(-1, 1)$, J. Approx. Theory **167** (2013), 65–93.

[2] **G. Mastroianni and I. Notarangelo**, Polynomial approximation with Pollaczek-type weights. A survey, Applied Numerical Mathematics (2019), https://doi.org/10.1016/j.apnum.2019.06.009

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• X Jaen Conference on Approximation Theory
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Fejér sums and Chebyshev polynomials*

Jorge Bustamante, Juan Jesús Merino and José María Quesada

Abstract

Let $L^p_w[-1,1]$ be the weighted Lebesgue space on $[-1,1]$ with $w(t) = (1-t^2)^{-1/2}$. We prove that the rate of convergence in $L^p_w[-1,1]$ of the Féjer sums is equivalent to a fractional modulus of smoothness of order $1/2$.

Keywords: polynomial approximation, Chebyshev polynomials, Fejér operator, direct results, strong converse results, fractional modulus of smoothness.
AMS Classification: 41A28, 41A40, 41A60.

Bibliography

[1] **G. Alexits**, Sur l'ordre de grandeur de l'approximation d'une fonction périodique par les sommes de Fejér, Acta Math. Acad. Sci. Hungar. 3 (1952), 29–42.

[2] **P. L. Butzer and R. L. Stens**, Chebyshev transform methods in the theory of best algebraic approximation, Abh. Math. Sem. Univ. 45 (1976), 165–190.

[3] **P. L. Butzer and R. L. Stens**, *Chebyshev transform methods in the solution of the fundamental theorem of best algebraic approximation in the fractional case*, In Fourier Analysis and Approximation Theory (Colloquia Math. Soc., János Bolyai, G. Alexits ans P. Turan eds., Budapest), 1976, 191–212.

[4] **Z. Ditzian and K. Ivanov**, Strong converse inequalities, J. Analyse Math. 61 (1993), 61–111.

[5] **T. Xie and X. Zhou**, Lower bounds of some singular integrals and their applications, Arch. Math. 88 (2007), 249–258.

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Beyond super-resolution

Hrushikesh Mhaskar

Abstract

The problem of super-resolution in general terms is to recuperate a finitely supported measure μ given finitely many of its coefficients $\hat{\mu}(k)$ with respect to some orthonormal system. The interesting case concerns situations, where the number of coefficients required is substantially smaller than a power of the reciprocal of the minimal separation among the points in the support of μ.

In this paper, we consider the more severe problem of recuperating μ approximately without any assumption on μ beyond having a finite total variation. In particular, μ may be supported on a continuum, so that the minimal separation among the points in the support of μ is 0. A variant of this problem is also of interest in machine learning as well as the inverse problem of de-convolution.

We define an appropriate notion of a distance between the target measure and its recuperated version, give an explicit expression for the recuperation operator, and estimate the distance between μ and its approximation. We show that these estimates are the best possible in many different ways. We also explain why for a finitely supported measure the approximation quality of its recuperation is bounded from below if the amount of information is smaller than what is demanded in the super-resolution problem.

Keywords: super-resolution, machine learning, de-convolution, data defined spaces, widths.

AMS Classification: 41A30, 28A33.

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Weighted approximation on the ball*

Miguel Piñar

Abstract

In this talk we explore the best approximation on the ball by means of orthogonal polynomials associated with weight functions that are invariant under reflection groups. A theory of orthogonal polynomials in this context can be developed in analogy to that for the orthogonal polynomials associated to standard spherical harmonics. Here, the standard first order partial differential operators are replaced by a family of commuting first order difference-differential operators: the so called Dunkl operators.

Keywords: best approximation, orthogonal polynomials, unit ball.
AMS Classification: 33C50, 42C10.

Bibliography

[1] **F. Dai, Y. Xu**, *Approximation theory and harmonic analysis on spheres and balls*, Springer Monographs in Mathematics, Springer, New York, 2013.

[2] **C. F. Dunkl, Y. Xu**, *Orthogonal polynomials of several variables*, Encyclopedia of Mathematics and its Applications 115, Cambridge University Press, 2014.

[3] **M. A. Piñar, Y. Xu**, Best Polynomial Approximation on the Unit Ball, IMA J. Numer. Anal. **38** (2018), 1209–1228.

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Riemann zeta function and the generalized Bernoulli polynomials of level m*

Yamilet Quintana and Héctor Torres-Guzmán

Abstract

In this work we show some relations between the Riemann zeta function and the generalized Bernoulli polynomials of level m. Our approach is based on the use of Fourier expansions for the periodic generalized Bernoulli functions of level m, as well as quadrature formulae of Euler-Maclaurin type. Some illustrative examples involving such relations are also given.

Keywords: Bernoulli polynomials, generalized Bernoulli polynomials of level m, Euler-Maclaurin quadrature formulae, quadrature formula, Riemann zeta function.
AMS Classification: 65D32, 41A55, 65B15, 33F05.

Bibliography

[1] **V. Lampret**, The Euler-Maclaurin and Taylor formulas: Twin, elementary derivations, Math. Mag. **74**(2) (2001), 109–122.

[2] **P. Natalini and A. Bernardini**, A generalization of the Bernoulli polynomials, J. Appl. Math. **2003**(3) (2003), 155–163.

[3] **Y. Quintana and H. Torres-Guzmán**, Some relations between the Riemann zeta function and the generalized Bernoulli polynomials of level m, arXiv:1901.03700 [math.CA].

[4] **Y. Quintana and A. Urieles**, Quadrature formulae of Euler-Maclaurin type based on generalized Euler polynomials of level m, Bull. Comput. Appl. Math. **6**(2) (2018), 43–64.

[5] **H. M. Srivastava and J. Choi**, *Zeta and q-Zeta Functions and Associated Series and Integrals*, Elsevier, London, 2012.

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Jackson-Favard type problems for the weight $\exp(-|x|)$ on the real line*

József Szabados

Abstract

It is known that the Favard inequality on polynomial approximation with respect to the weight $\exp(-|x|)$ fails to hold [7]. We show that this phenomenon holds for higher derivatives as well, while a modified version suggested by Lubinsky can be established.

Keywords: weight, modulus of smoothness, Jackson-Favard type estimate, polynomial approximation.
AMS Classification: 40A10, 41A17.

Bibliography

[1] **Z. Ditzian and V. Totik**, *Moduli of Smoothness*, Springer-Verlag, 1987.

[2] **G. Freud**, On Markov-Bernstein type inequalities and their applications, J. Approx. Theory **19** (1977) 22-37.

[3] **G. Freud, A. Giroux and Q. I. Rahman**, Sur l'approximation polynomiale avec poids $\exp(-|x|)$, Canad. J. Math. **30** (1978) 358-372.

[4] **Eli Levin and D. S. Lubinsky**, Canonical products and the weights $\exp(-|x|^\alpha)$, $\alpha > 1$, with applications, J. Approx. Theory **49** (1987) 149-169.

[5] **D. S. Lubinsky**, Jackson and Bernstein theorems for the weight $\exp(-|x|)$ on $\mathbb{R}$, Israel J. Math. **153** (2006) 193-220.

[6] **D. S. Lubinsky**, A survey of weighted polynomial approximation with exponential weights, Surveys in Approx. Theory **3** (2007) 1-105.

[7] **D. S. Lubinsky**, Which weights on $\mathbb{R}$ admit Jackson theorems?, Israel J. Math. **155** (2006) 253-280.

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Hardy spaces with variable exponents and applications in summability*

Ferenc Weisz

Abstract

Let $p(\cdot): \mathbb{R}^n \to (0, \infty)$ be a variable exponent function satisfying the globally log-Hölder condition. We introduce the variable Hardy spaces $H_{p(\cdot)}(\mathbb{R})$ and $H_{p(\cdot)}[0,1)$ and give their atomic decompositions. It is proved that the maximal operator of the Fejér means of the Fourier transforms and Walsh-Fourier series is bounded on these spaces. This implies some norm and almost everywhere convergence results for the Fejér-means, amongst others the generalization of the well known Lebesgue's theorem.

Keywords: variable Hardy spaces, variable Hardy-Lorentz spaces, atomic decomposition, Fourier transforms, Fejér-summability, Walsh-Fourier series.
AMS Classification: Primary 42B08; Secondary 42C10, 42A38, 42A24, 42B25.

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Multidimensional discrete PDE splines*

M.D. Buhmann, J. Jódar and M.L. Rodríguez

Abstract

Radial basis function (RBF) methods have emerged as an important and effective tool for the numerical solution of partial differential equations (PDE) in any number of dimensions and for the approximation of an unknown multivariate function by interpolation at scattered sites [2, 5], entering in a field traditionally tackled by finite element methods (FEM) [4].

Also, PDE surfaces, which are surfaces whose behaviour is governed by PDE [1], have been shown to possess many modelling advantages in a wide range of fields. A combination of conditions of interpolation and approximation can be used for the PDE method of surface design: on one hand, the surface has to approximate a given data set, and on the other hand, it has to be modelled by a partial differential equation. In addition, the surface has to satisfy some boundary conditions that are included along with the equation as a boundary value problem. Moreover, this 2-dimensional approximation problem may be generalized to the d-dimensional case, for any positive integer d.

We present the recent results of our work [3] in which, by using RBF techniques, we study the existence and the uniqueness of the solution of the generalized problem in a Lipschitz domain and arbitrary dimension. We discretize the solution of the problem in terms of RBF, we show the convergence of this solution to a function from which the data values are obtained and we establish some estimations of the error.

Keywords: approximation, interpolation, radial basis functions, PDE.
AMS Classification: 41A30, 41A63, 65N15.

Bibliography

[1] **M. I. G. Bloor and M. J. Wilson**, Representing PDE Surfaces in Terms of B-splines, Comput. Aided Des. **22**(6) (1990), 324–331.

[2] **M. D. Buhmann**, *Radial Basis Functions: Theory and Implementations*, Cambridge University Press, 2003.

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[3] **M.D. Buhmann, J. Jódar, and M.L. Rodríguez**, Radial discrete PDE splines on Lipschitz domains, Journal of Mathematical Analysis and Applications (2019) DOI: 10.1016/j.jmaa.2019.06.024.

[4] **P. G. Ciarlet**, *The Finite Element Method for Elliptic Problems*, North–Holland, 1978.

[5] **H. Wendland**, *Scattered data approximation*, Cambridge monographs on applied and computational mathematics, Cambridge University Press, 2005.

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